



Charles University Prague
Mathematics and Physics Faculty

FP 2B 2022

Electron collisions

Collision Cross section

Collision rate coefficient

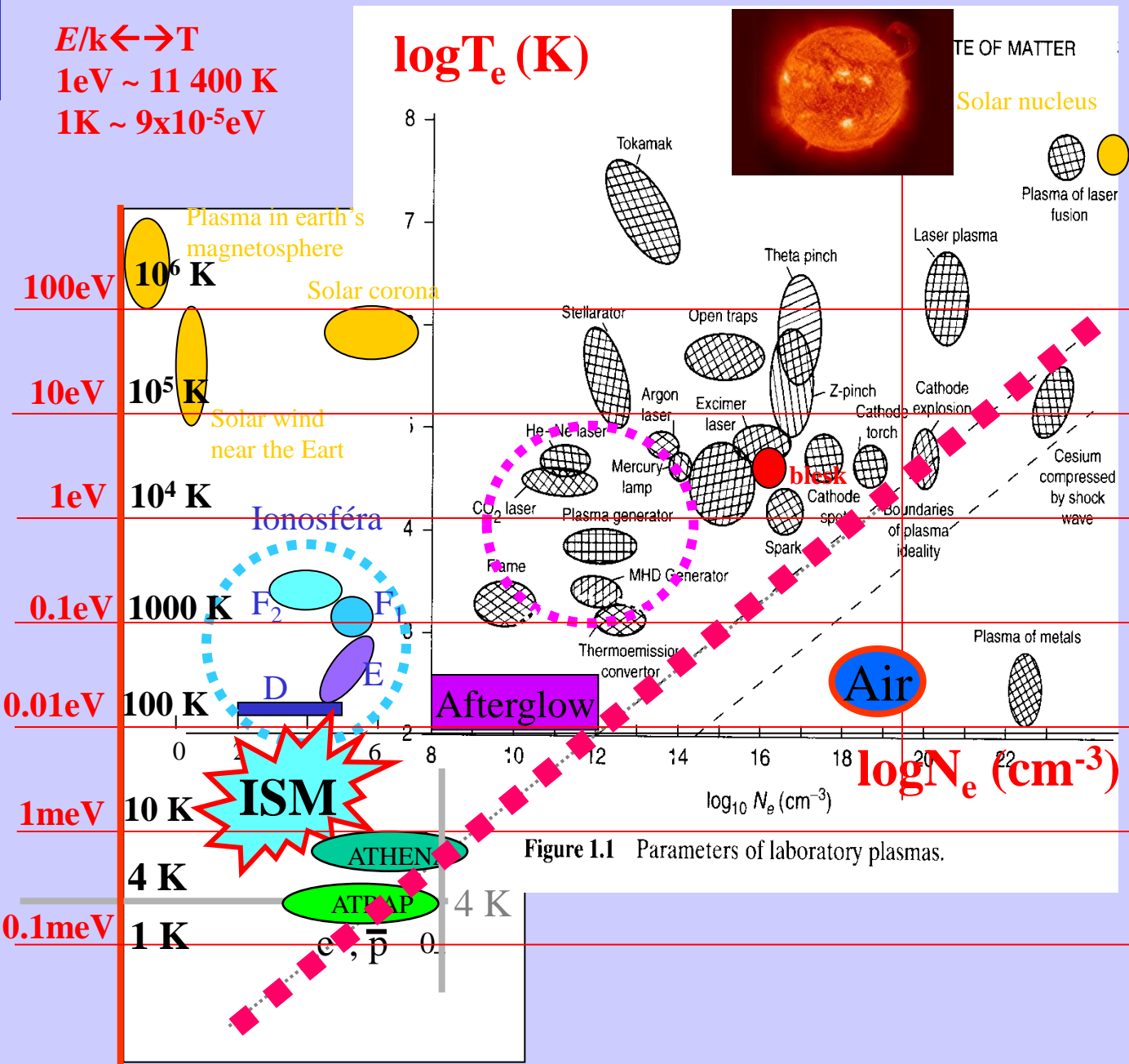
Reaction Cross section

Reaction rate coefficient

Temperatures and kinetic energies



$E/k \leftrightarrow T$
 $1\text{eV} \sim 11\,400\text{ K}$
 $1\text{K} \sim 9 \times 10^{-5}\text{eV}$



