
Atomic Plasma and Quantum control

Elementary processes in plasma

Plasma categorisation

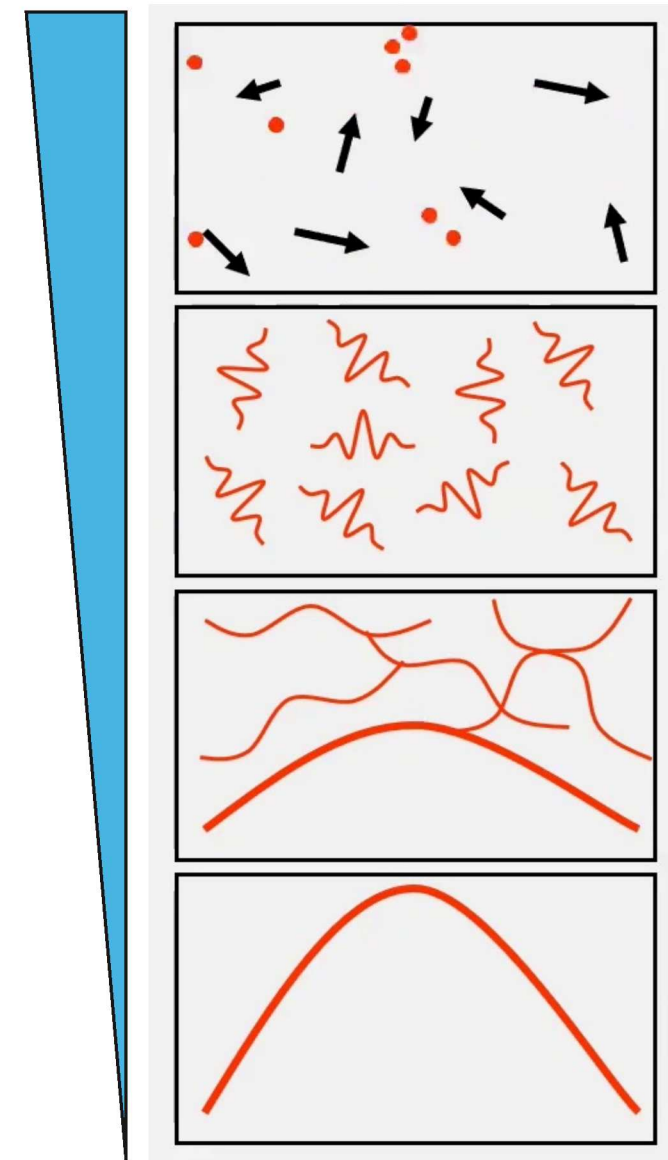
- Classical vs Quantum
- Coupled Strongly or Weakly

Classical vs Quantum

- Thermal de Broglie wavelength

- For an electron at 1 mK, $\lambda_{th} = 2 \mu\text{m}$
- For a Ca^+ ion $\lambda_{th} = 9 \text{ nm}$
- For Rb atom at $T = 10^{-7} \text{ K}$, 600 nm, BEC
- Quantum if $\lambda_{th} > \text{mean free path}$

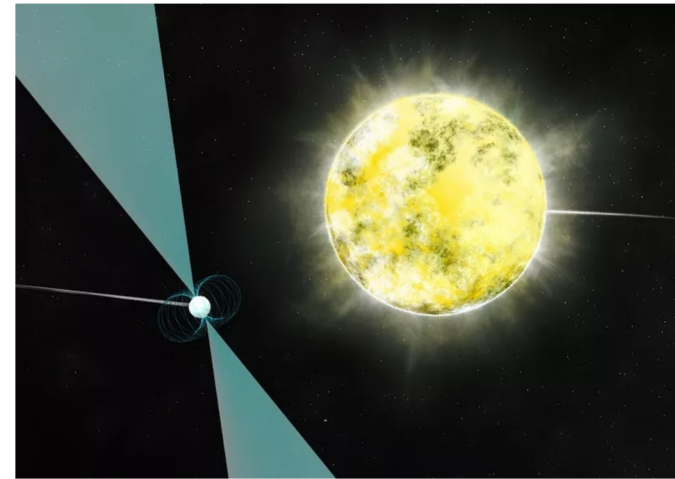
$$\lambda_{th} = \sqrt{\frac{2\pi\hbar^2}{m_e k_B T_e}}$$





Coupling parameter

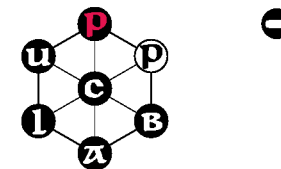
$$\Gamma = \frac{Z^2 e^2}{4\pi\epsilon_0 a_{\text{ws}}} \frac{1}{k_{\text{B}} T}, \quad a_{\text{ws}} = (3/4\pi n_i)^{1/3}$$



An artist's impression of the white dwarf star orbiting with the pulsar PSR J2222-0137. (Image credit: B. Saxton (NRAO/AUI/NSF))

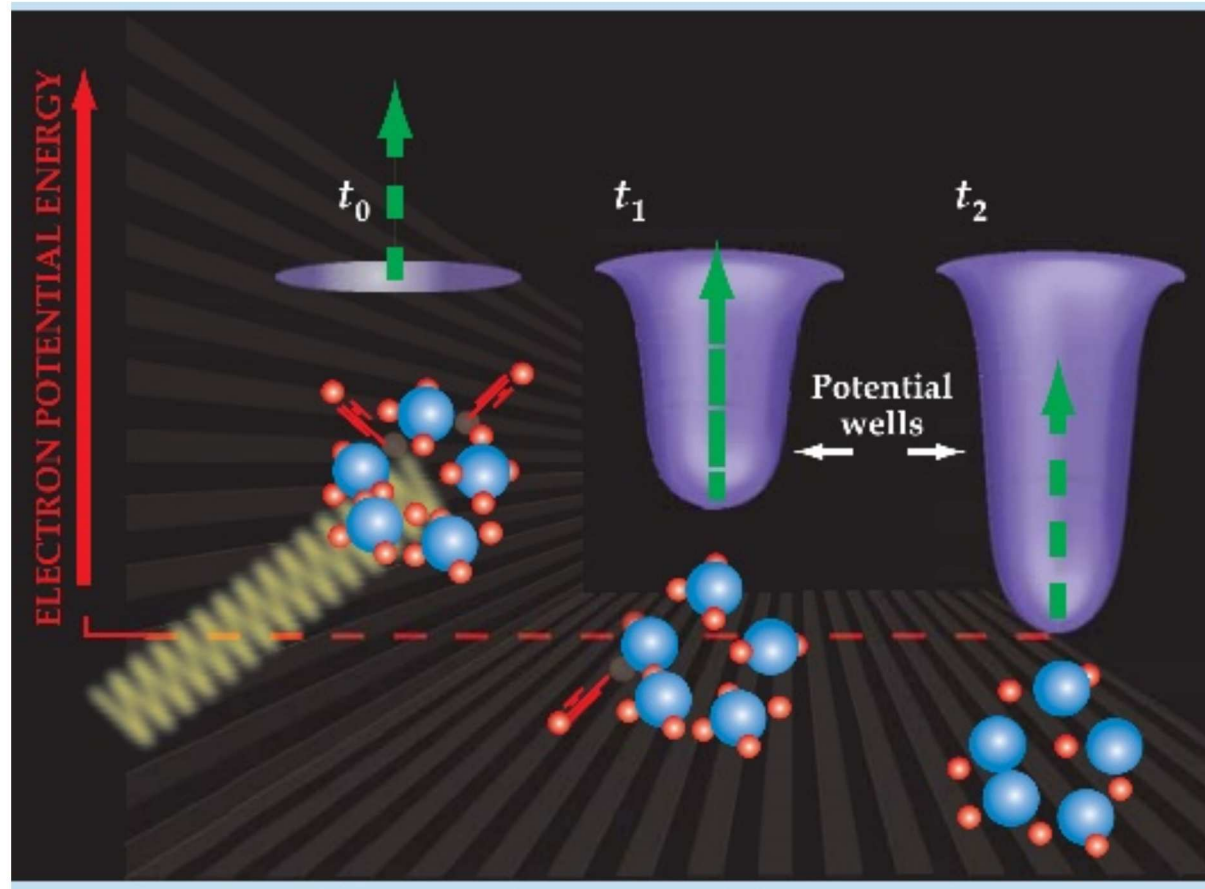


Kroker, T. et al. Ultrafast electron cooling in an expanding ultracold plasma. Nat Commun 12, 596 (2021).

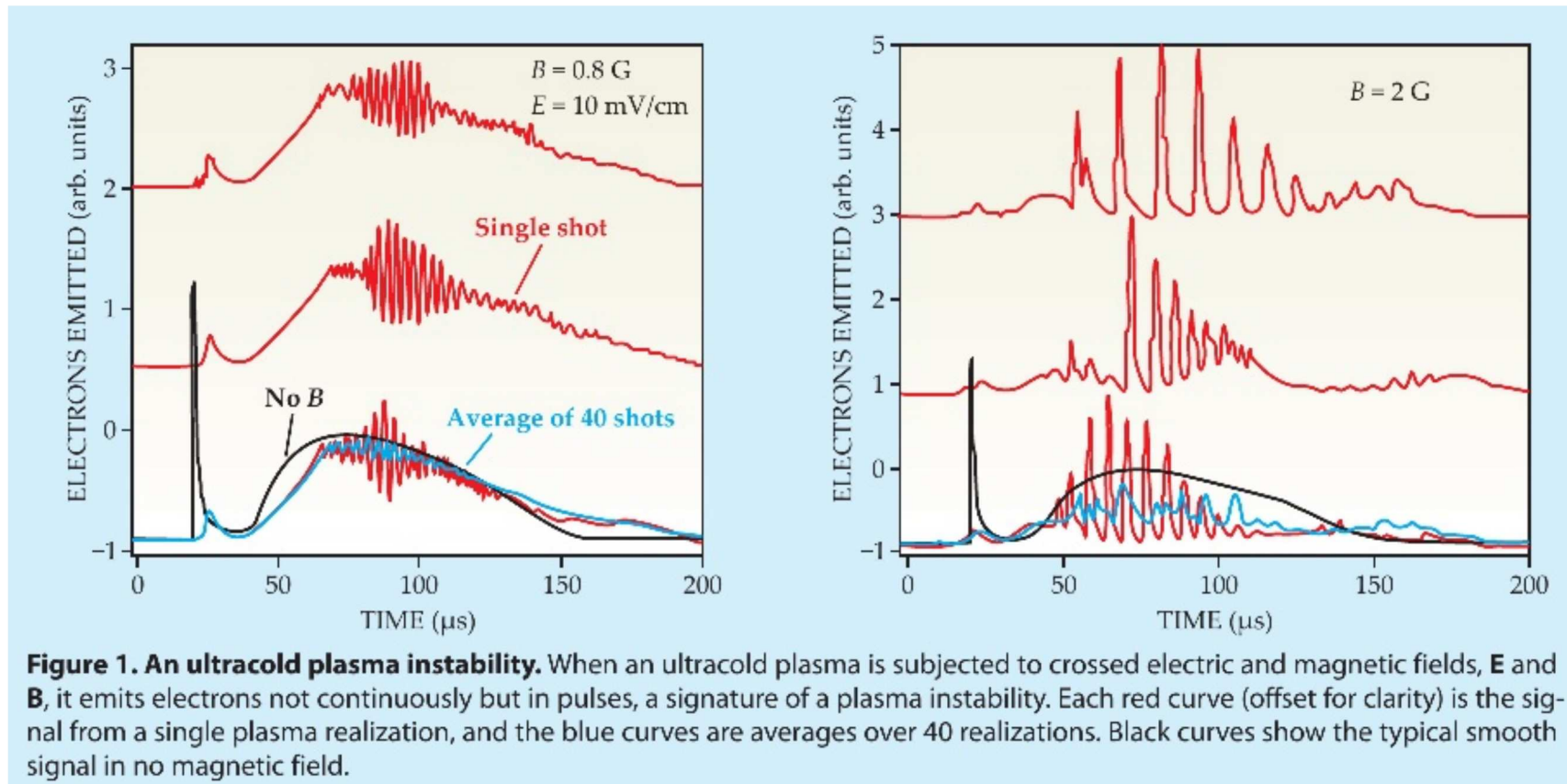


ultra-cold
plasma lab
prague

Generation



Behaviour I



Behaviour II

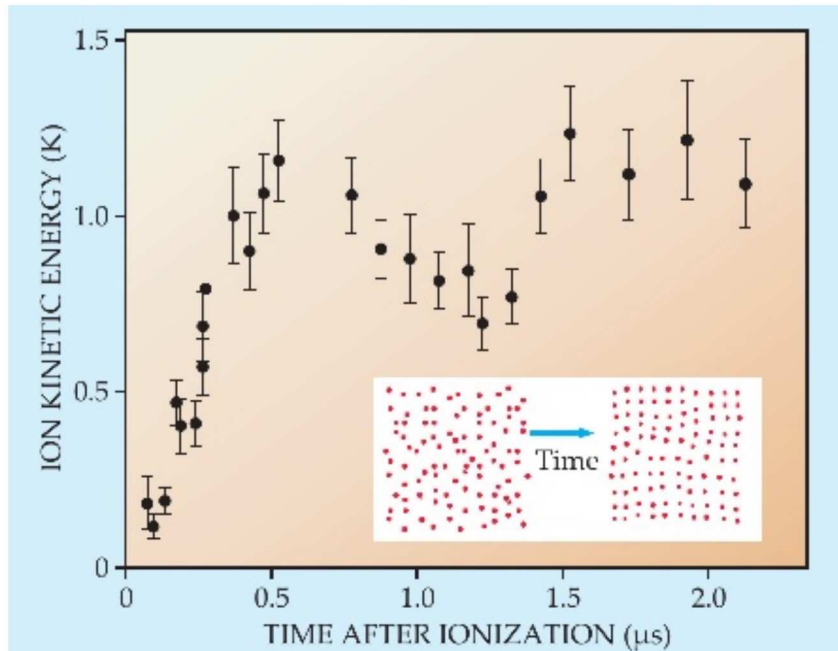


Figure 2. Equilibration dynamics in an ultracold plasma. In the first 0.5 μs after ionization, the plasma experiences rapid disorder-induced heating as the ions' excess potential energy is converted into kinetic energy. Thereafter, the kinetic energy exhibits damped oscillations about its equilibrium value as the cloud of ions settles into its potential-energy minimum and adopts a liquidlike short-range order, as shown in the inset.

