Psychophysics

Psychometric functions

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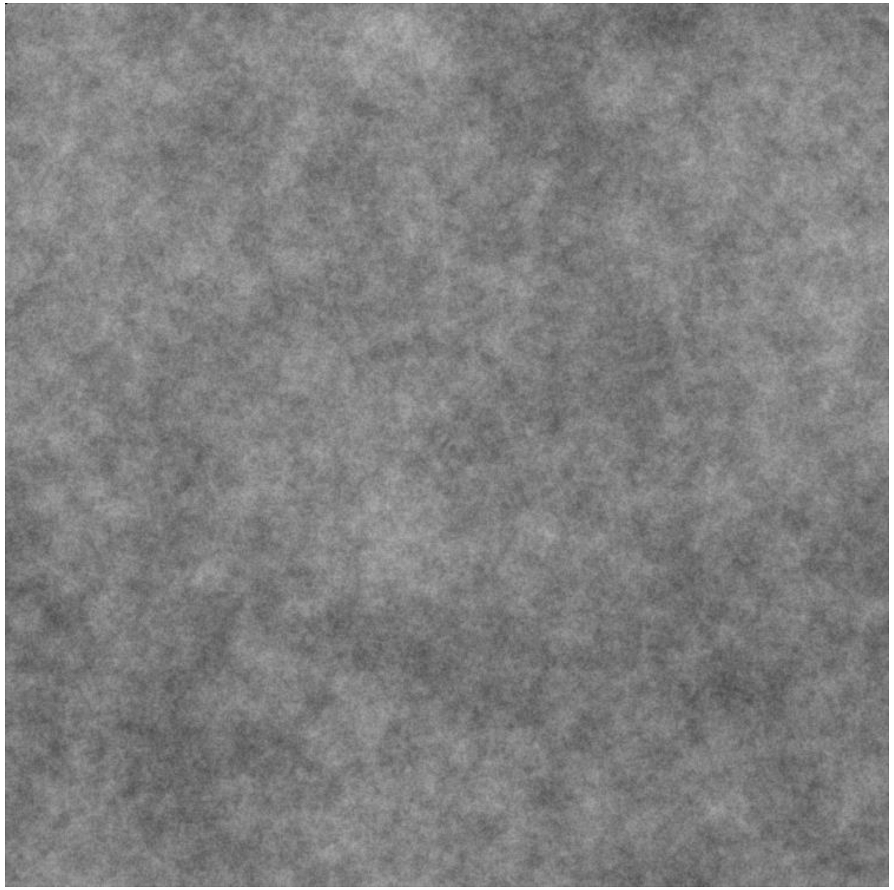
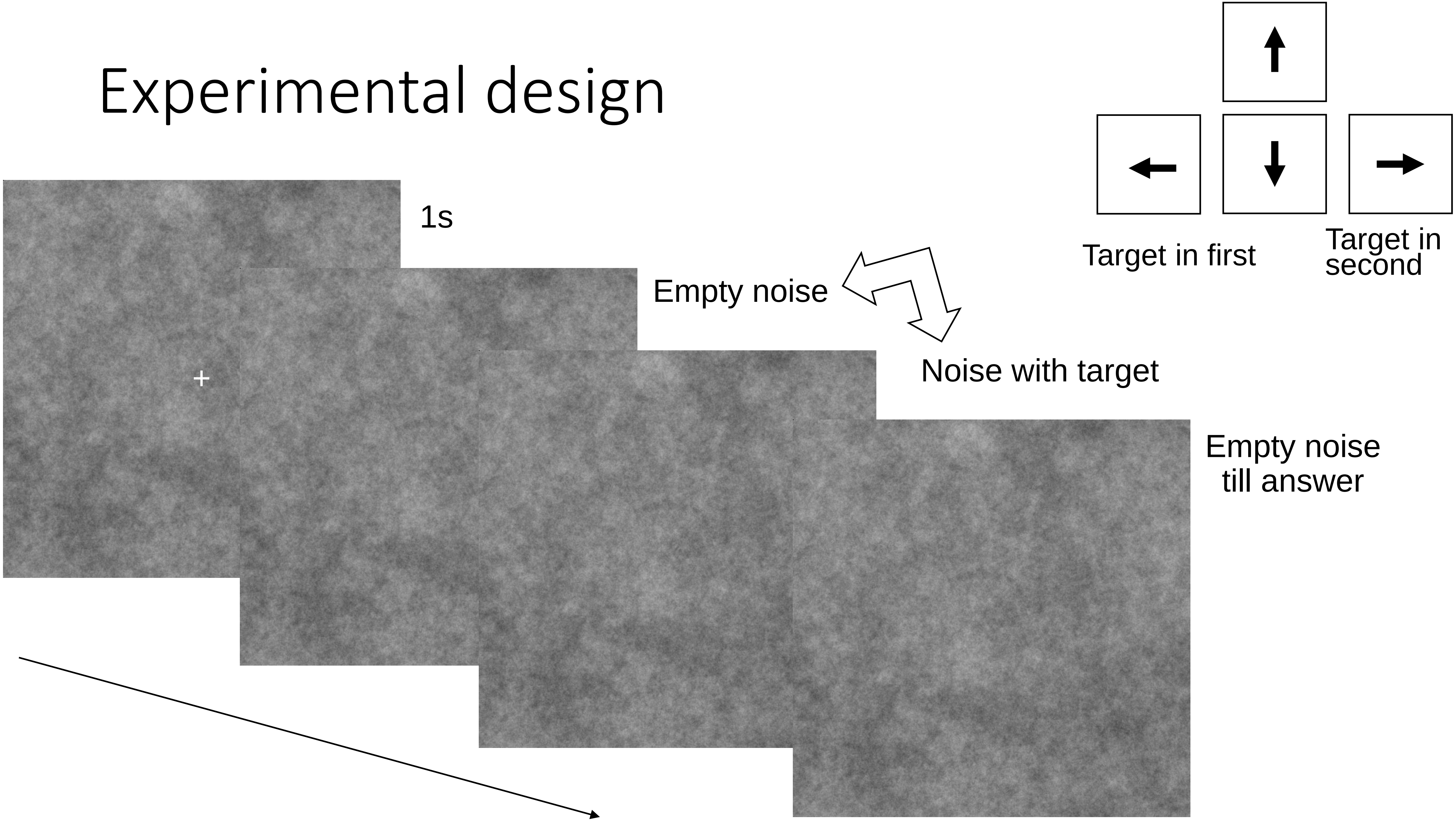
Psychometric function