De Broglie wave length

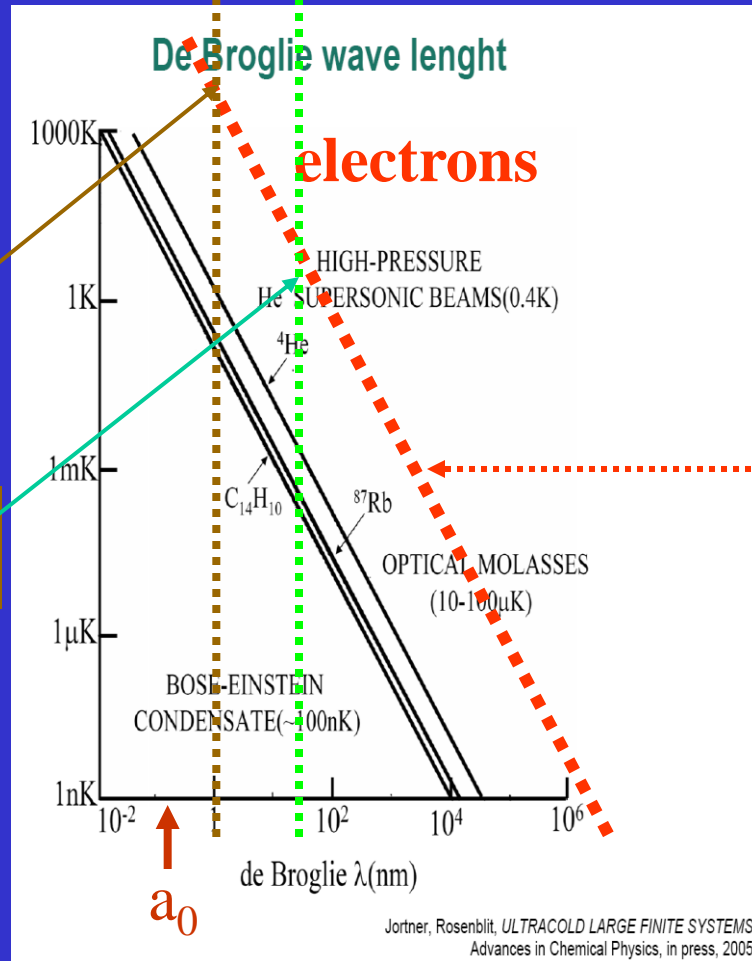
$$\lambda_{DB} = \frac{h}{p} = \frac{h}{Mv} \sim \frac{h}{\sqrt{MT}}$$