Debye shielding

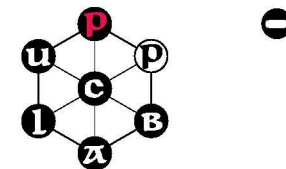
$$\lambda_D = \sqrt{\epsilon_0 k_b T / n e^2},$$

Strong coupling

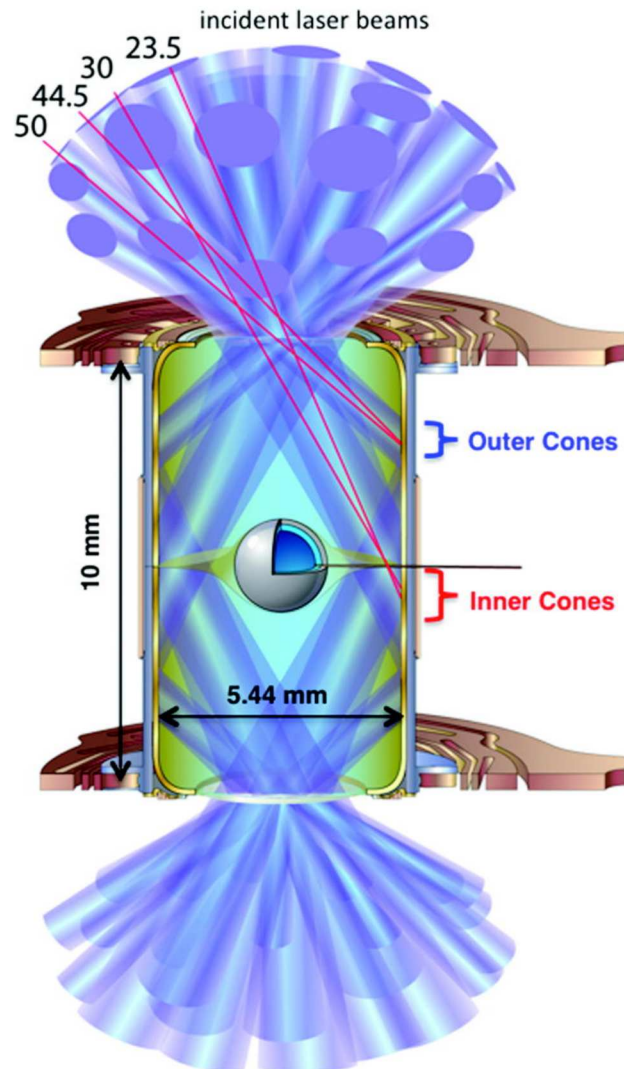
$$n\lambda_D^3 < 1$$

Disorder-induced heating

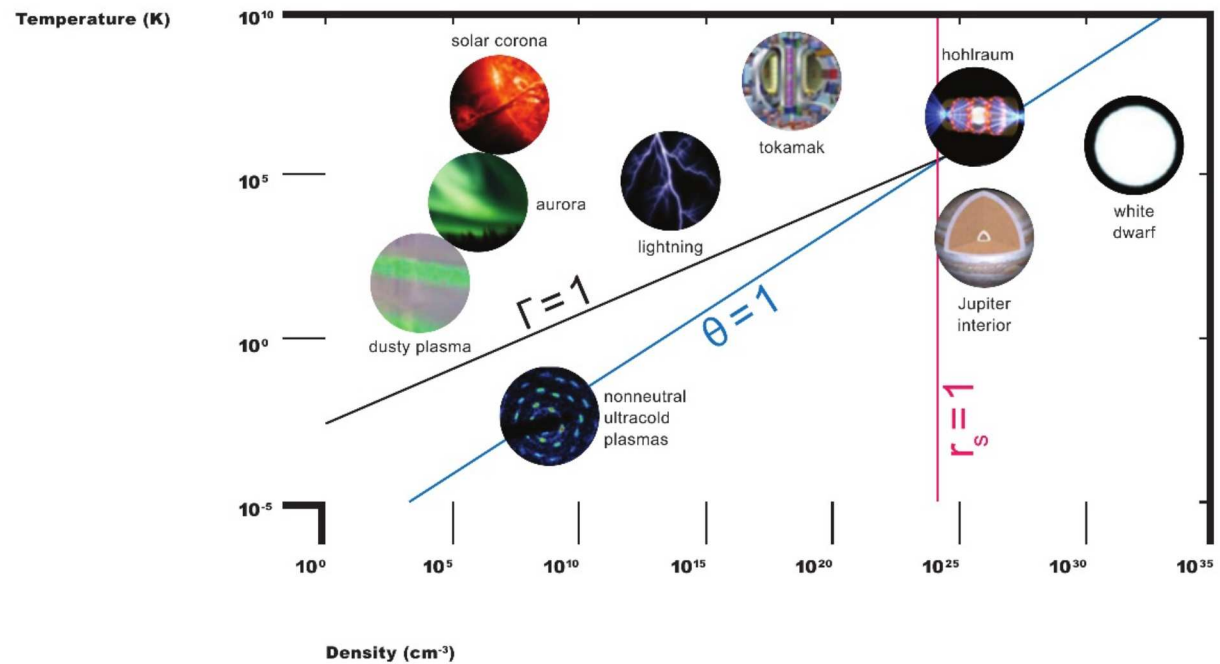
Contrast to cooling of coffee mug



Relevance



Crossovers

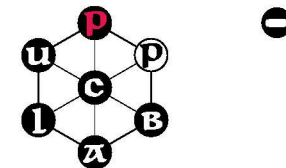


Report of the Panel on Frontiers of Plasma Science, Plasma: at the frontier of scientific discovery, US Department of Energy, 2016

Degeneracy parameter $\theta = (de\text{ Broglie } wvl.)^2 / a_{ws}^2$

Brueckner parameter

$$r_s = \frac{a_{ws}}{a_B}$$



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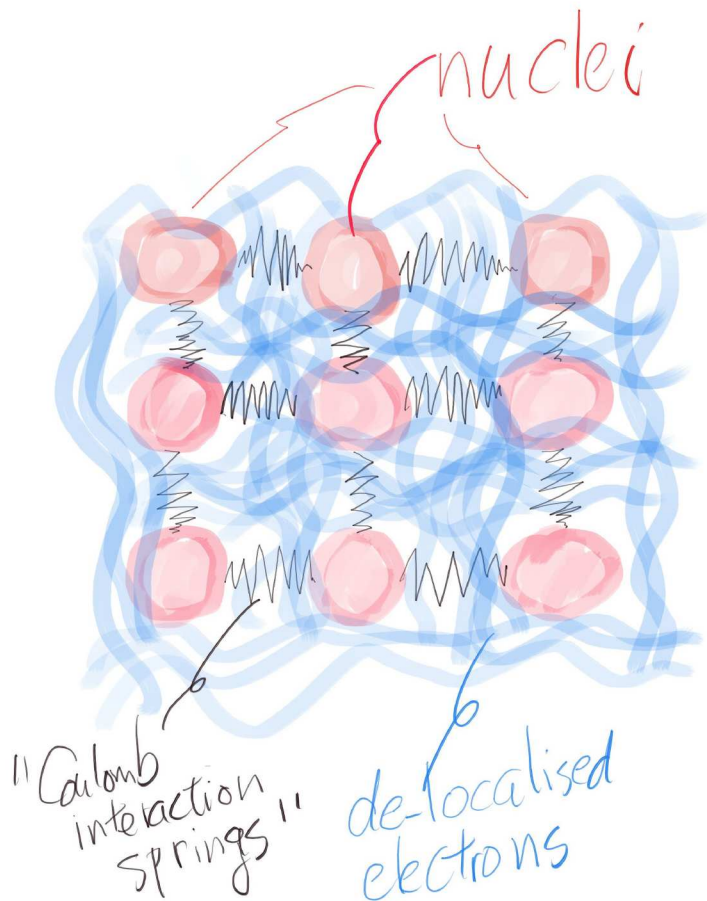
D. E. Hinkel, M. D. Rosen, E. A. Williams, A. B. Langdon, C. H. Still, D. A. Callahan, J. D. Moody, P. A. Michel, R. P. J. Town, R. A. London, S. H. Langer; Stimulated Raman scatter analyses of experiments conducted at the National Ignition Facility. Phys. Plasmas 1 May 2011; 18 (5): 056312. <https://doi.org/10.1063/1.3577836>



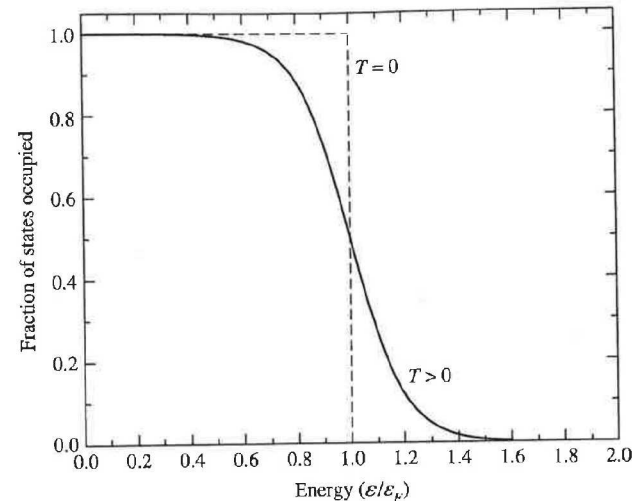
White Dwarf Star Core

Electron delocalisation

$$\lambda_{th} = \sqrt{\frac{2\pi\hbar^2}{m_e k_B T_e}}$$



e: neutralising background



thermal energy < Fermi energy

$$\frac{3}{2} kT < \frac{\hbar^2}{2m_e} \left[3\pi^2 \left(\frac{Z}{A} \right) \frac{\rho}{m_H} \right]^{2/3}$$

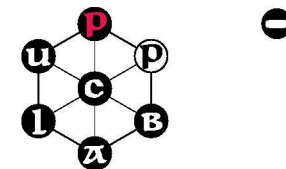
or

$$\frac{T}{\rho^{2/3}} < \frac{\hbar^2}{3m_e k} \left[\frac{3\pi^2}{m_H} \left(\frac{Z}{A} \right) \right]^{2/3} = 1261 \text{ K m}^2 \text{ kg}^{-2/3}$$



$$V_{ij}(r) = \frac{e^2 Z_i Z_j}{r_{ij}} e^{-r_{ij}/\lambda} \quad (1)$$

with inter-particle separation r_{ij} and Thomas-Fermi screening length $\lambda^{-1} = 2k_F \sqrt{\alpha/\pi}$ (electron Fermi momentum $k_F = (3\pi^2 \langle Z \rangle n)^{1/3}$, ion



ultra-cold
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PRAQUE

White Dwarf Stars

clocks of the universe, cooling rate influenced by composition

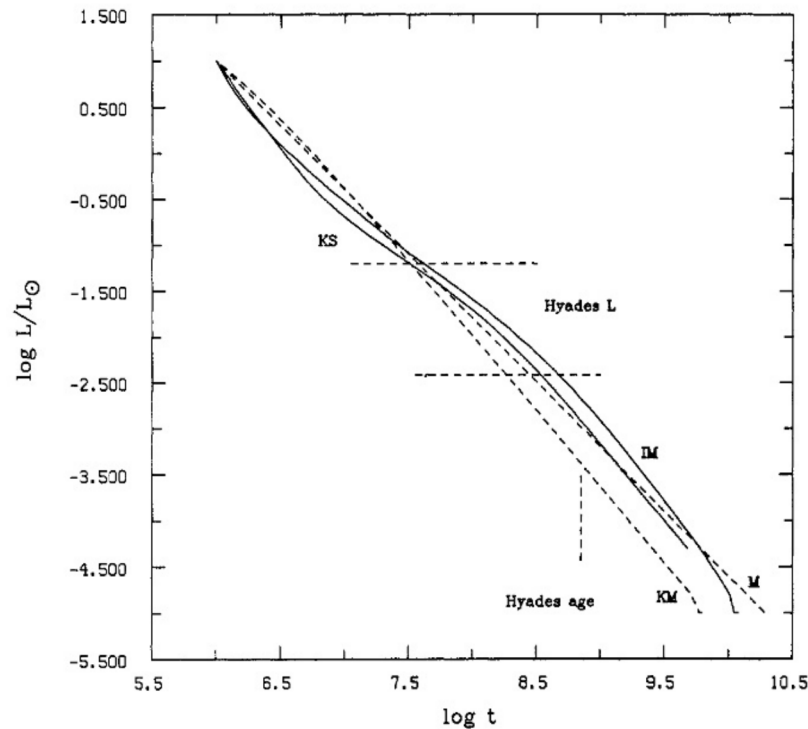


Figure 5. Cooling of white dwarfs with $0.6 M_{\odot}$. Continuous lines: IM, Iben and MacDonald (1985); KS Koester and Schönberner (1986). Broken lines: simple models based on Mestel (1952) (M) and equation (4.20) with numbers a and b from Koester (1976). See text for further explanations.

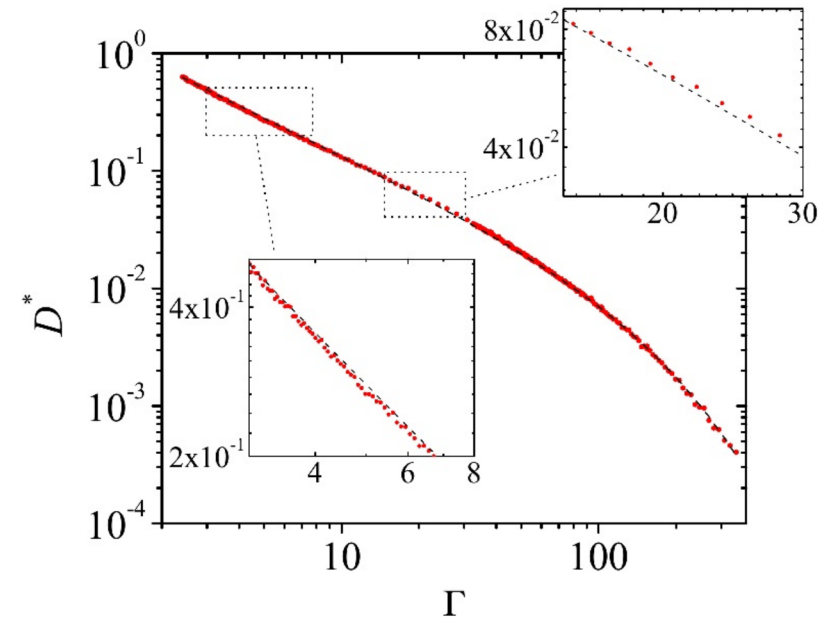
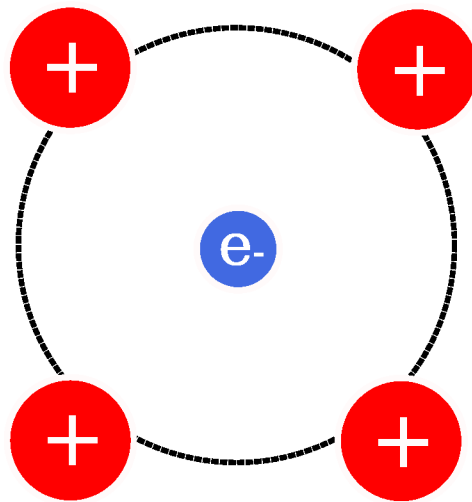


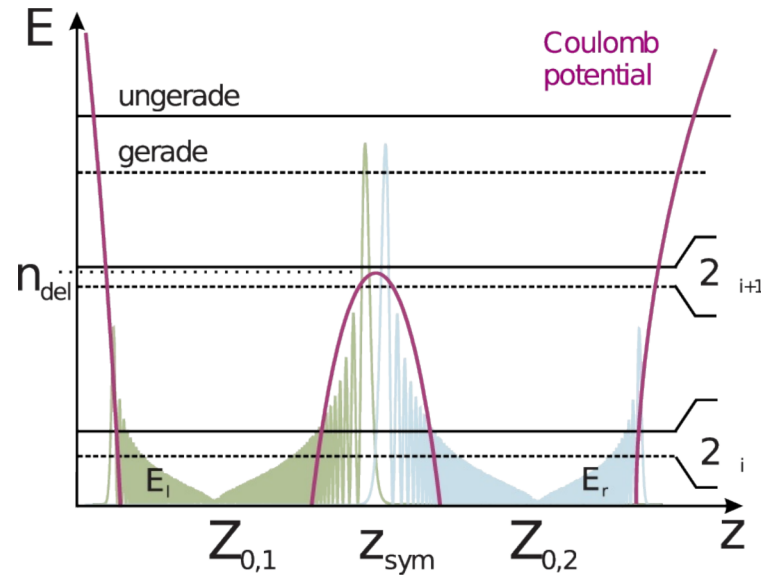
Figure 1. D^* for ^{16}O from MD (red) with fit from equation (4) (dashes) using best-fitting parameters $C = 0.7323$ and $B = 0.006937$. The fit is good to about 5 per cent for all Γ , but it is clear that it systematically overpredicts for $\Gamma \lesssim 10$ (bottom inset) and underpredicts for $10 \lesssim \Gamma \lesssim 100$ (top inset). Normalized residuals for all runs are shown in Fig. 2.

L ... luminosity
Rep. Prog. Phys. 53 (1990) 837-915.