- In the experiment, we show Gabor's patch in the noise

- We are working with very low intensities



Experimental design



Data

subject_id	trial_id	target contrast	target location	response	correct
1	1	0.04	first	first	1
1	2	0.02	first	second	0
1	3	0.08	second	second	1
1	4	0.02	first	first	1
1	5	0.16	second	second	1
1	6	0.08	first	second	0
1	7	0.16	second	second	1
1	8	0.04	second	first	0

Aggregated data

- We aggregate accuracies for individual levels

subject_id	target contrast	accuracy
1	0.02	58%
1	0.04	63%
1	0.08	78%
1	0.16	85%
1	0.32	95%

Visualization

- We are interested in threshold, here as stimulus level for 75% accuracy

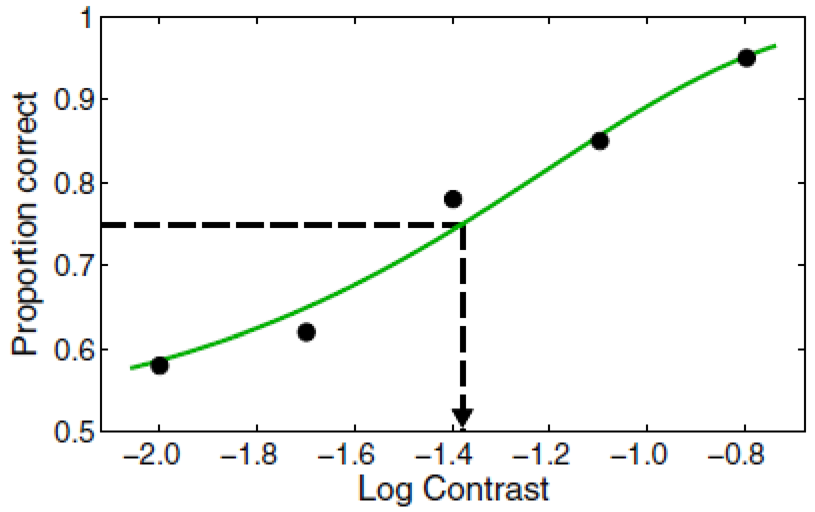


FIGURE 4.1 Example of a PF from a hypothetical experiment aimed at measuring a contrast detection threshold. The threshold is defined here as the stimulus contrast at which performance reaches a proportion correct equal to 0.75. Data are fitted using a log-Quick function.