1eV

$$\lambda_{eDB}(1eV) \sim 11.6 \text{ \AA} \sim 1.16 \times 10^{-9} \text{ m}$$

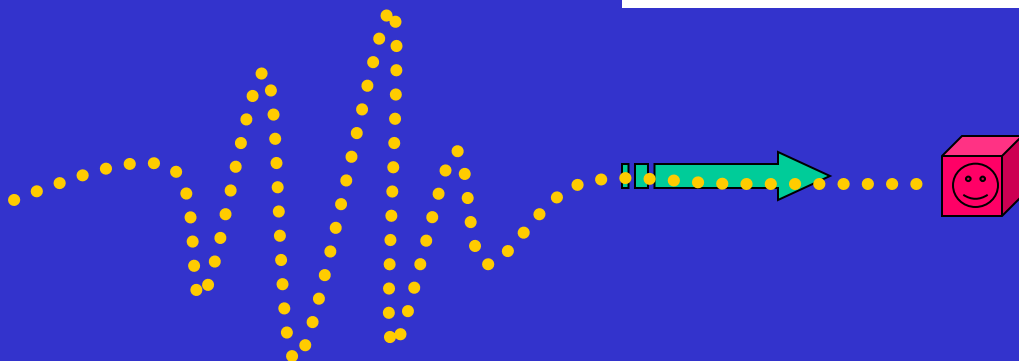
4K

$$\lambda_{eDB}(4K) \sim 540 \text{ \AA} \sim 54 \times 10^{-9} \text{ m}$$

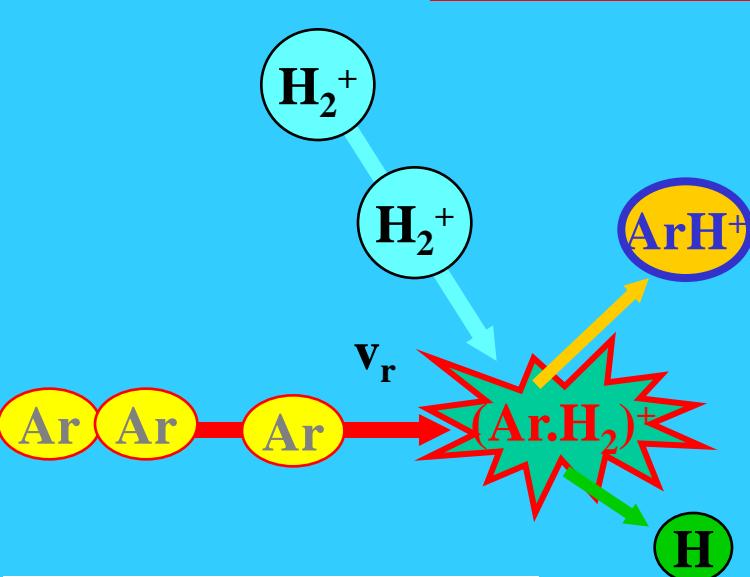
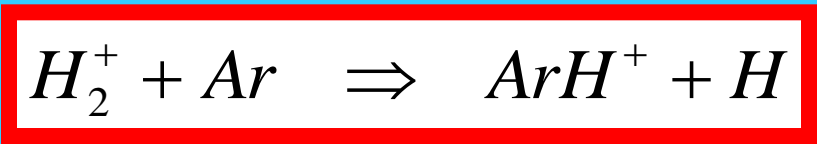