Benefits of light mass, low temperatures

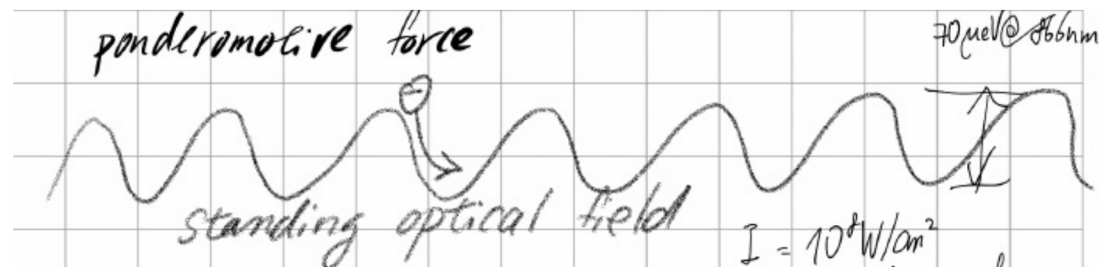


If $\lambda_{th} \approx a$, diffraction



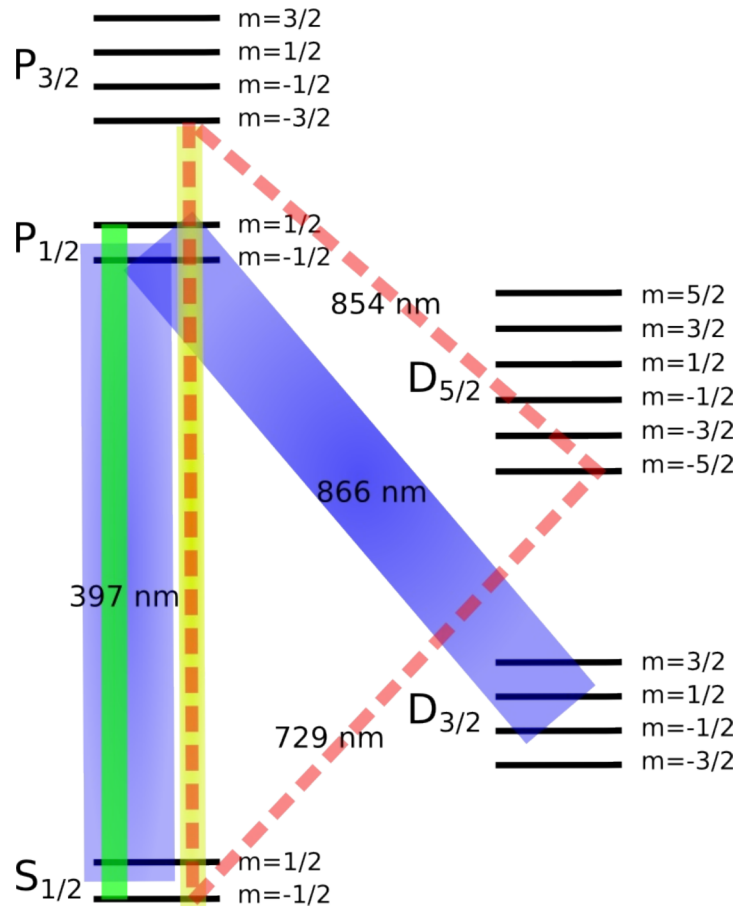
Schroedinger cat state

Lesanovsky, I., Müller, M. & Zoller, P. Trap-assisted creation of giant molecules and Rydberg-mediated coherent charge transfer in a Penning trap. Phys. Rev. A 79, 010701 (2009).

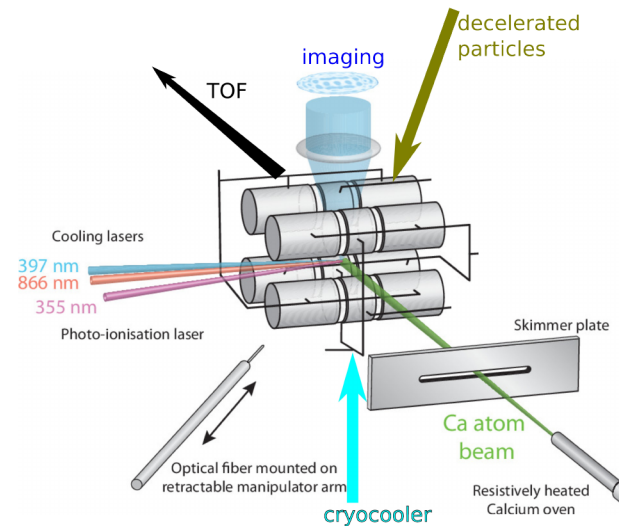


Interaction with optical field

Production of ion Coulomb crystal



Doppler cooling limit: 0.5 mK for calcium S-P transition



Interaction with radiation - macroscopic

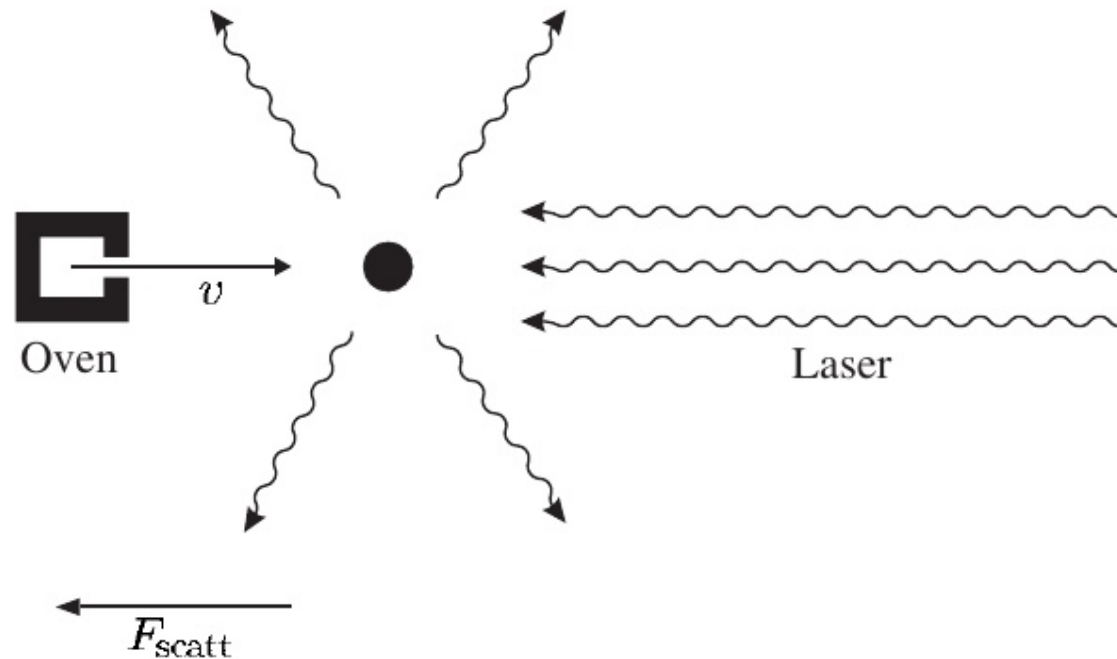
Radiation pressure



Crookes radiometer (!!!!)

Laser cooling by scattering force

Action on atoms



How large is the scattering force on atoms?

Force = (photon momentum) x (scattering rate)

$$F = \hbar k \times R_{scatt}$$

$$k = \frac{2\pi}{\lambda}$$

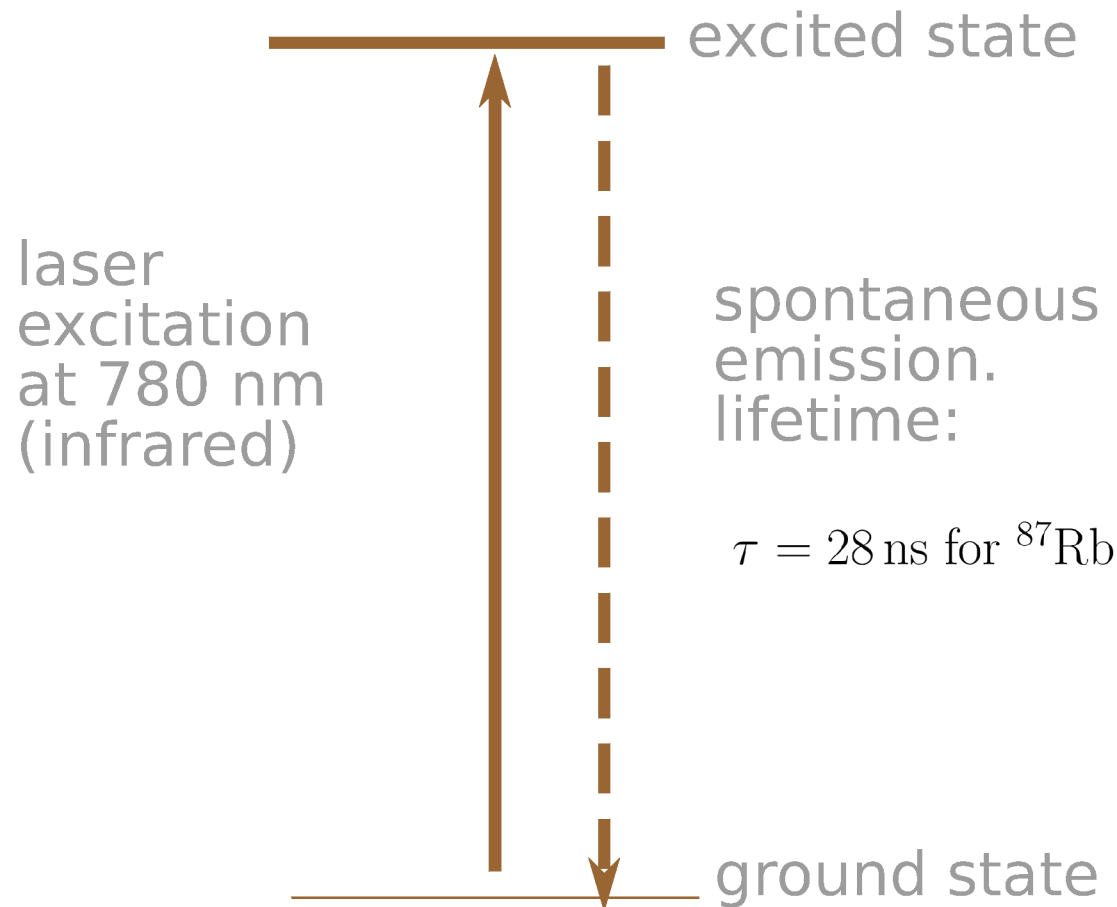
deceleration = Force / mass

$$a_{max} = \frac{F_{max}}{m} = \frac{\hbar k}{m} \times R_{scatt} = v_{rec} \times \frac{1}{2\tau}$$

Recoil velocity of Rb atom = 6 mm /s

Many photons

Two-level atom



How fast can we scatter?

$$a_{max} = 10^5 \text{ m/s}^2 = 10^4 g$$

Stopping distance

$$v^2 = 2as$$

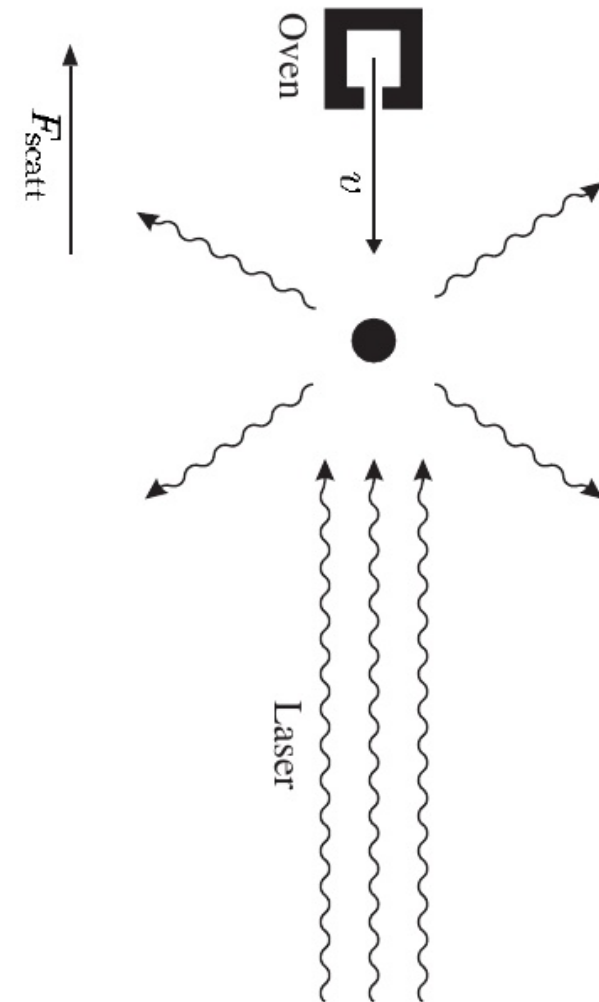
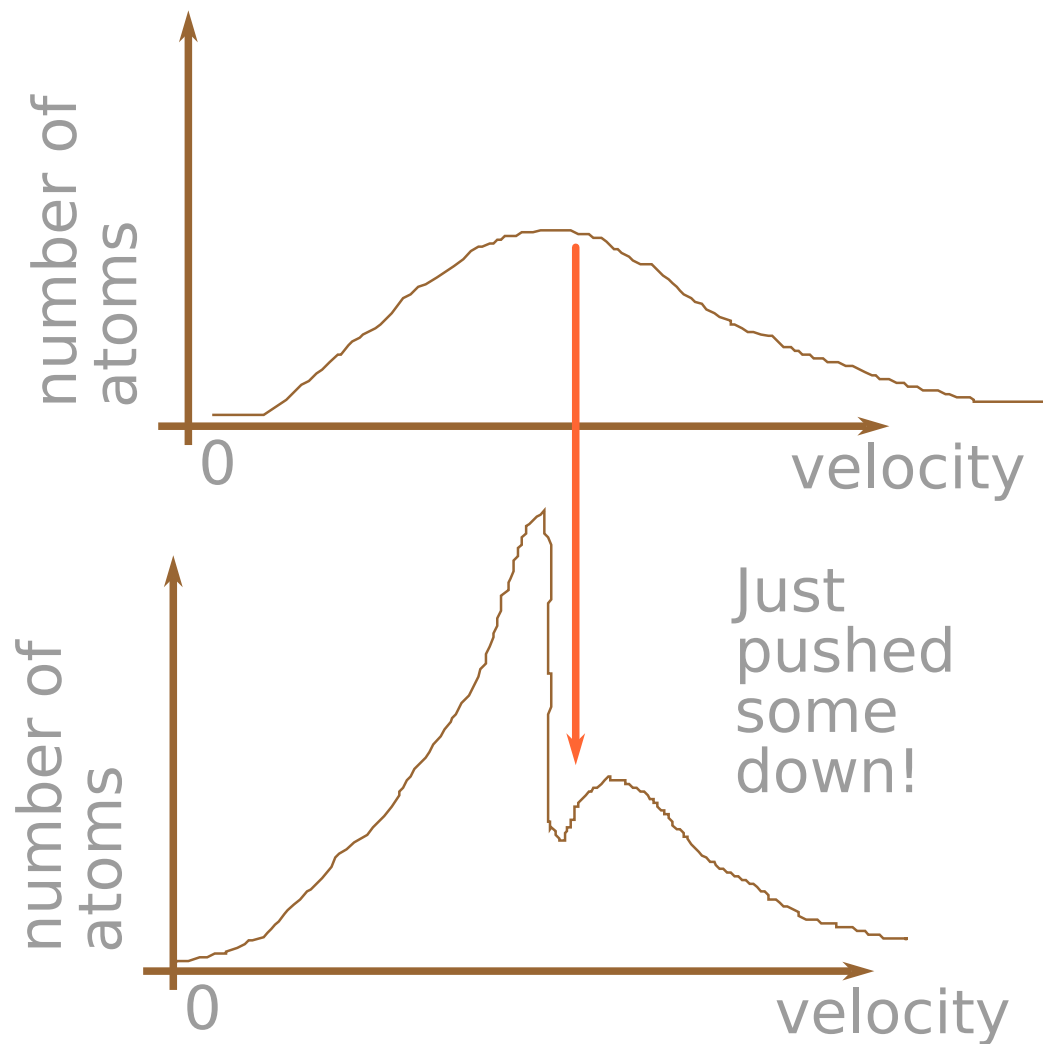
$$s = 1 \text{ m}$$

$$\text{for } v(0) = 300 \text{ m/s,}$$

$$a = a_{max}/2$$

Similar distance for other alkali metals: Na, K, Cs

Slowing atoms with laser light



Doppler shift much greater than natural width

$$\frac{\Delta f}{f_0} = \frac{v}{c}$$

Doppler width = velocity / wavelength

$$\Delta f_{Doppler} = \frac{v}{\lambda} = \frac{300}{7.8 \times 10^{-7}} = 380 \text{ MHz} \quad (\text{estimate})$$

Natural width (lifetime broadening)