How to describe the curve

- Usually 2 parameters
  - Threshold / PSE – intensity value where the state changes /when it subjectively looks the same to us
  - Slope – function growth rate (typically in the threshold)

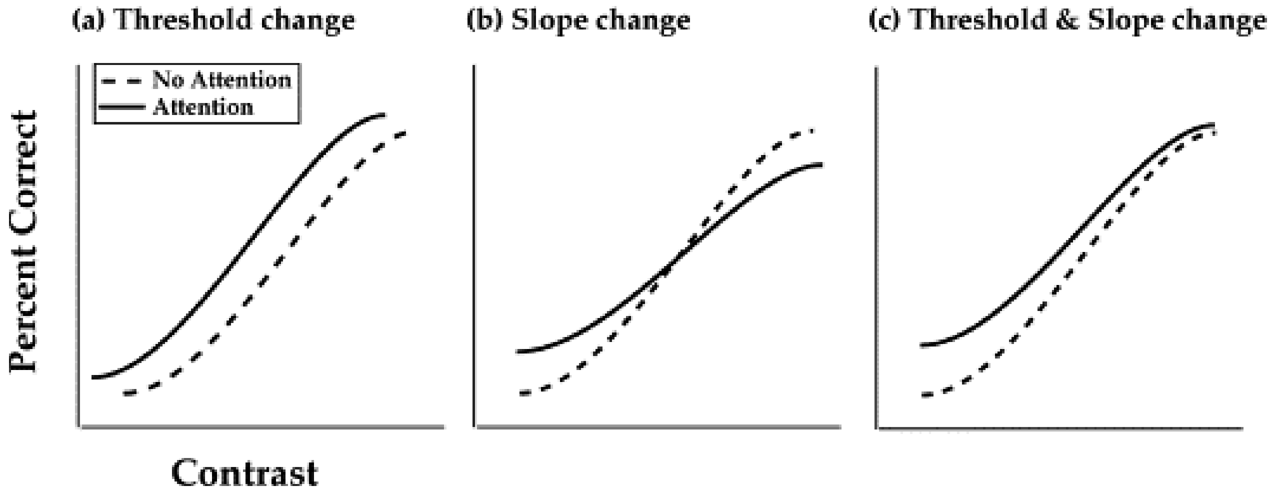
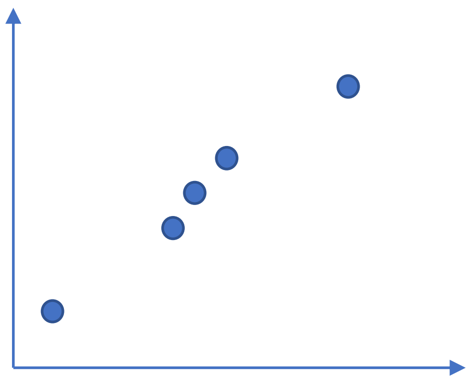


TABLE 4.1 Six values describing a fitted psychometric function

Threshold $\hat{\alpha}$	Slope $\hat{\beta}$	SE threshold	SE slope	Deviance	*p*-value
−1.3789	0.9079	0.0704	0.1582	1.1728	0.7652

What kind of data do we want

ideal



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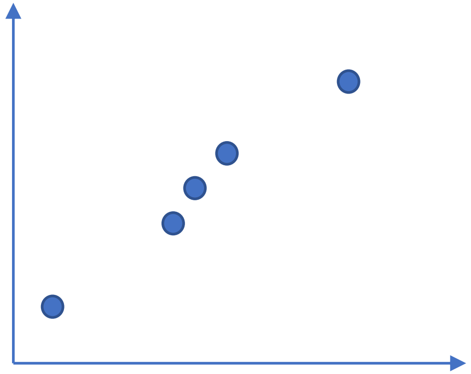
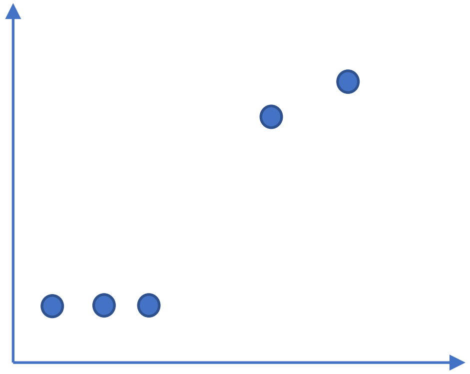


sort of



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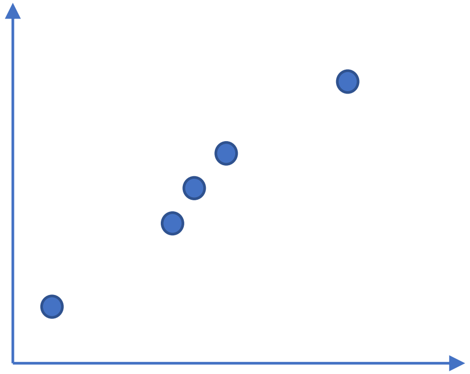