$$\lambda_{DB} = \frac{h}{m_e v_e} \sim \frac{h}{\sqrt{m_e T}}$$

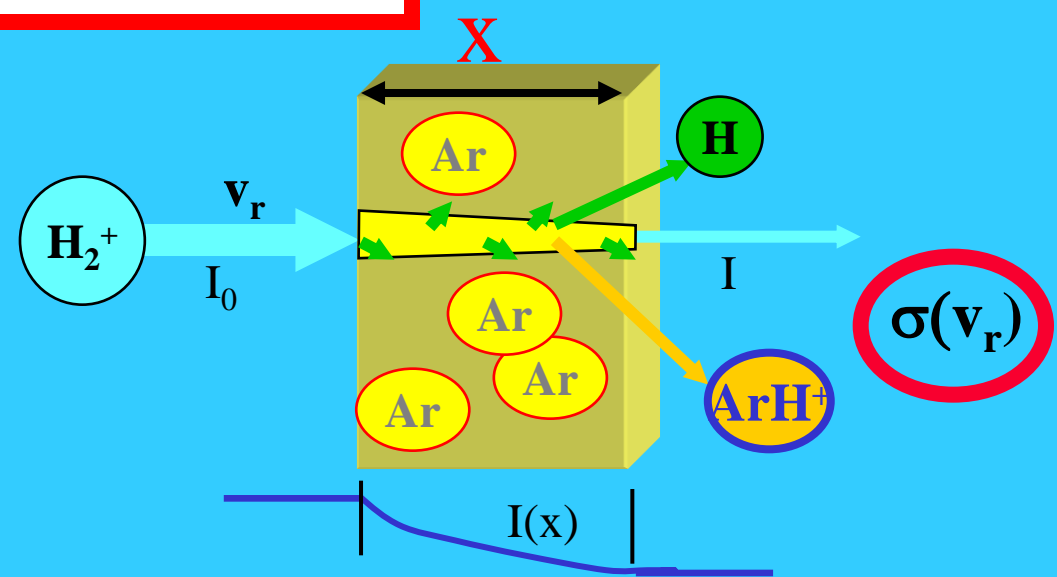
$$T \sim \frac{1}{\lambda_{DB}^2}$$



Single collision



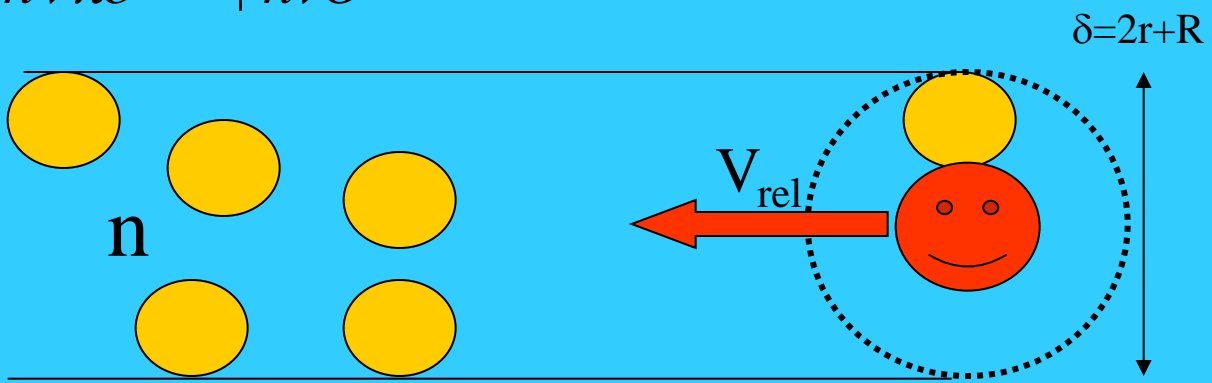
reaction cross section



$$I=I_0\exp(-\sigma n_{Ar}x)$$

$$v_{coll} = +nV_{rel} = +nvS = +nv\pi\delta^2 = \mp nv\sigma$$

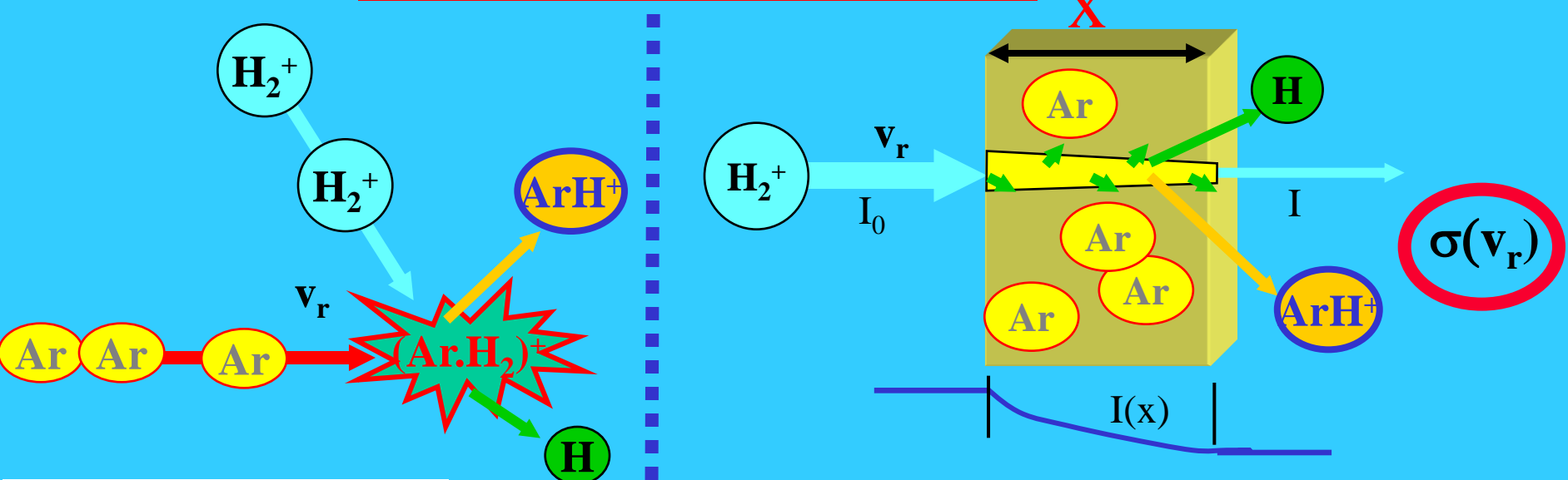
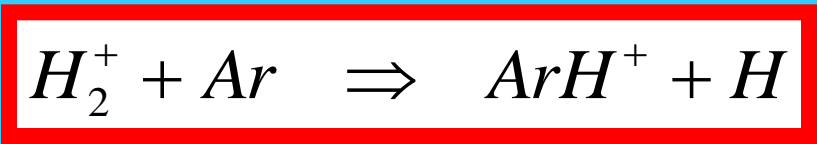
$$\frac{dI}{dt} = -\frac{I}{\tau_{coll}} = -Iv_{coll}$$