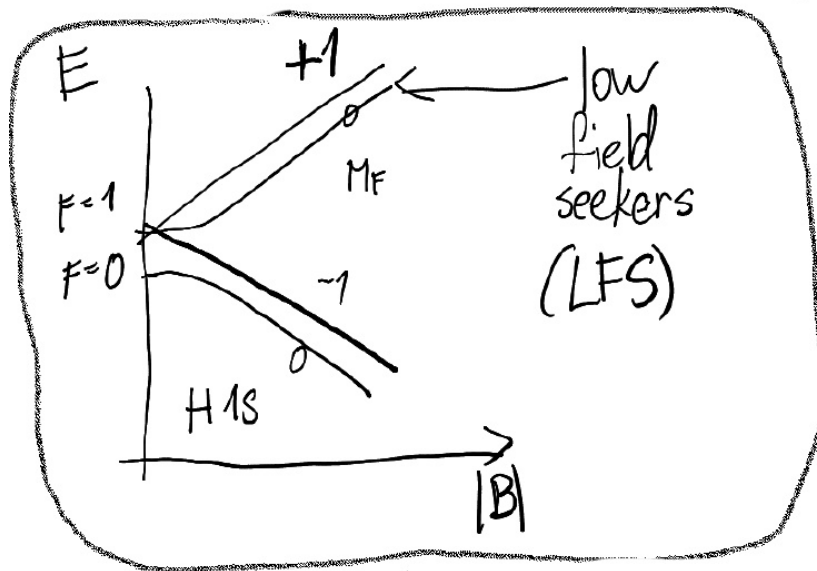
$$\tau = 28 \text{ ns for } ^{87}\text{Rb}$$

$$\Delta f_N = \frac{1}{2\pi\tau} = 6 \text{ MHz}$$

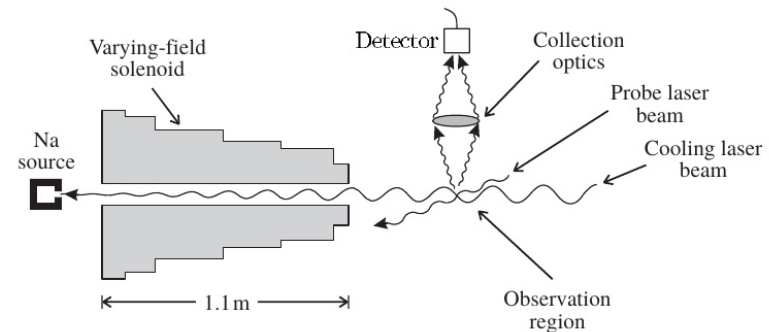
Compensate for the Doppler shift

Shift the wavelength in time: chirp cooling - not popular - cannot do it continuously (pulsed laser)

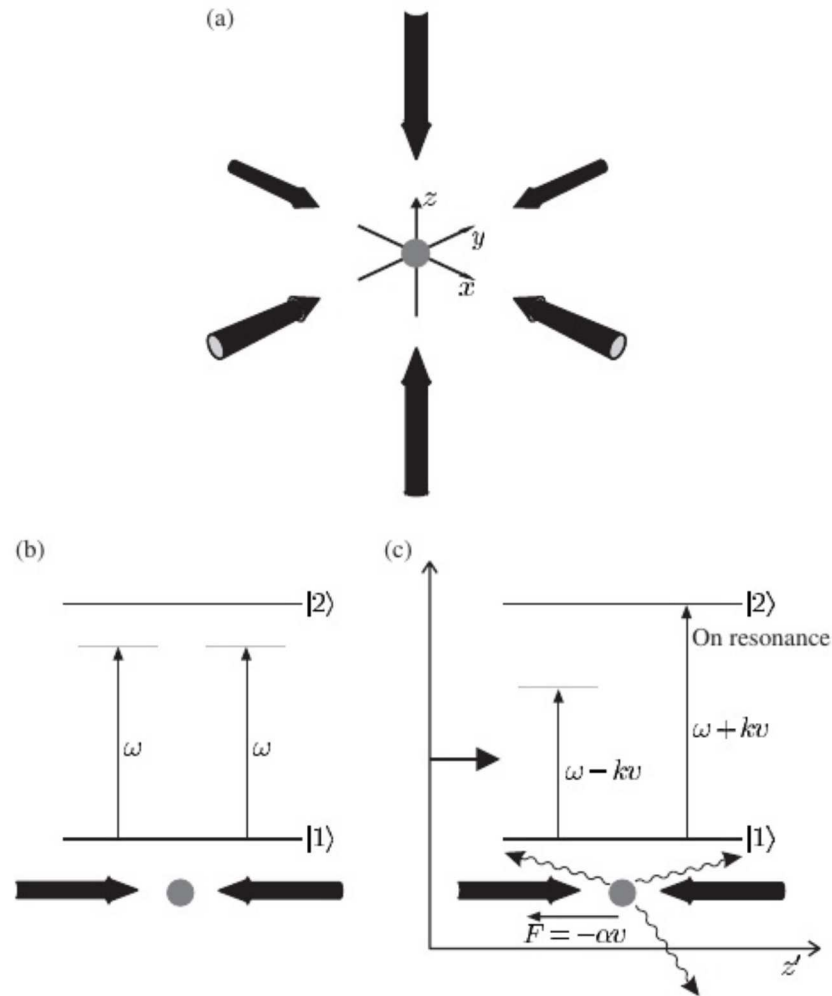
Use Zeeman shift (frequency shift in magnetic field)



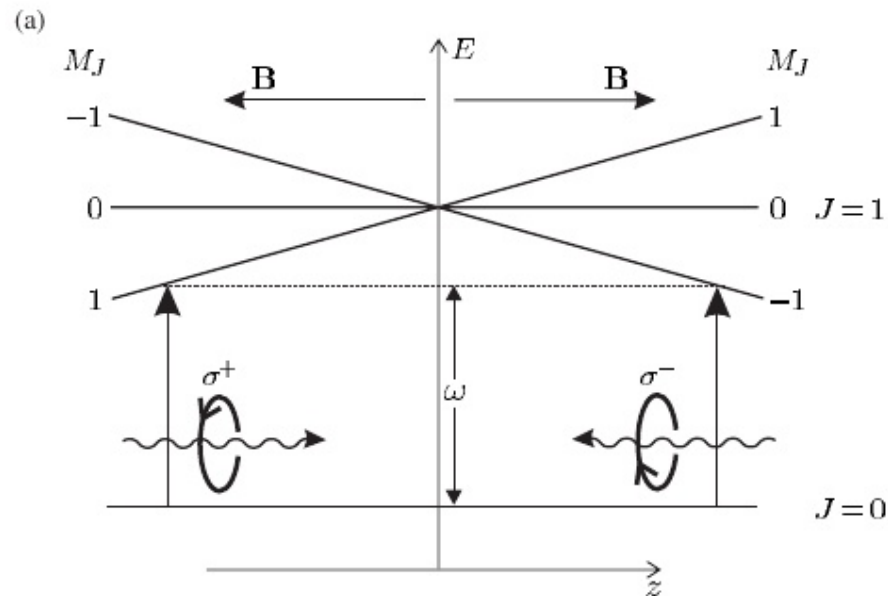
Zeeman slower: space-dependent magnetic field



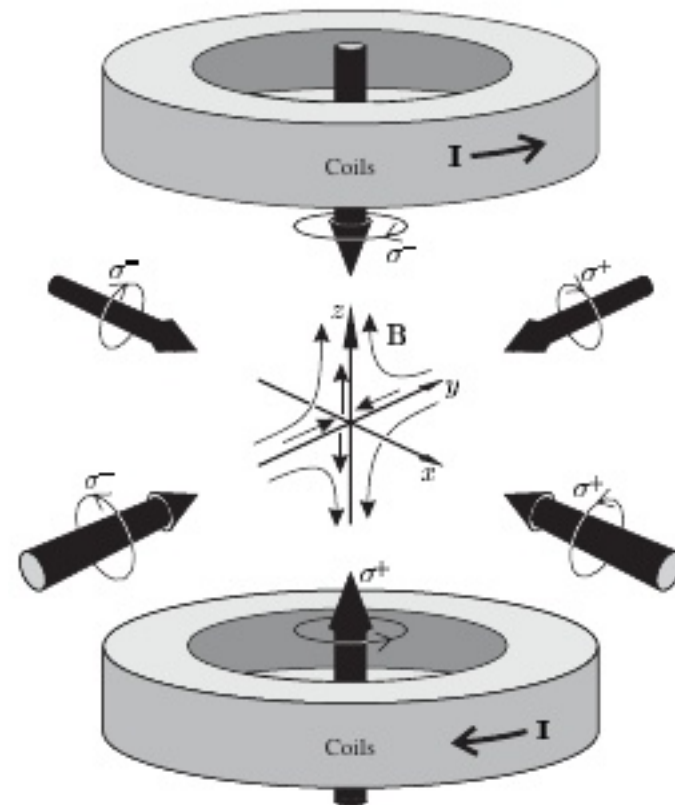
Optical molasses



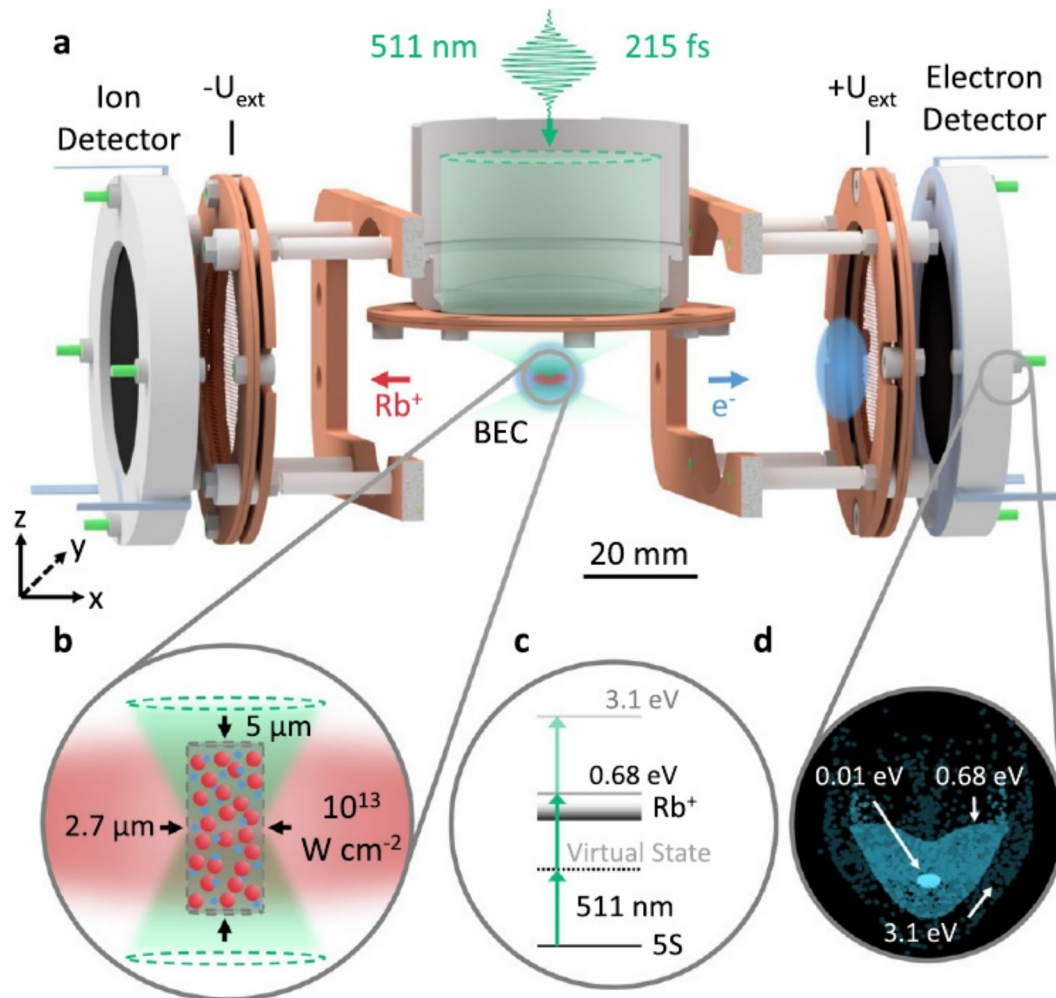
$$k_B T_D = \frac{\hbar \Gamma}{2} = \frac{\hbar}{\tau}$$



Gas (atoms) pushed to the centre due to force imbalance.

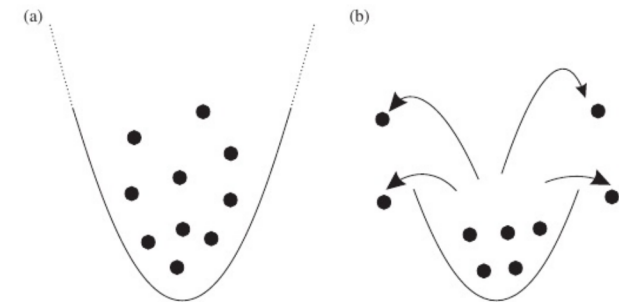


Experiment with Bose-Einstein condensate



$$\lambda_{th} = \sqrt{\frac{2\pi\hbar^2}{mk_B T}}$$

Evaporative cooling



Kroker, T. et al. Ultrafast electron cooling in an expanding ultracold plasma. Nat Commun 12, 596 (2021).

Atoms do have a magnetic moment

$$\mu_F = g_F \mu_B \sqrt{f(f+1)}$$

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

$$g_F \approx g_J \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)}$$

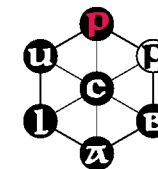
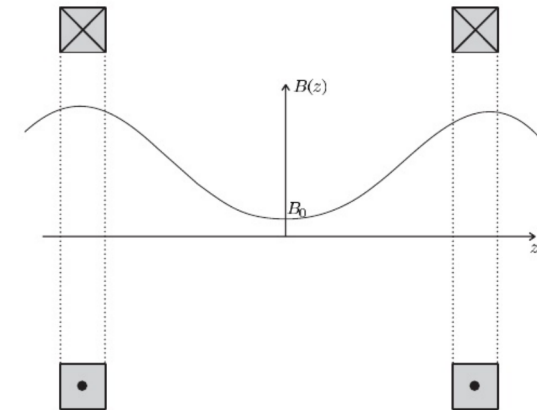
(Landé g-factor)

Energy in magnetic field: $V = -\mu_F \cdot B = m_F g_F \mu_B B$ (Zeeman energy)

Force acting in an inhomogeneous magnetic field:

$$F = -m_F g_F \mu_B \nabla B$$

Larger the J, L, F, stronger the interaction!

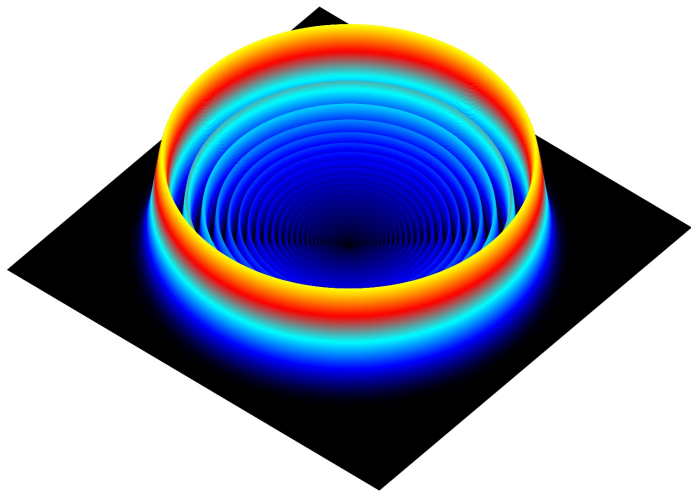


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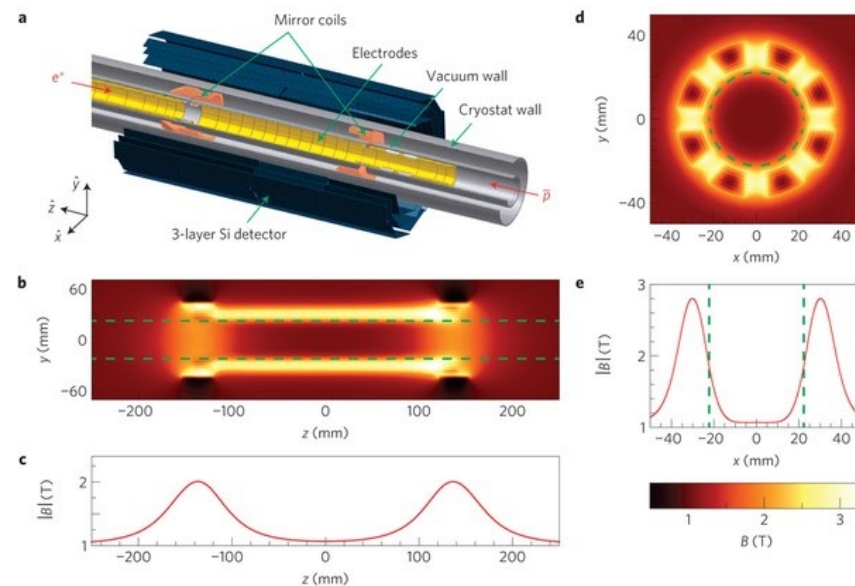
Rydberg atoms

Born from mutual love - Coulomb interaction
- between an electron and an ion.