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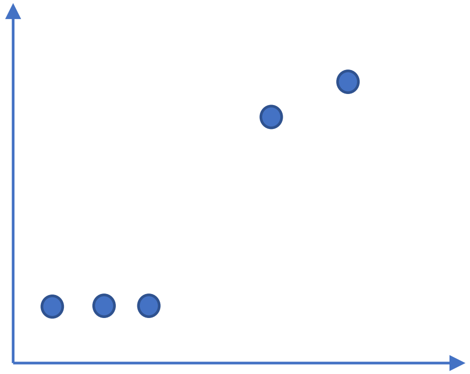
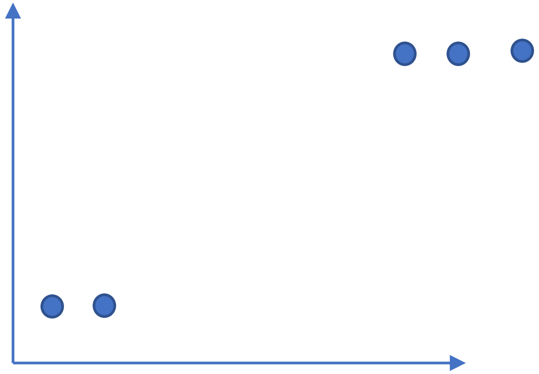
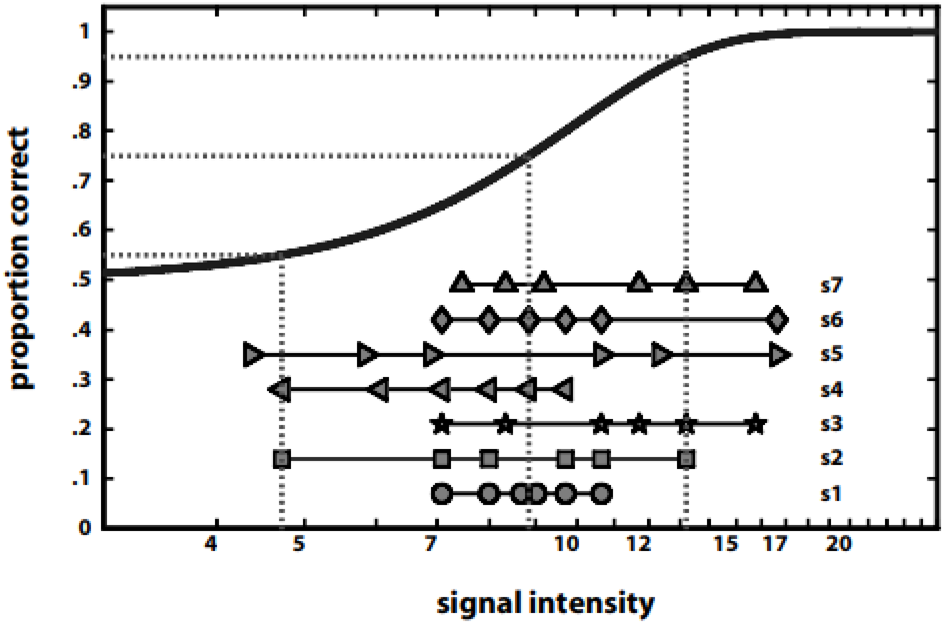


completely wrong

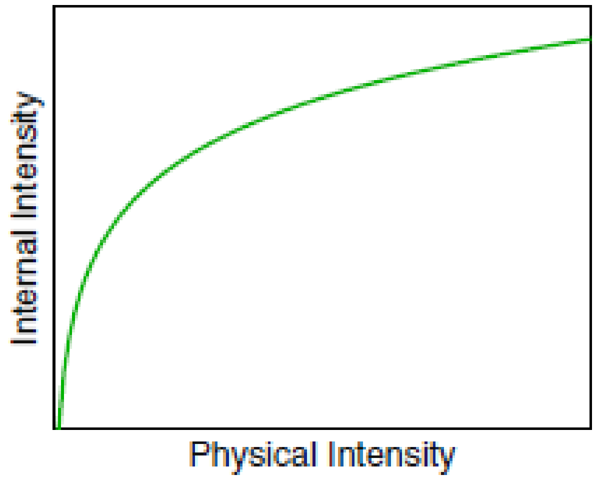


In evaluations of methods for PF



Amount of data and level of stimuli

- Usually, we want data around threshold
- Approximate 400 trials per subject
  - 5 levels
  - 80 trials per level
- Transducer function
  - Function converting real intensity and subjective intensity
- Usually logarithmical scale