$$I(t) = I_0 \exp(-v_{coll}t) = I_0 \exp(-\sigma n v_{rel}t)$$

$$I=I_0\exp(-\sigma n_{Ar}x)$$

Single collision



reaction cross section

$$I = I_0 \exp(-\sigma n_{Ar} x)$$

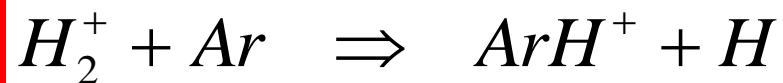
Proportionality factor

$$\frac{dI}{dx} \sim -INx \qquad \frac{dI}{dx} = -\sigma INx$$

$$\frac{dI}{Idx} = \frac{d \ln(I)}{dx} = -\sigma Nx$$

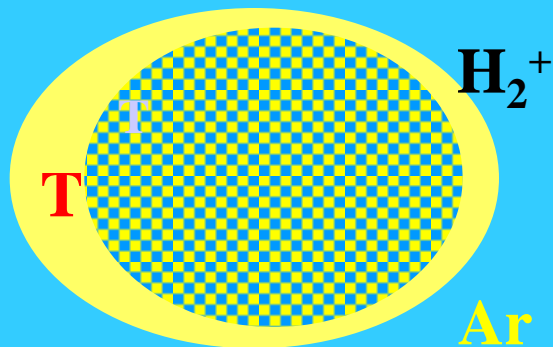
$$I(x) = I_0 \exp(-\sigma Nx)$$

Kinetics of elementary process



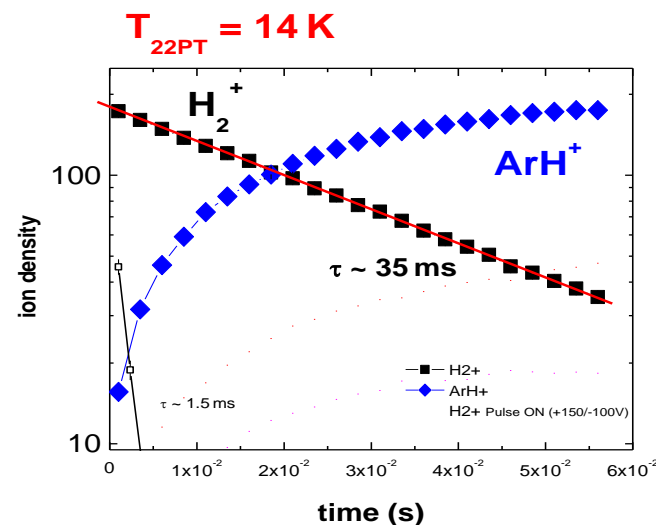
Multiple collision

@ T



reaction rate coefficient

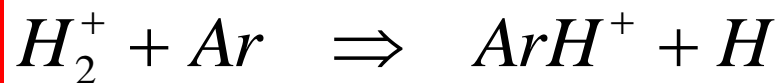
$$d(n_{H_2^+})/dt = -k n_{H_2^+} \cdot n_{Ar}$$