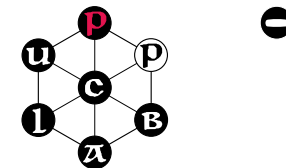
Or, between an positron and an antiproton.



35s state of hydrogen



CERN Alpha trap, trapping antimatter in a magnetic bottle



Interesting things about Rydberg atoms

Dimensions:

$$r = \frac{n^2 \hbar^2}{k e^2 m}$$

microns (like bacteria)

Lifetimes:

$$\tau \propto n^3 (l + 1/2)^2$$

hundreds of microseconds

Dipole moments:

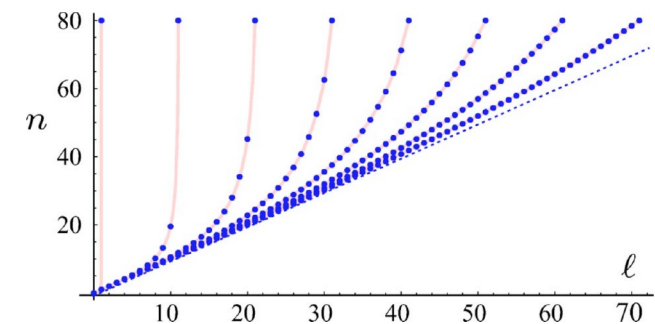
$$d \propto n^2$$

Radiative cascade, only changes of l by ± 1 allowed.

Energy spacing:

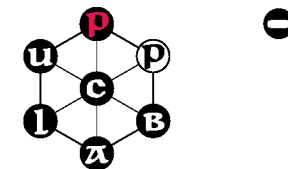
$$\Delta E \propto \frac{1}{n^3}$$

And remember the magnetic moment! Circular states with $l=n-1$.



$$\mu_F = g_F \mu_B \sqrt{f(f+1)}$$

Flannery, M. R. & Vranceanu, D. Quantal and classical radiative cascade in Rydberg plasmas. Phys. Rev. A 68, 030502 (2003).



Antimatter plasma

3-body recombination

$$k \propto T^{-9/2}$$

High Rydberg - reionization

Low-Rydberg - stabilization

Guiding centre atom in magnetic field

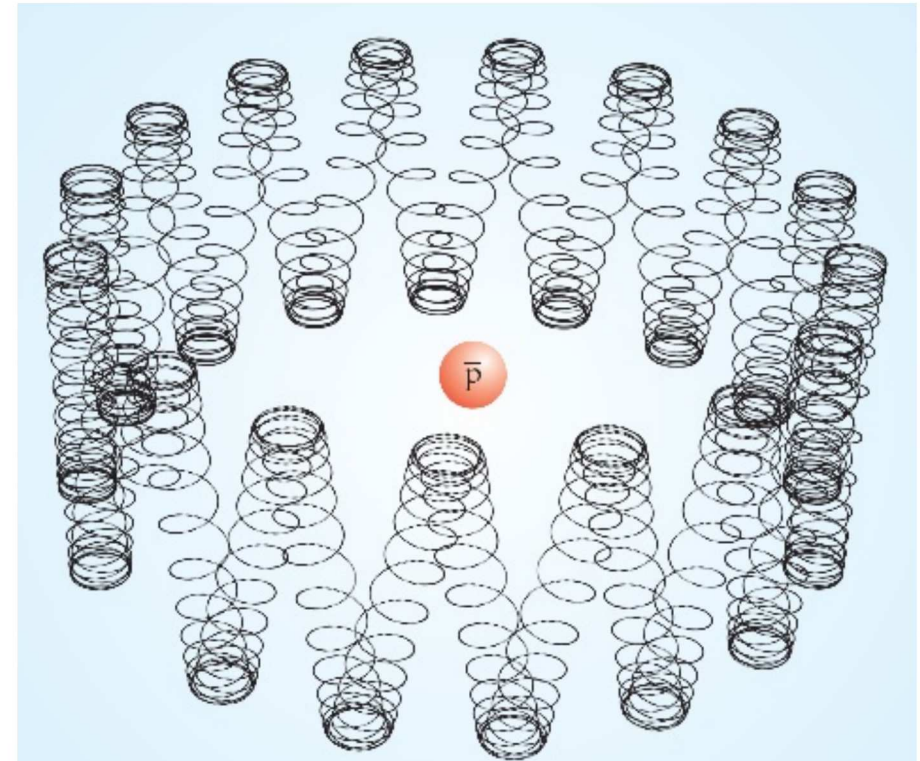
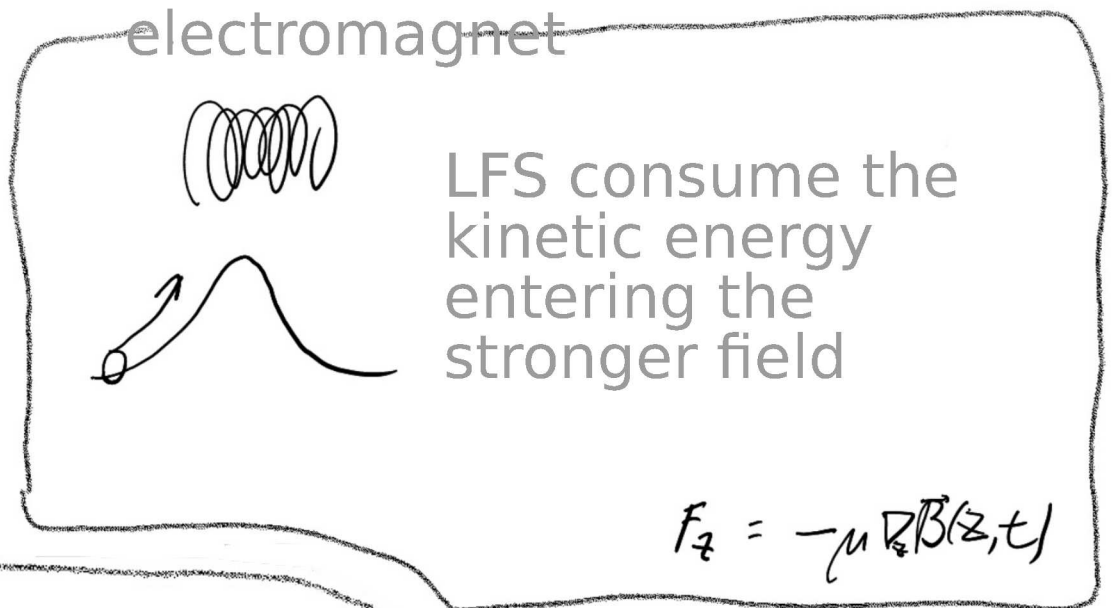
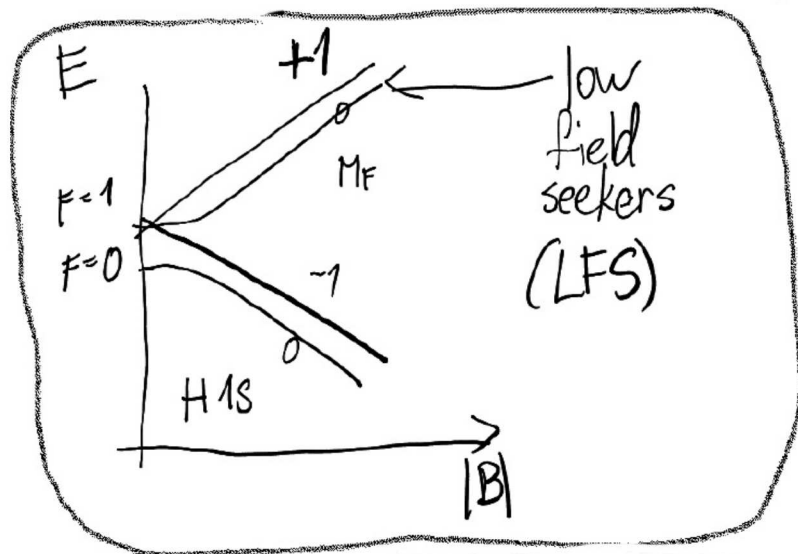


Figure 3. A classical trajectory of a positron in an anti-hydrogen Rydberg atom in a strong magnetic field. The positron undergoes cyclotron motion with small Larmor radius, bounces back and forth along a field line, and drifts around the antiproton. The complicated motion alters the dynamics of an antimatter plasma containing such atoms.

Deceleration without lasers



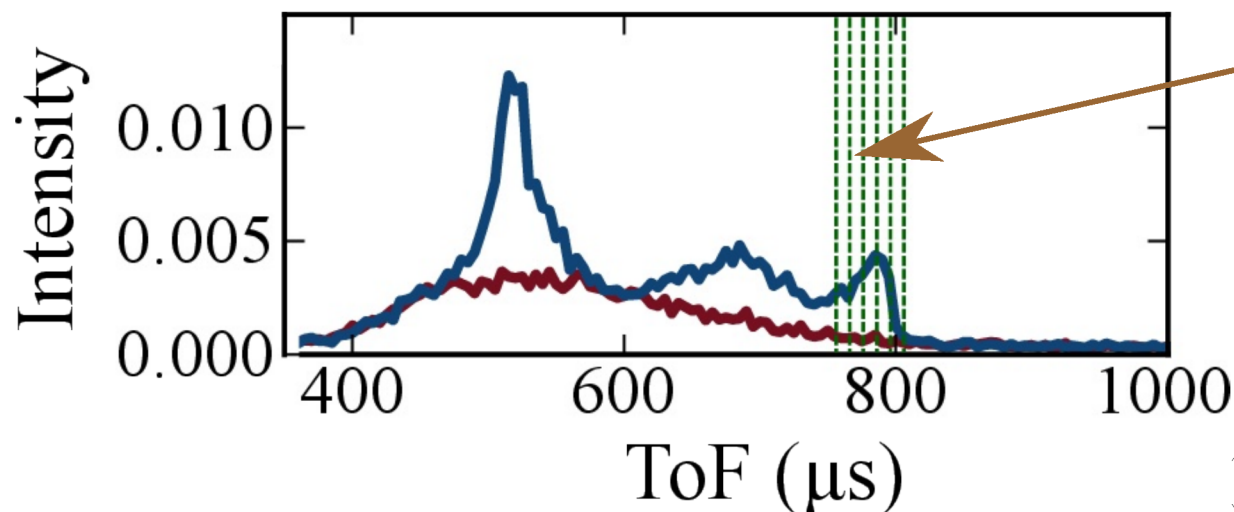
switching of coils



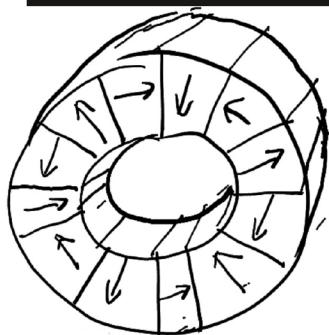
gradual deceleration

There is a catch.

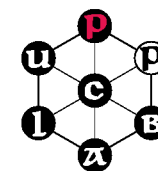
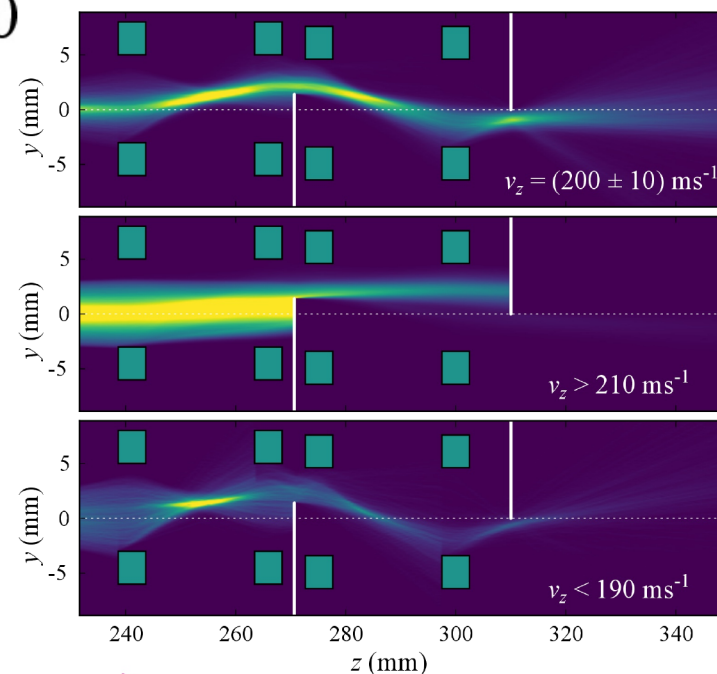
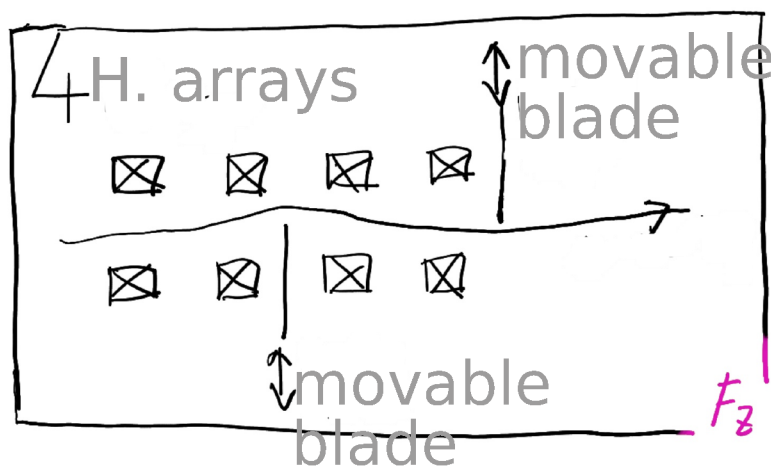
Catch: filtering the velocity



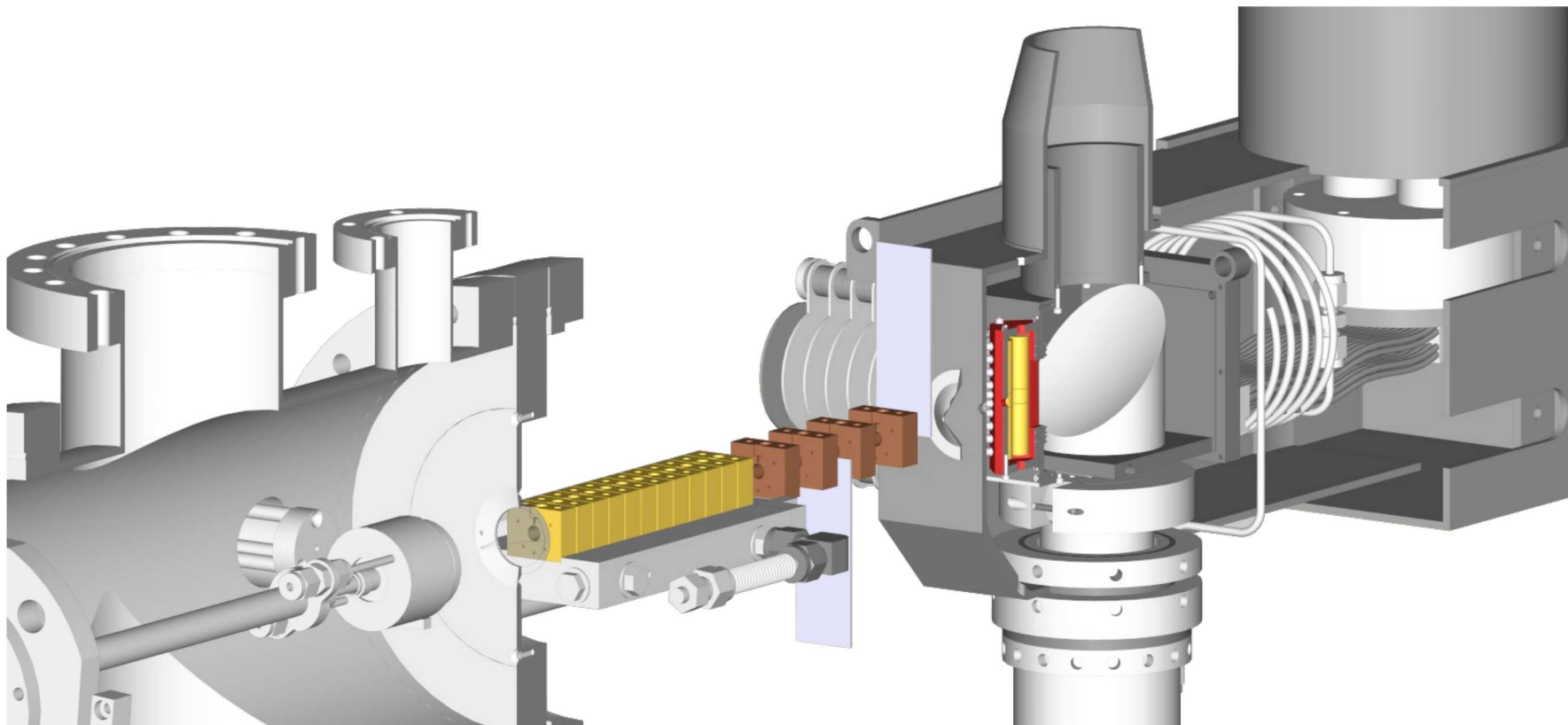
Obstacle course for hydrogen atoms



Hallbach array
for localised
magnetic field



Putting it all together



What about the ions?