Number of levels

$$x_i = 10^{[\log a + (i-1)\log(b/a)/(n-1)]}$$

Lower limit

Upper limit

Dipper function

- Not all transducer look the same

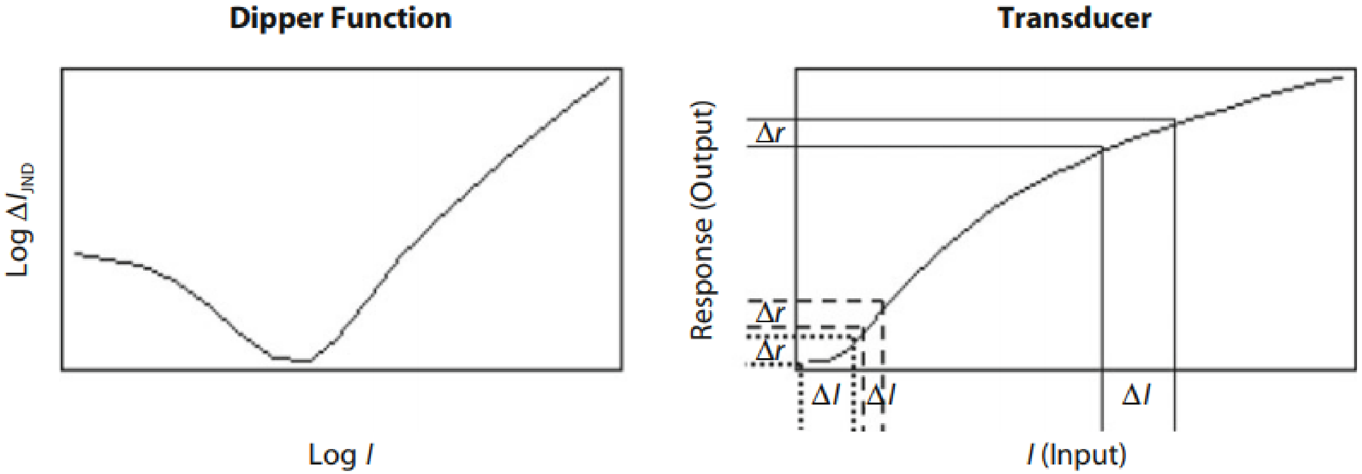


Figure 6. Nonlinear transduction. The transducer panel on the right shows the relationship between stimulus magnitude (on an arbitrary dimension) and the response of one sensory mechanism. In general, two stimuli can be discriminated if and only if the responses they elicit differ by more than some criterion Δr. Negative masking (indicated by the dipper function on the left) happens whenever the sensory response increases faster than the stimulus magnitude. Consider what happens when the pedestal is zero: The just noticeable difference (JND) for detection is represented as the distance between the two vertical dotted lines. The vertical dashed lines are closer together, and thus the JND for small pedestals is smaller. Because the thin solid lines are farther apart, the JND for large pedestals is larger.

Solomon, J. A. (2009). The history of dipper functions. *Attention, Perception, & Psychophysics*, *71*(3), 435-443.