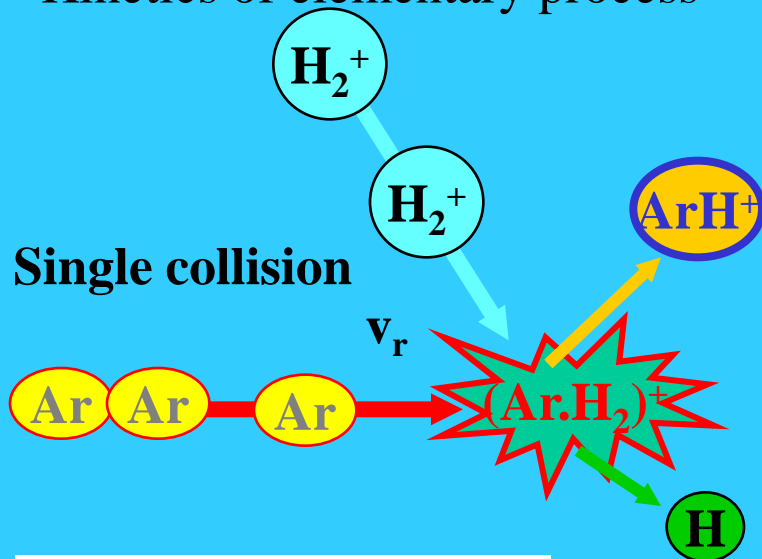
k(T)

$$n_{H_2^+} = (n_{H_2^+})_0 \exp(-k n_{Ar} t)$$

Kinetics of elementary process



Single collision

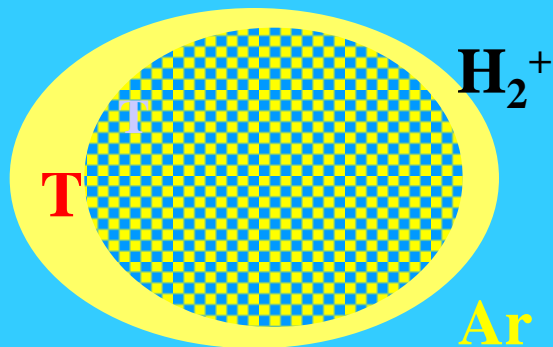


reaction cross section

$$I = I_0 \exp(-\sigma n_{Ar} x)$$

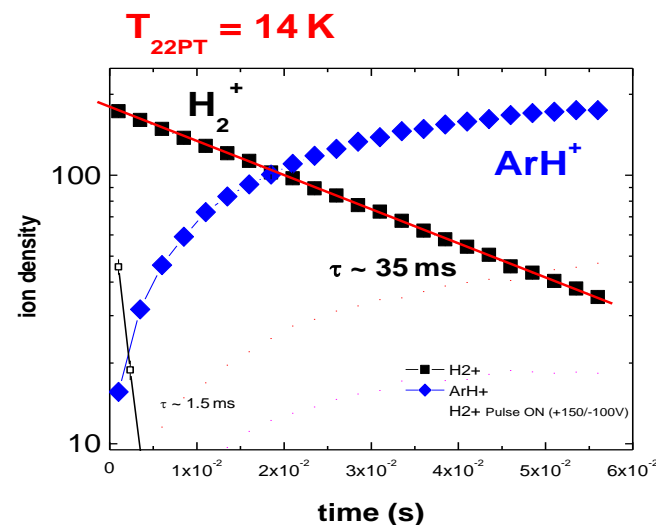
Multiple collision

@ T



reaction rate coefficient

$$d(n_{H_2^+})/dt = -k n_{H_2^+} \cdot n_{Ar}$$



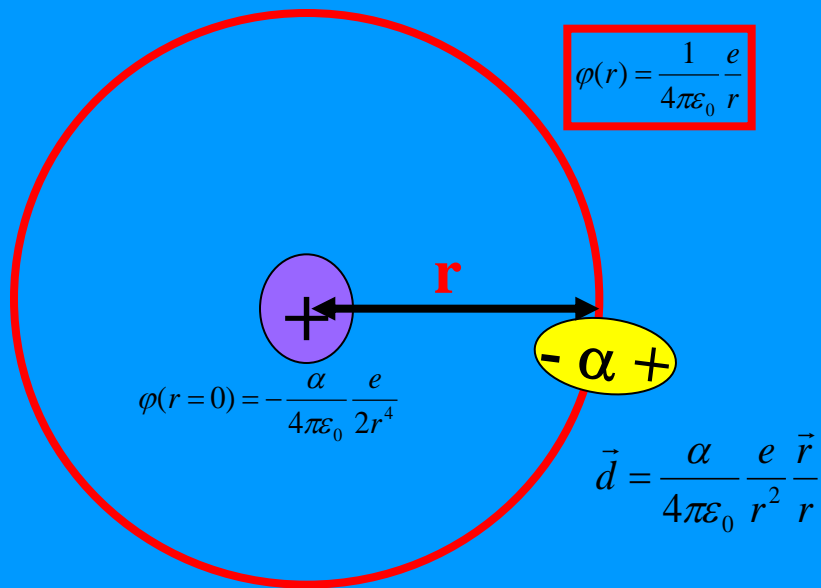
$$n_{H_2^+} = (n_{H_2^+})_0 \exp(-k n_{Ar} t)$$