What we have learnt

It is possible to cool, decelerate and trap neutral atoms. Especially easy if they are hydrogen-like.

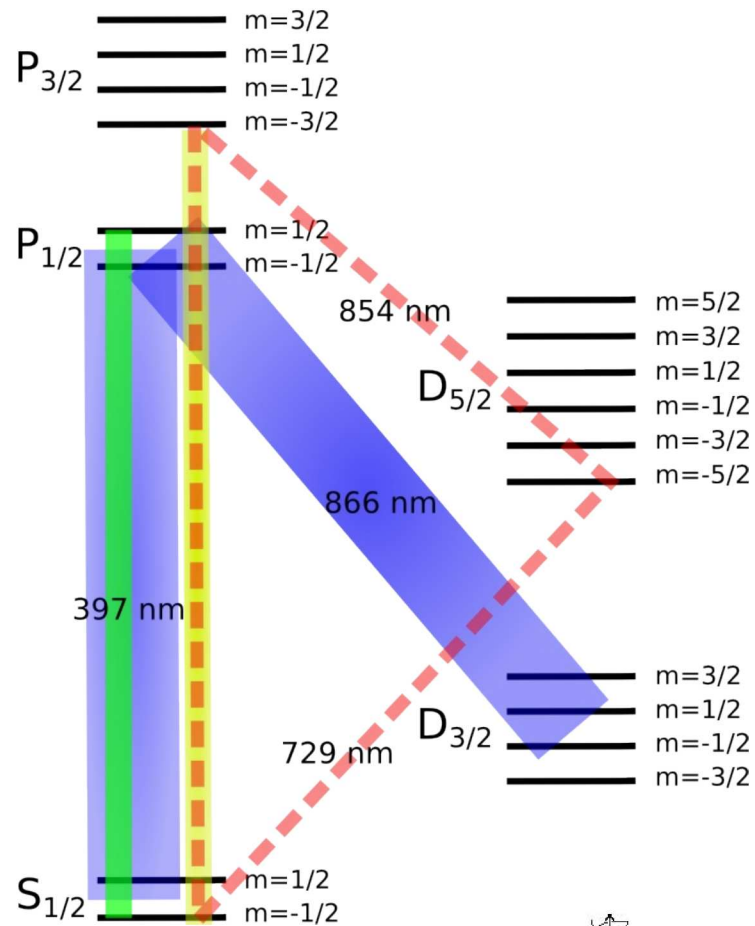
1 H																	2 He															
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne															
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar															
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr															
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe															
55 Cs	56 Ba	*	71 Lu	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn														
87 Fr	88 Ra	**	103 Lr	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Fl	115 Uup	116 Lv	117 Uus	118 Uuo														
																		*	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb
																		**	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No

1 H																	2 He	
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne	
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar	
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr	
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe	
55 Cs	56 Ba	*	71 Lu	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	**	103 Lr	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Fl	115 Uup	116 Lv	117 Uus	118 Uuo
		*	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb		
		**	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No		

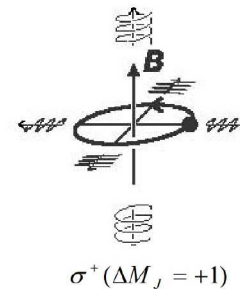
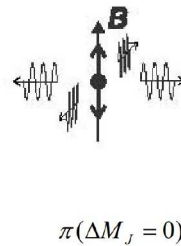
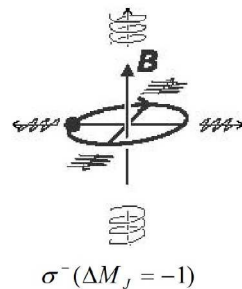
Atoms of alkaline earth metals that have lost one electron are also hydrogen-like.

Ions are friends with benefits: the electric charge allows them to be confined.

Cooling of calcium ions



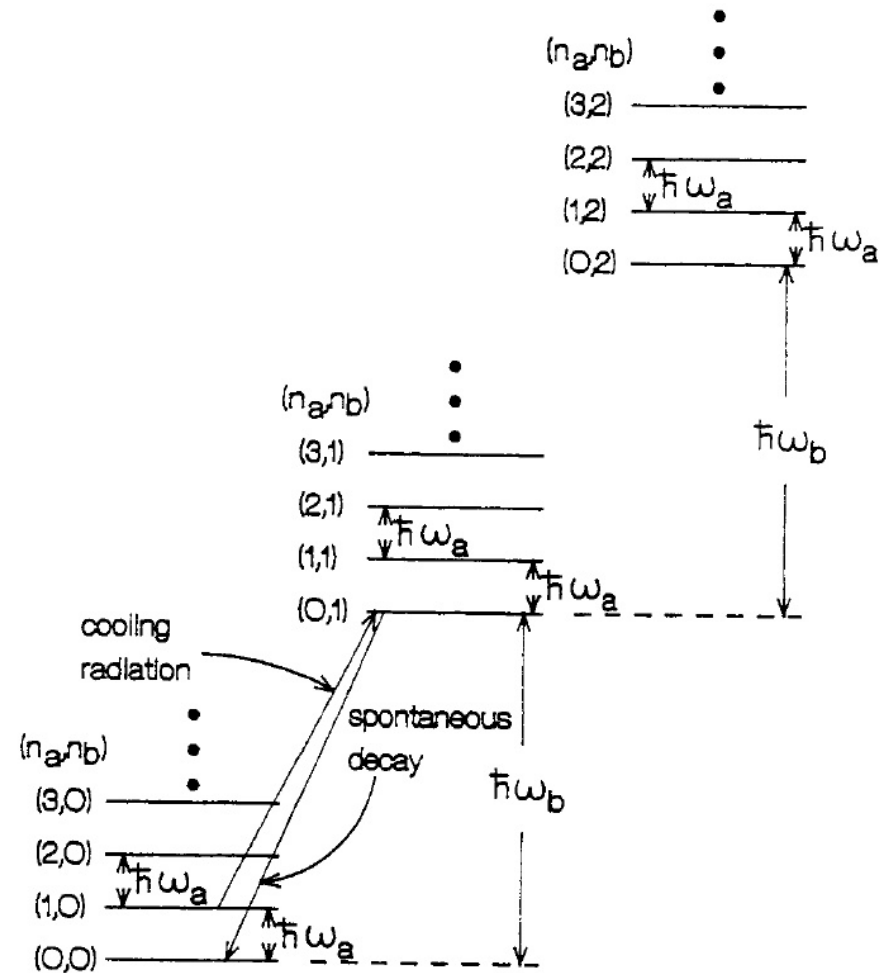
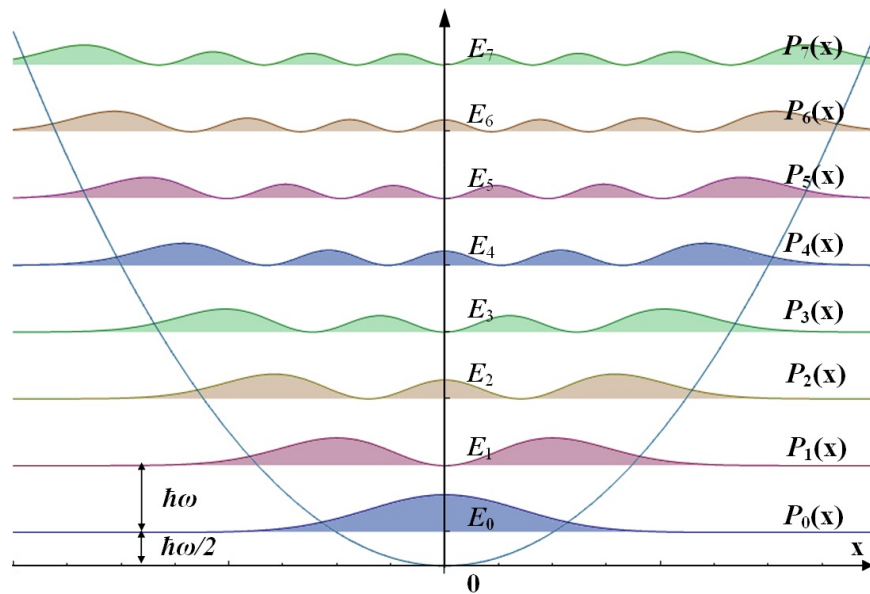
Allowed transitions		Electric dipole (E1)	Magnetic dipole (M1)
Rigorous rules	(1)	$\Delta J = 0, \pm 1$ ($J = 0 \leftrightarrow 0$)	
	(2)	$\Delta M_J = 0, \pm 1$ ($M_J = 0 \leftrightarrow 0$ if $\Delta J = 0$)	
	(3)	$\pi_f = -\pi_i$	$\pi_f =$
LS coupling	(4)	One electron jump $\Delta L = \pm 1$	No electron jump $\Delta L = 0,$ $\Delta n = 0$
	(5)	If $\Delta S = 0$ $\Delta L = 0, \pm 1$ ($L = 0 \leftrightarrow 0$)	If $\Delta S = 0$ $\Delta L = 0$
Intermediate coupling	(6)	If $\Delta S = \pm 1$ $\Delta L = 0, \pm 1, \pm 2$	



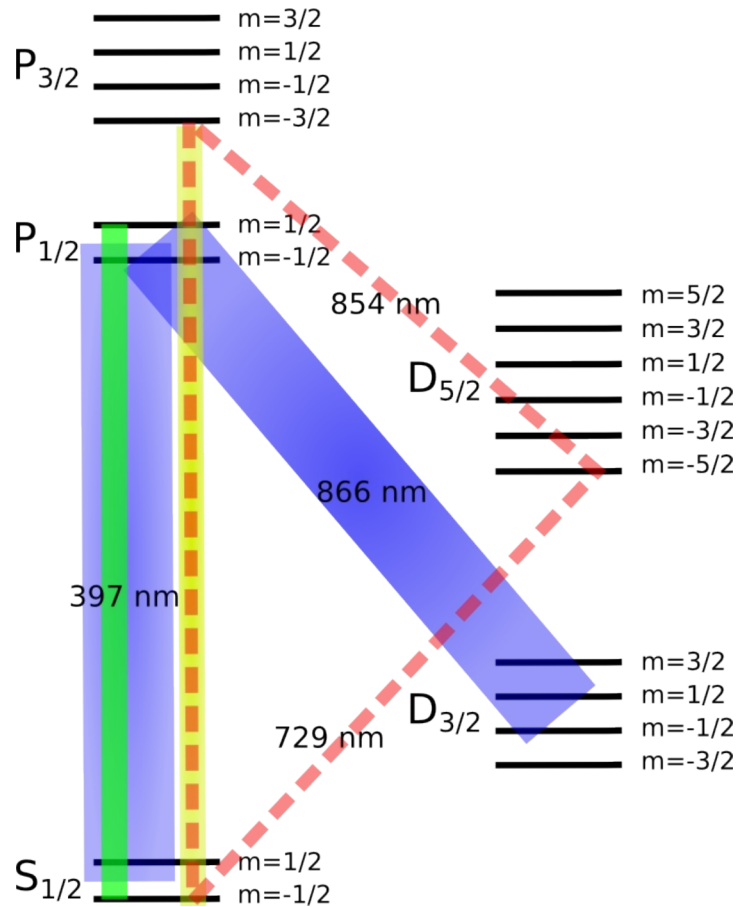
polarisation matters

Quantum states of trapped ions

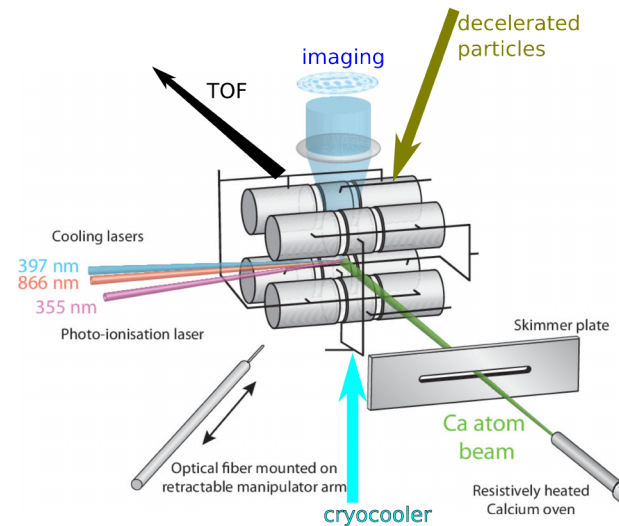
The trap generates a harmonic potential well.



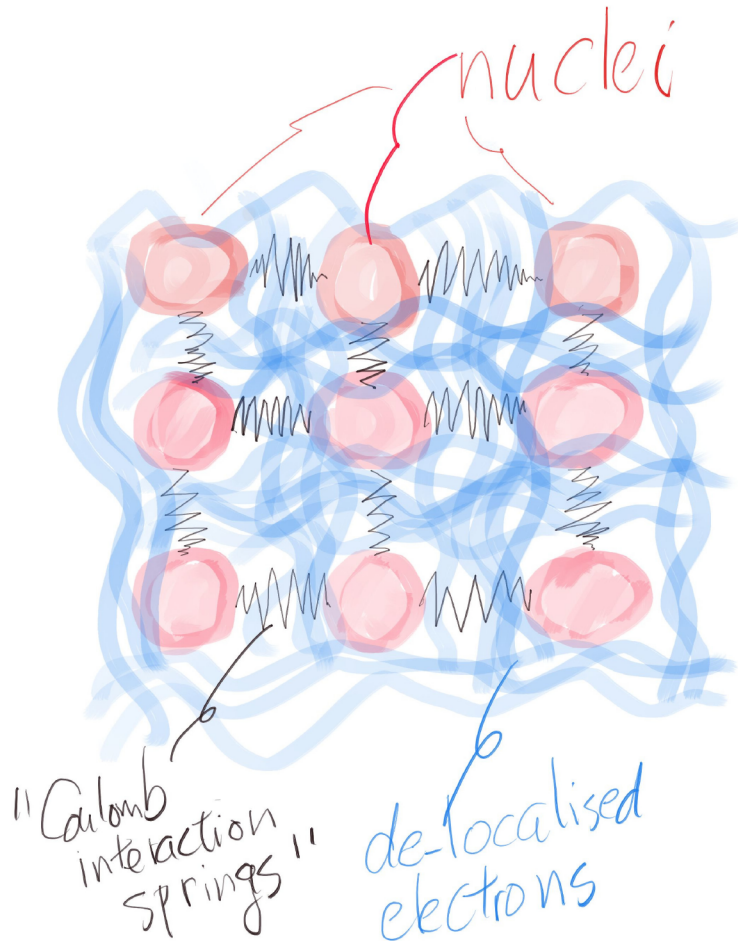
Production of ion Coulomb crystal



Doppler cooling limit: 0.5 mK for calcium S-P transition



What we are up to: Promiscuous electrons



Electron delocalisation

$$\lambda_{th} = \sqrt{\frac{2\pi\hbar^2}{m_e k_B T_e}}$$

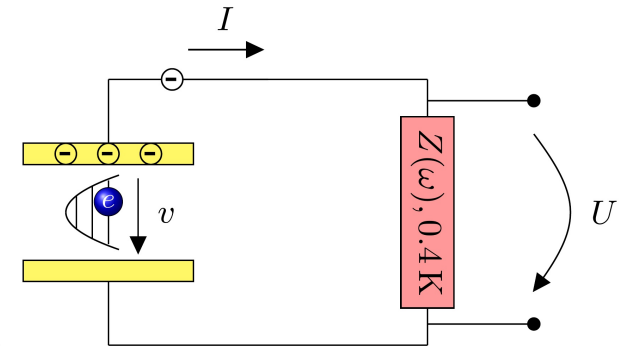
A diagram illustrating electron delocalization. A blue circle labeled "e-" is at the top, with two red circles labeled "+" below it. Wavy lines connect the electron to each ion. A speech bubble above the electron says "Here or here?".