Dipper function

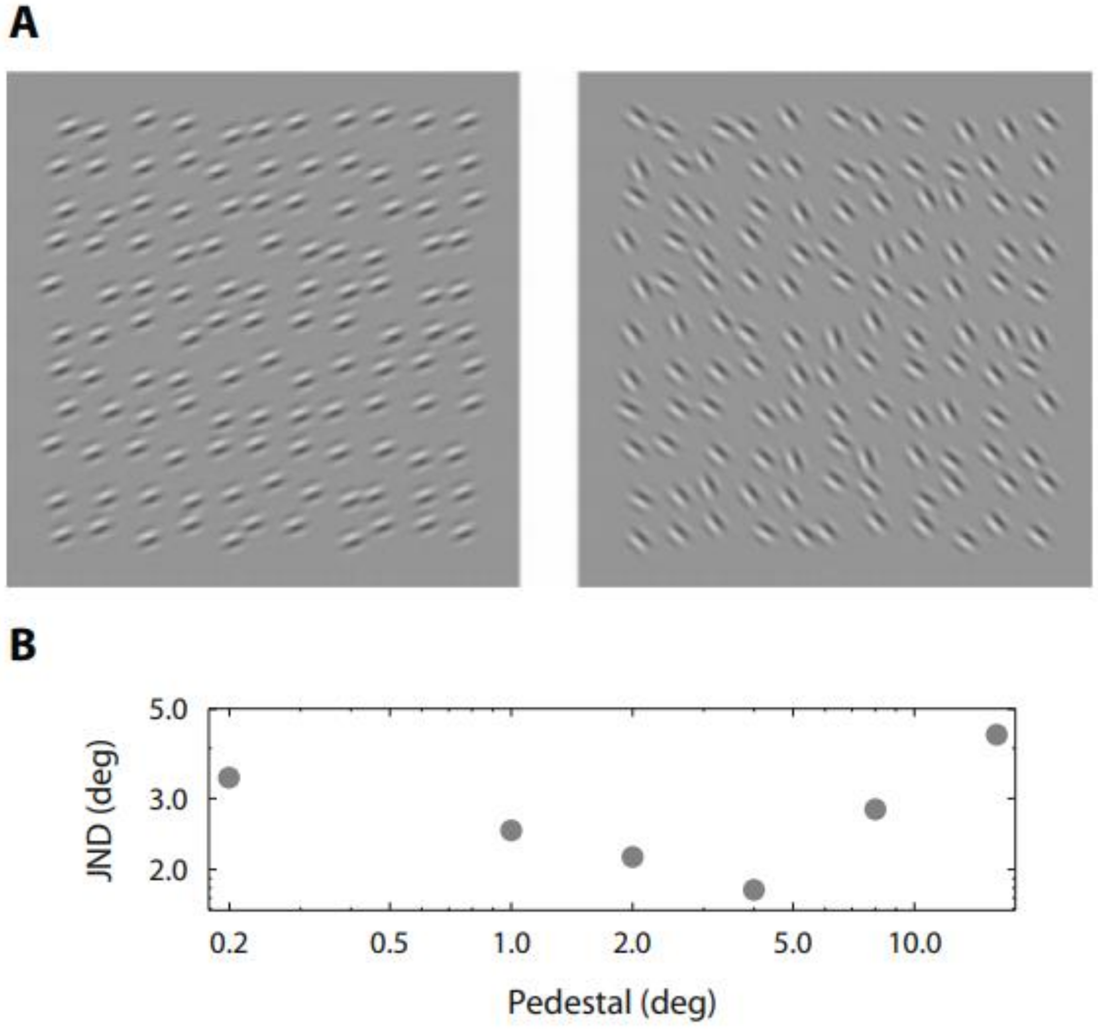


Figure 2. Example stimuli and example results from Morgan et al. (2008). The observers' 2AFC task was to report which of two images had higher variability in orientation. In the example here (panel A), there is zero variability in the image on the left. Orientations on the right were sampled from a Gaussian distribution with an 8º standard deviation. Each point in panel B shows observer M.M.'s (82% correct) just noticeable difference (JND) in the standard deviation.

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Types of functions

- Several types of functions
- Specific function is selected based on:
  - theory
  - „nicely fits data“

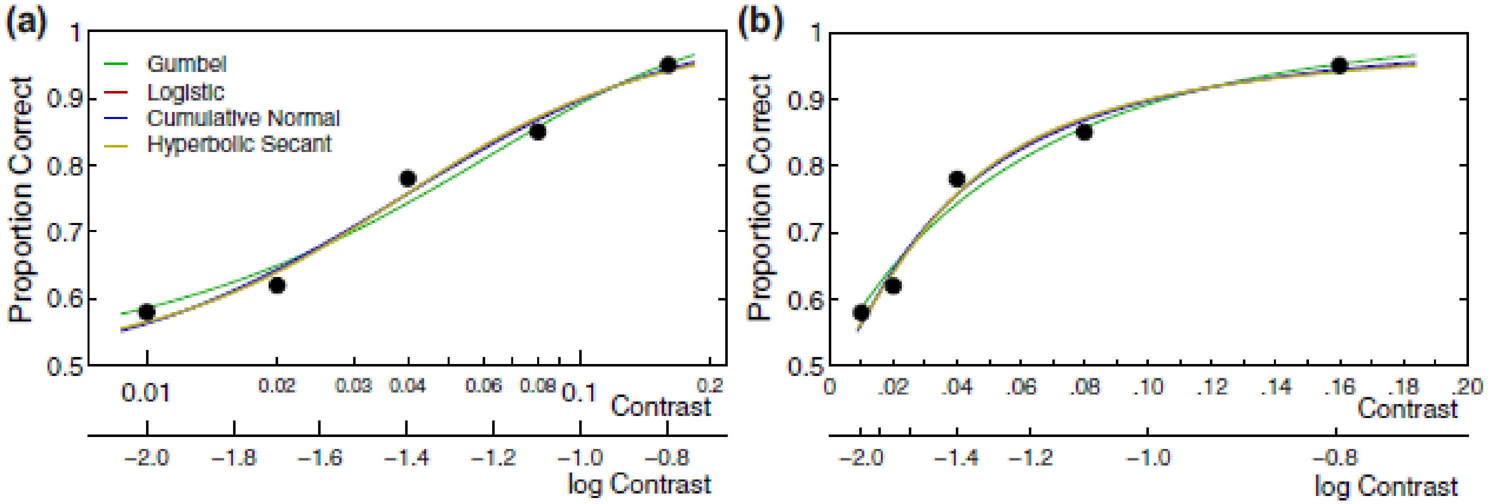


FIGURE 4.3 Example fits of four different PF functions. In (a) stimulus contrast is logarithmically spaced and in (b) contrast is linearly spaced.