$\sigma(v_r)$

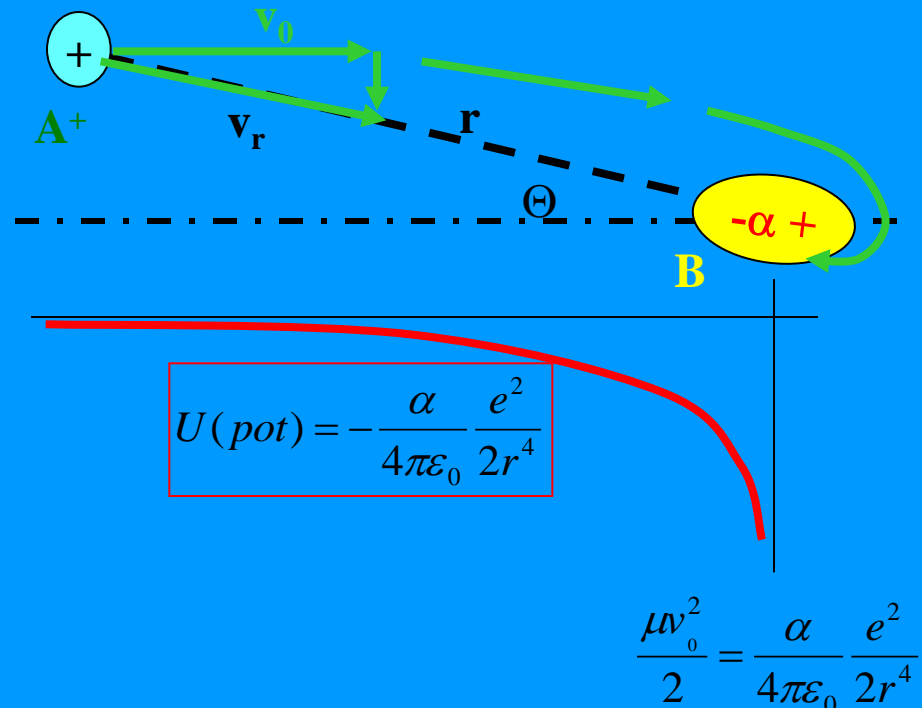
$k(T) = \langle v \sigma \rangle$

$k(T)$

Collision cross section of IMR

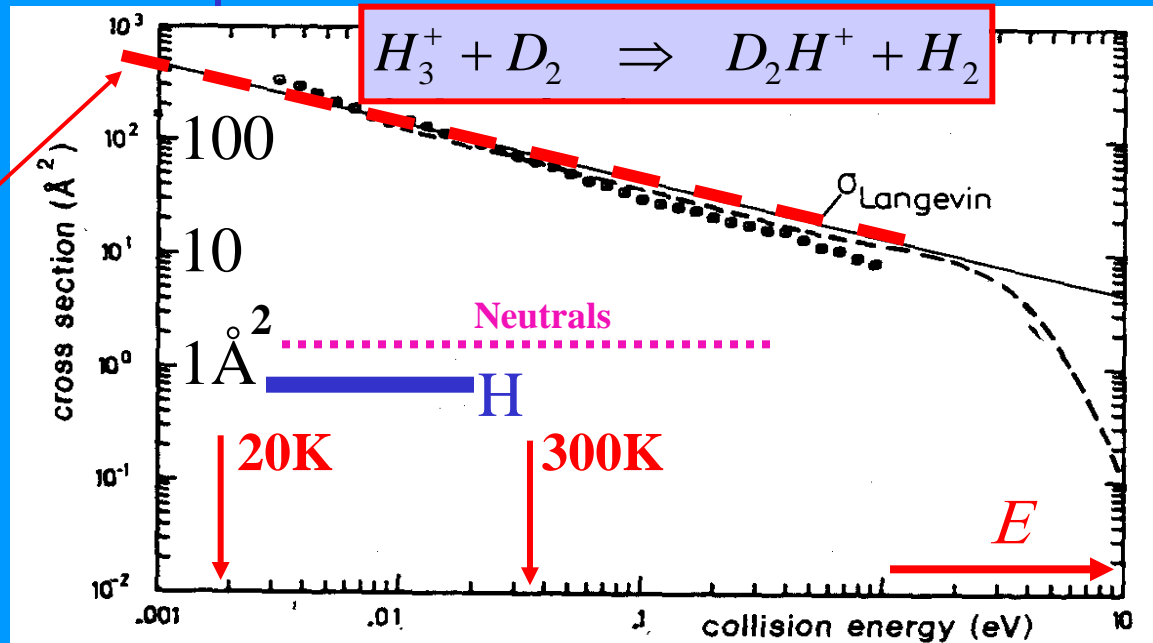


α - polarizability



$$\sigma = \pi \rho_0^2 = \frac{2\pi e}{v_0 (4\pi\epsilon_0)} \sqrt{\frac{\alpha}{\mu}}$$

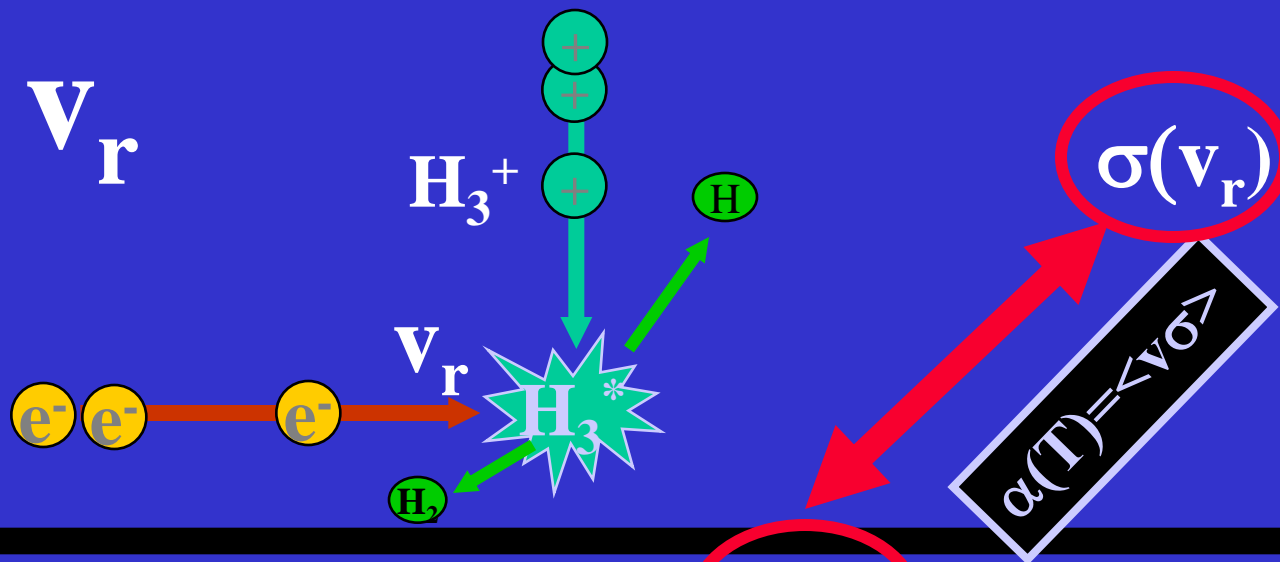
$$\sigma = \pi \rho_0^2 \sim \frac{1}{v_0} \sqrt{\frac{\alpha}{\mu}} \sim \frac{1}{\sqrt{E}}$$



Recombination



Cross section



$\alpha(T)$ Rate coefficient

Multiple collisions

$\text{H}_3^+ + \text{He}, \text{H}, \text{H}_2, h\nu \dots$

$$\frac{dn_e}{dt} = -\alpha n_i n_e = -\alpha n_e^2$$