So, how do we cool the electrons?

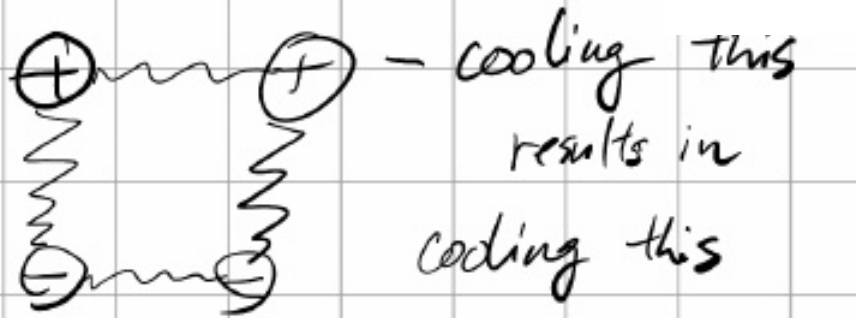
Production of low-energy electrons

Near-threshold photo-ionisation of trapped ion

Cooling in interaction with super-conducting (cryogenic) circuits



Noguchi's idea

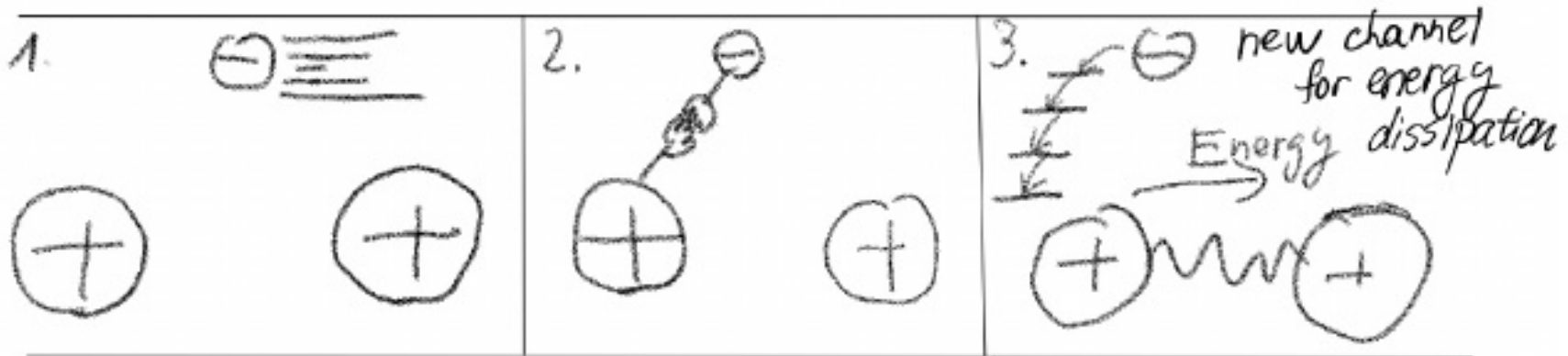
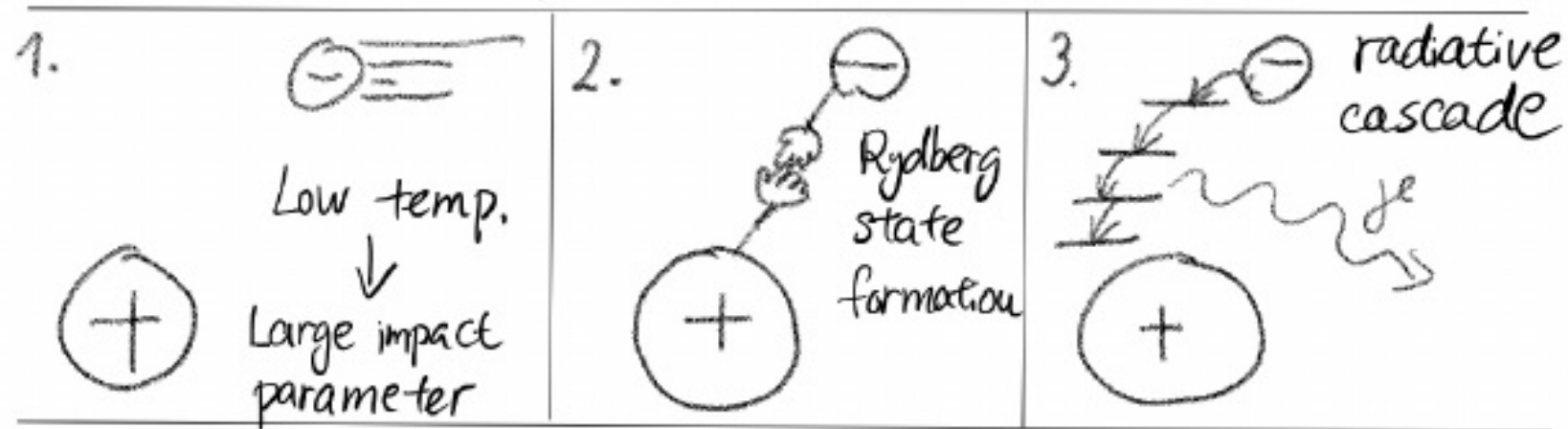


Sympathetic cooling in interaction with ions

Electrons are "invisible", ions fluoresce!

Recombination is the unsolved problem

Recombination problem



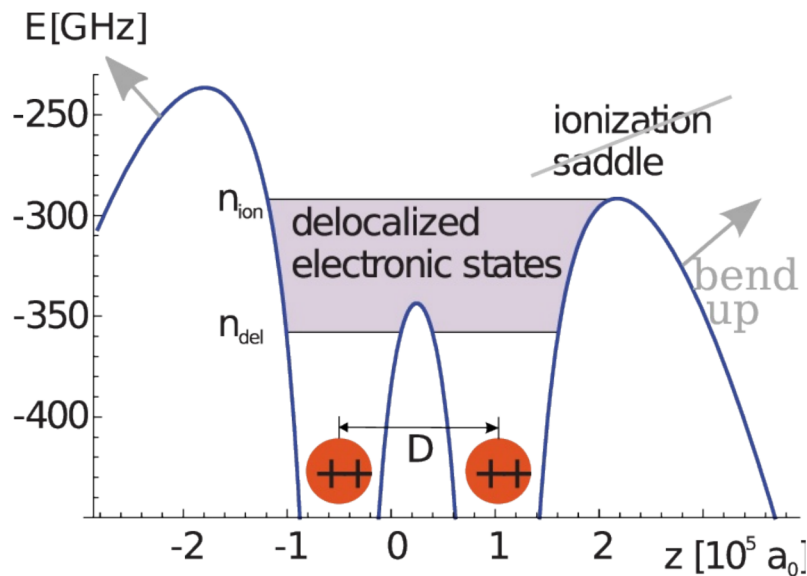
Experimental approach

Confinement in trap

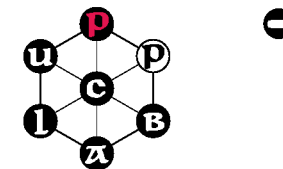
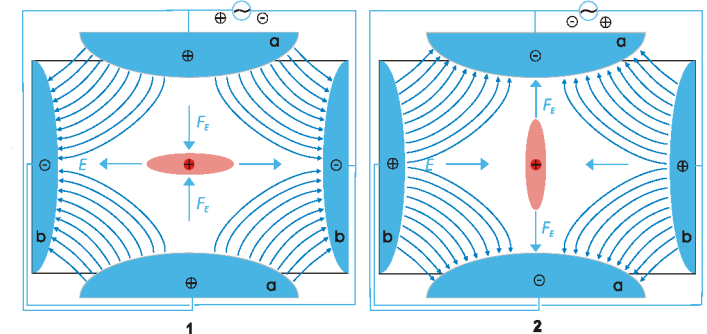
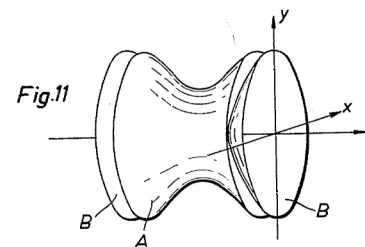
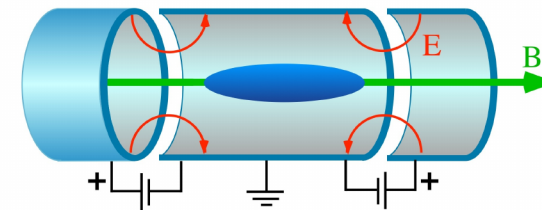
Not Penning but Paul

Penning: mass-agnostic

Paul: charge-agnostic
(ponderomotive potential)



Lesanovsky, I., Müller, M. & Zoller, P. Trap-assisted creation of giant molecules and Rydberg-mediated coherent charge transfer in a Penning trap. Phys. Rev. A 79, 010701 (2009).

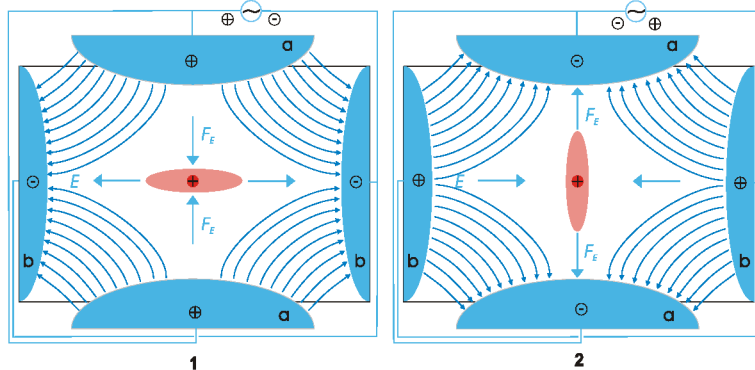


Confinement in Paul trap

Not directly applicable: large m:q ratio difference between ions and electrons

Two-frequency Paul trap

$$V(t) = V_e \cos \Omega_e t + V_i \cos \Omega_i t$$

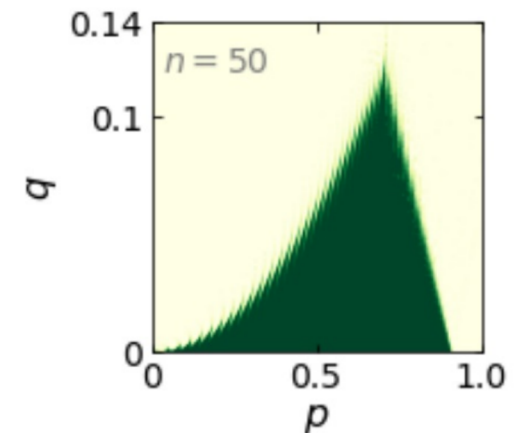


Requirements for stability

$$V_e \gg V_i \quad \Omega_e \gg \Omega_i$$

Our choice:

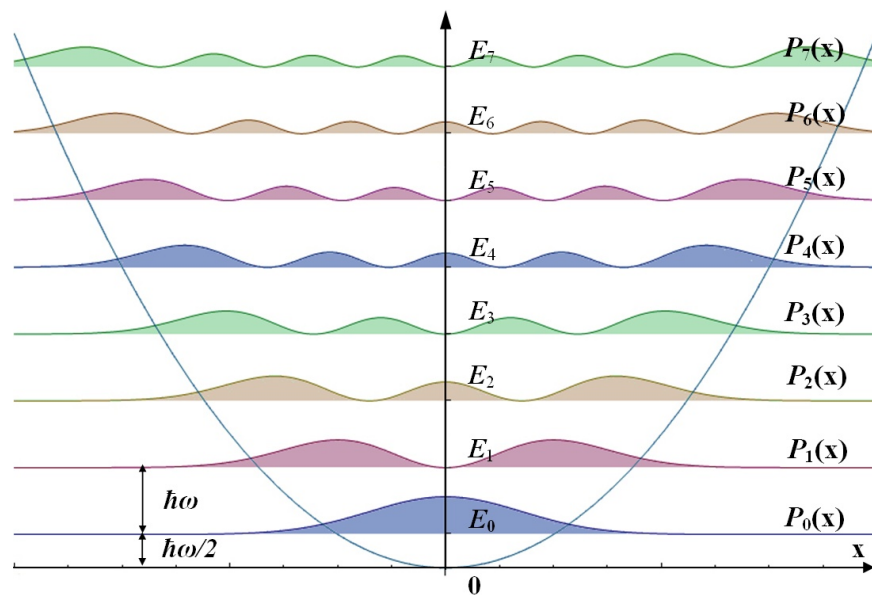
$$V_e = 100 \text{ V}, f_e = 2.5 \text{ GHz}, V_i = 1 \text{ V}, f_i = 1 \text{ MHz}$$