How to describe curve + human behaviour

- Usually 4 parameters
  - Threshold / PSE – intensity value where the state changes /when it subjectively looks the same to us
  - Slope – function growth rate (typically in the threshold)
  - Guess rate – γ, what is the probability of success in guessing
  - Lapse rate – λ, procentage of trials, in which participant makes incorrect response

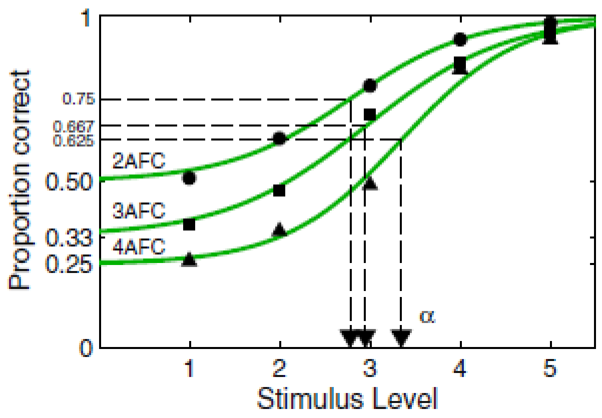


FIGURE 4.4 Example PFs for a 2AFC, 3AFC, and 4AFC task. A Logistic function has been fit to each set of data by using a different value for the guessing parameter, γ (0.5, 0.33, and 0.25, respectively). The lapse rate λ was set to 0 for all three. The threshold α corresponds to proportion correct of 0.75, 0.667, and 0.625, respectively.

PF for appearance tasks

- Guessing rate is not useful -> functions go from zero
- We are not looking for threshold, but point of subjective equivalence (PSE) – when both states seem equal
- Sometimes, y axis is on scale -1 to 1 (0 is PSE) – but this for reporting only, data must be fitted with values 0-1

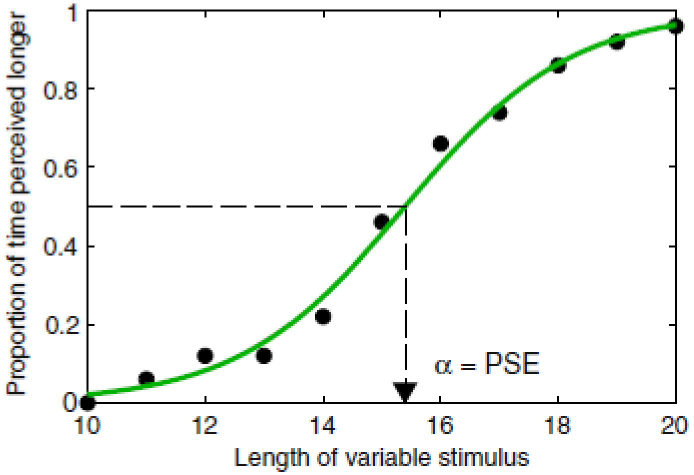


FIGURE 4.5 PF for an appearance-based task.

Theory behind PF

- If x is the stimulus intensity, $\psi(x)$ is the performance for stimulus x
- We're not as interested in performance as we are in the internal sensory response $F(x; \theta)$, where θ are the parameters of the function
- $F(x; \theta)$ we can't measure directly, only $\psi(x)$
- Two theories HTT and SDT

High threshold theory

- 2IFC experiment, Signal S and noise N
- abstraction: we have specialized neurons in brain reacting on S (like a Gaussian distribution)
- N also produces signal, distance is linear with respect to intensity
- We have an internal threshold, and if the sensory signal exceeds it, I'll register it
- I don't know how strong the signal was unless it reaches the threshold

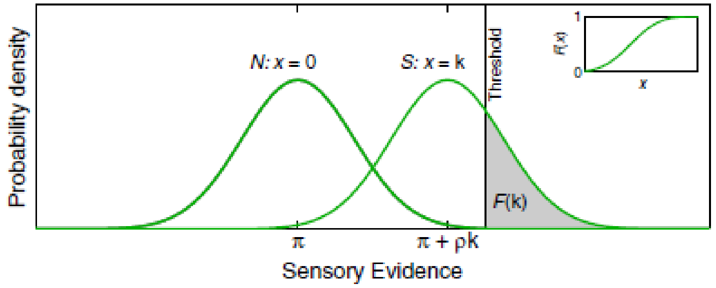


FIGURE 4.6 High-Threshold Theory and the PF.