p, q - normalised amplitudes

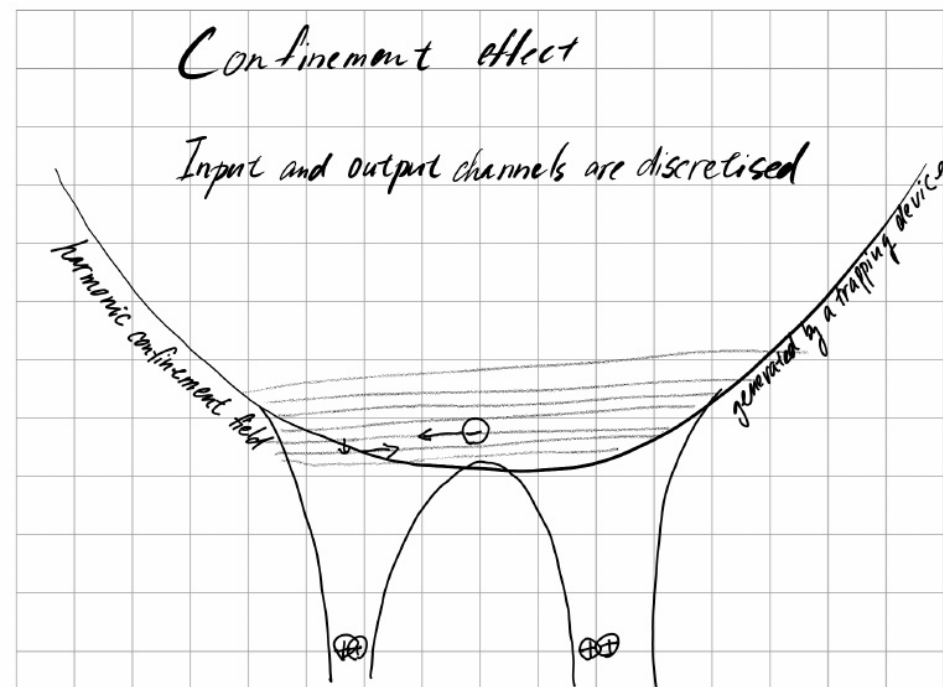
Confinement effect

In transversal dimension



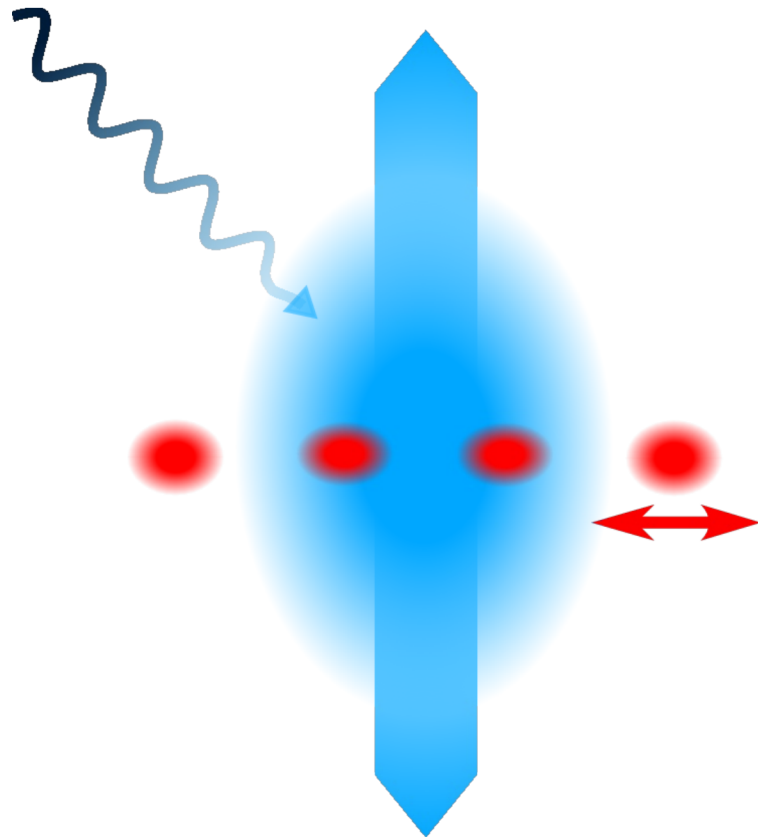
prominent for electrons

In longitudinal dimension

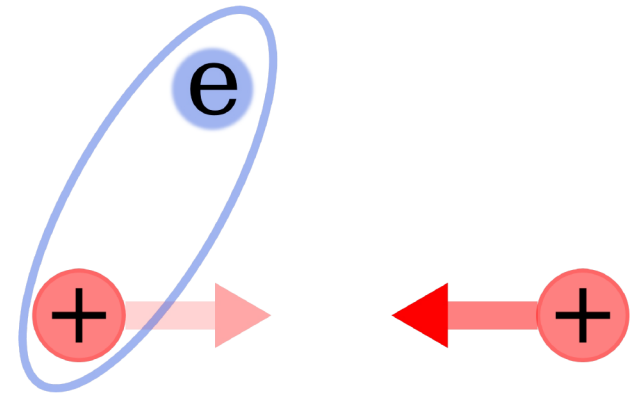


Exploitation of the system

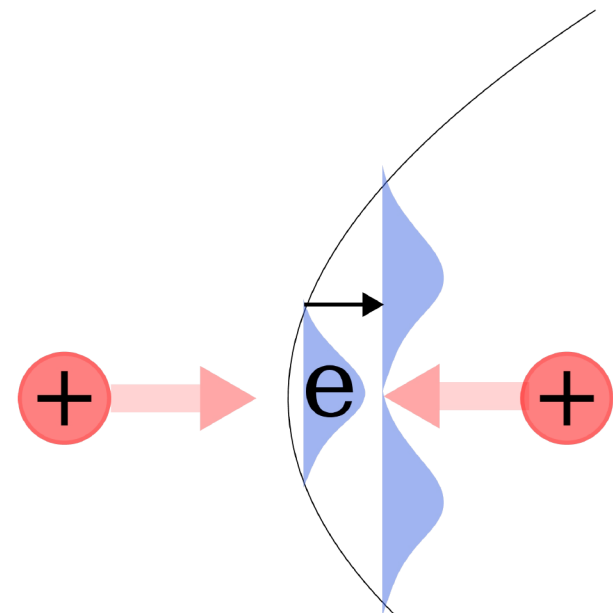
Single photon detection



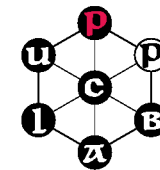
classical,
localised



quantum
mechanical,
delocalised



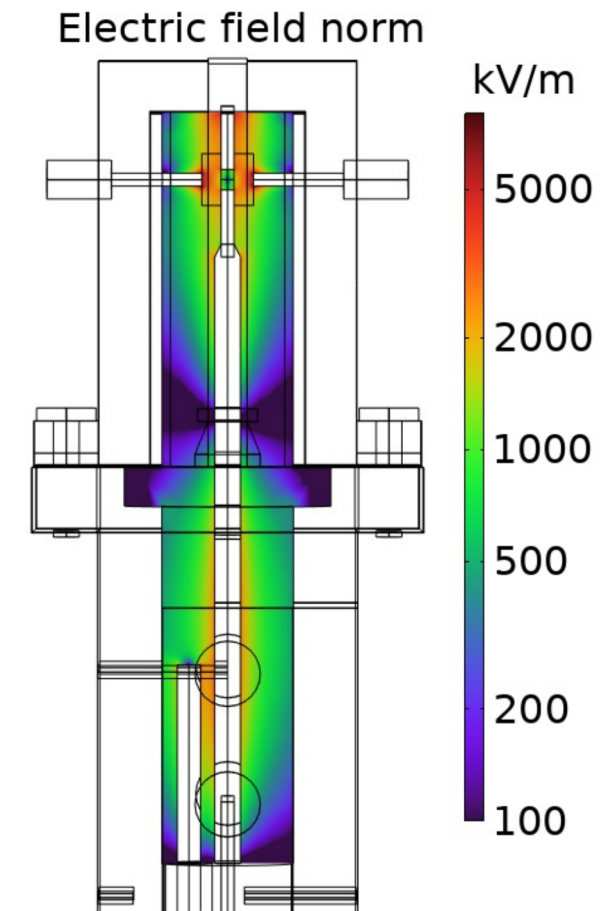
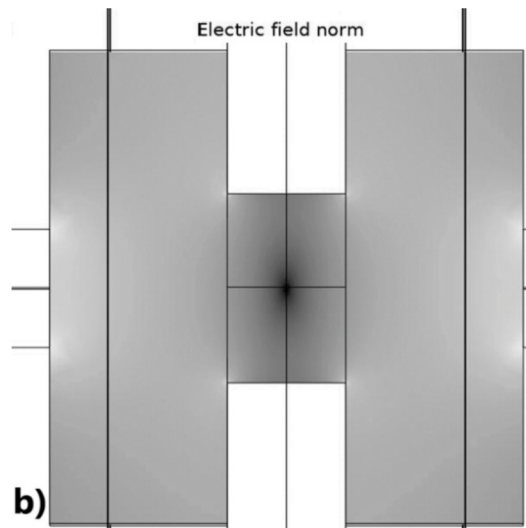
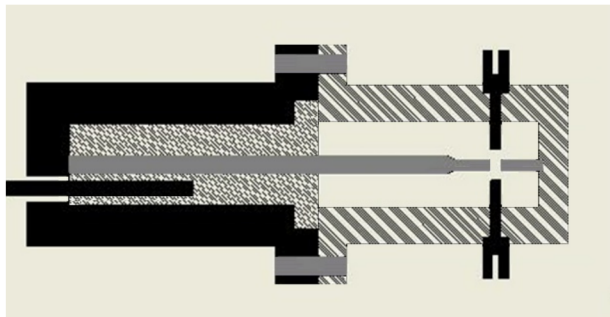
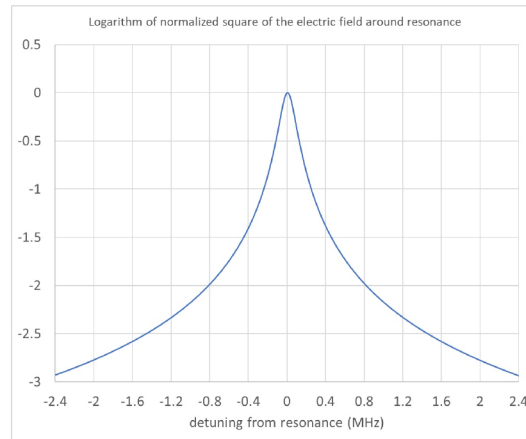
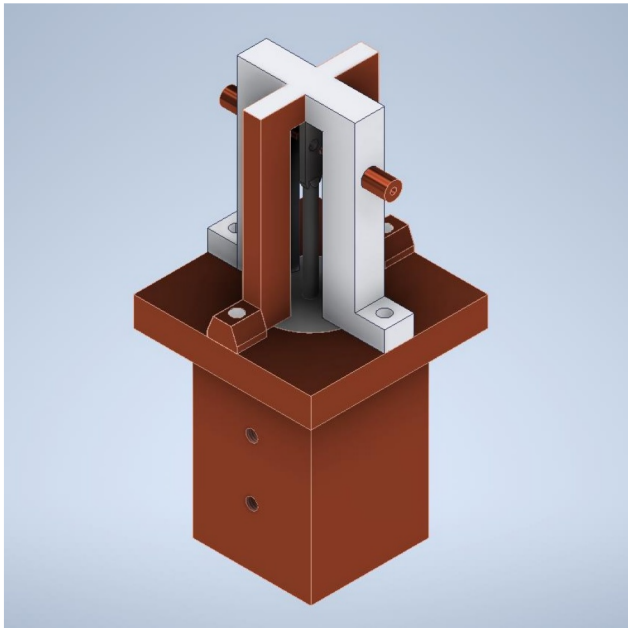
change in the screening



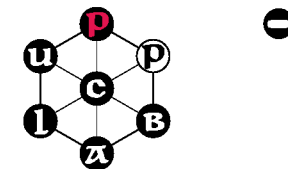
ultra-cold
plasma lab
prague

Realisation

Our choice: $V_e = 100$ V, $f_e = 2.5$ GHz, $V_i = 1$ V, $f_i = 1$ MHz



coaxial resonator

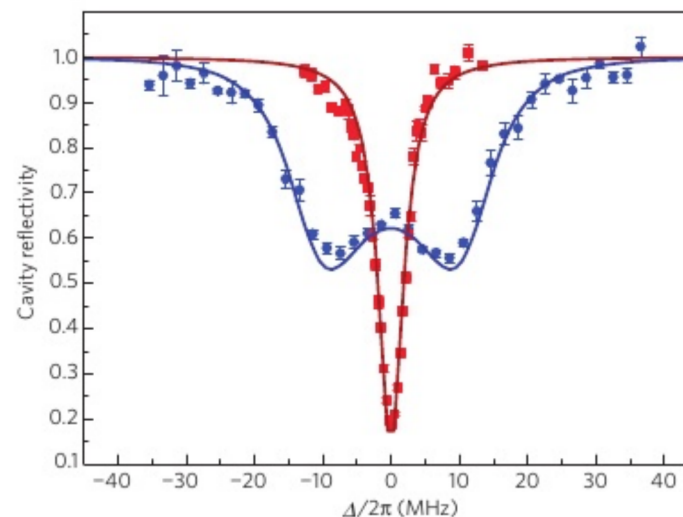
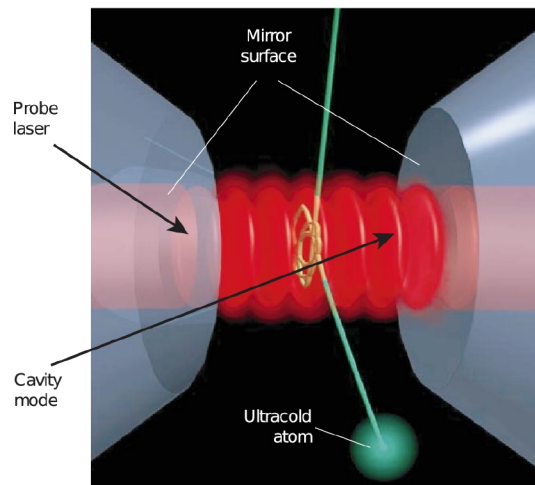


Ion vibration detection

Fluorescence imaging by microscope (slow, low resolution, possible to detect vibration modes in Coulomb crystal)

Interaction with an optical resonator (fast, high sensitivity, integral, state preparation necessary)

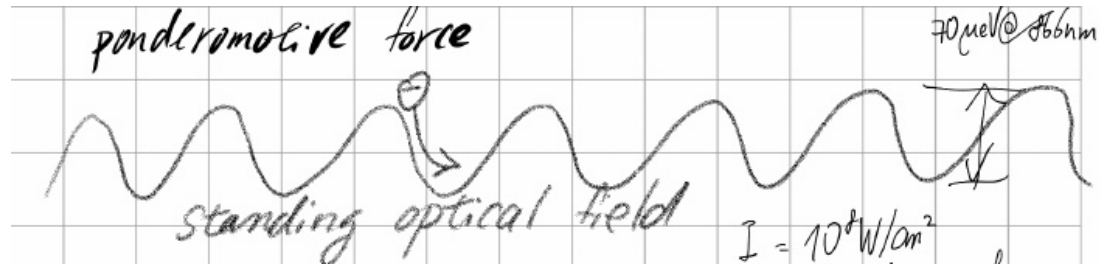
Photon collection using cavities, self-interference ...



Vahala, K. J. Optical microcavities. *Nature* 424, 839-846 (2003).

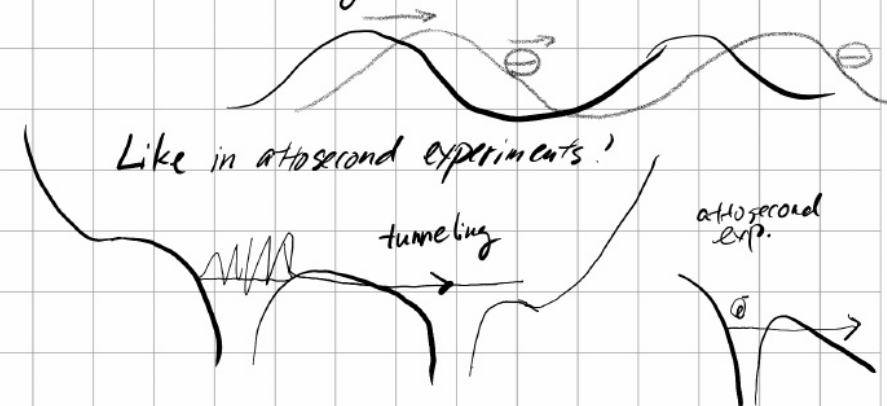
Herskind, P. F., Dantan, A., Marler, J. P., Albert, M. & Drewsen, M. Realization of collective strong coupling with ion Coulomb crystals in an optical cavity. *Nat. Phys.* 5, 494-498 (2009).

Other physics



How do we control the position?

By optical ponderomotive field
like a rolling matt?

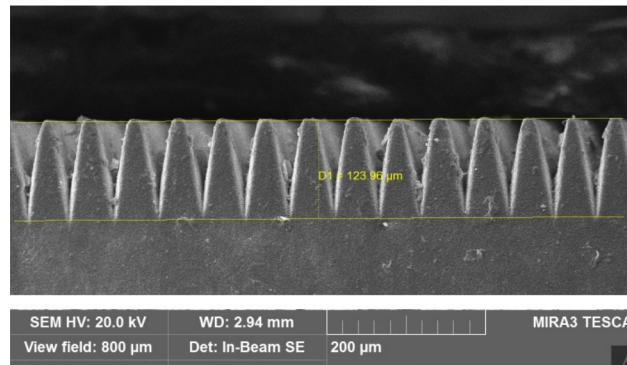


Preparing for application

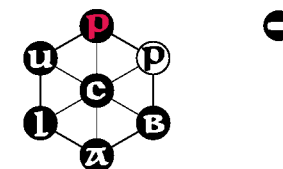
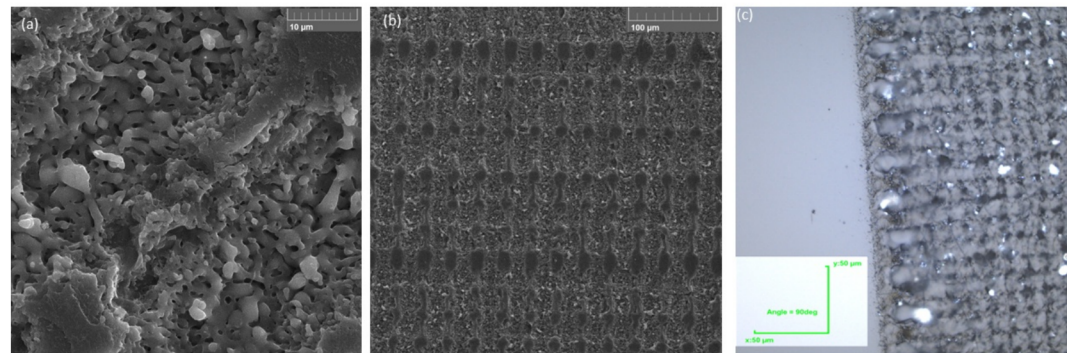
Fabrication of high-power microwave circuit boards



Surface
roughening,
metallization,
circuit carving



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ultra-cold
plasma lab
prague

Thank you

Theory:

Michal Tarana (J. Heyrovsky
Institute of Physical Chemistry)

Payman Mahmoudi (postdoc)

Trap design:

Niklas Lausti (doctoral student)

Experiment preparation:

Vineet Kumar (doctoral
student)

Fibre cavity development:

Pavel Honzátko (Institute of
Photonics and Electronics)

Ivan Hudák (master's degree
st.)

Circuit board:

Petr Hauschwitz (HiLase)

Ladislav Cvrček (Czech
Technical University)

Albin Antony (ex-postdoc)