HTT – guessing rate and lapse rate

- If I don't reach the threshhold, I have to guess (guessing rate)
- Lapse rate – probability of making an error (in theory two types)
  - Sensory inattention
  - Motor error
- Relationship between $\psi(x; \alpha,\beta,\lambda,\gamma)$ and $F(x;\alpha,\beta)$

Breakdown of answers

$$\psi(x;\alpha,\beta,\gamma,\lambda^*) = \gamma + (1-\gamma-\lambda^*+\lambda^*\gamma)F(x;\alpha,\beta)$$

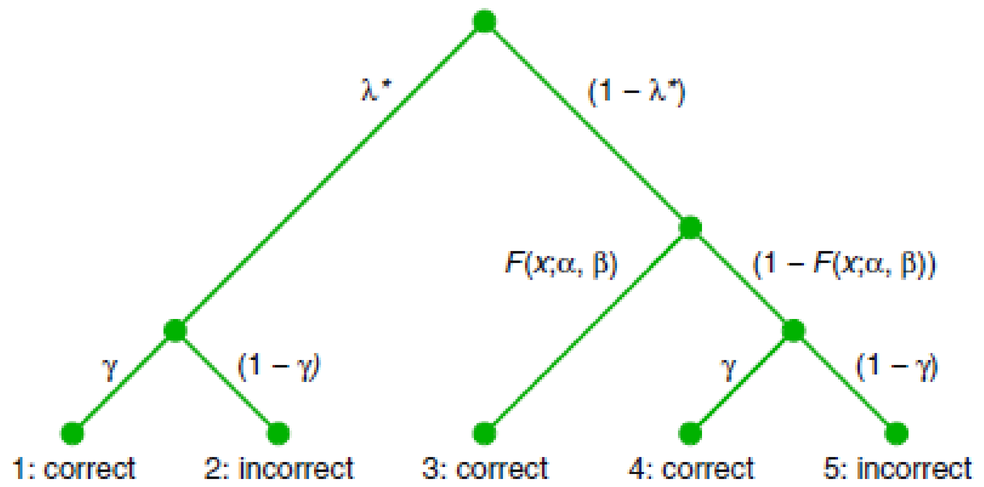


FIGURE 4.7 Relation between $F(x;\alpha,\beta)$ and $\psi(x;\alpha,\beta,\gamma,\lambda)$ according to high-threshold theory.

More common version

- Lapse can also occur when guessing
- So we define λ = λ*(1 – γ)
- λ – probability that the proband will make a mistake
- λ*- motor error

- If a proband sneezed every tenth trial during 2AFC:
  λ = 0.1(1 – 0.5) = 0.05

$$\psi(x;\alpha,\beta,\gamma,\lambda^*) = \gamma + (1-\gamma-\lambda^*+\lambda^*\gamma)F(x;\alpha,\beta)$$

↓

$$\psi(x;\alpha,\beta,\gamma,\lambda) = \gamma + (1-\gamma-\lambda)F(x;\alpha,\beta)$$

Range of PF

- 1-λ is upper assymptote, γ is lower asymptote

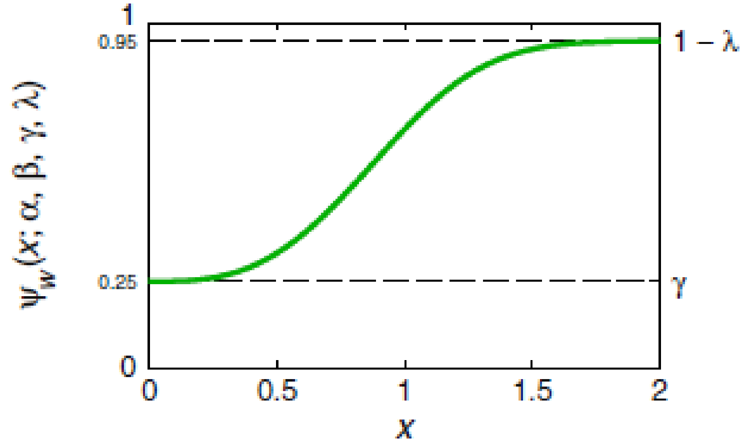


FIGURE 4.8 $\psi_W(x;\alpha,\beta,\gamma,\lambda)$, where F is modeled by the Weibull function $F_W(x;\alpha,\beta)$, threshold $\alpha = 1$, slope $\beta = 3$, guess rate $\gamma = 0.25$, and lapse rate $\lambda = 0.05$.

PF according SDT

- SDT does not have fixed threshold,so we need to take into account both signal and noise
- Every trial, we randomly select answer from N and S distribution and we answer based on higher value
- Difference is also normály distributed

Upper part of cumulative normal function -> different transducer is required

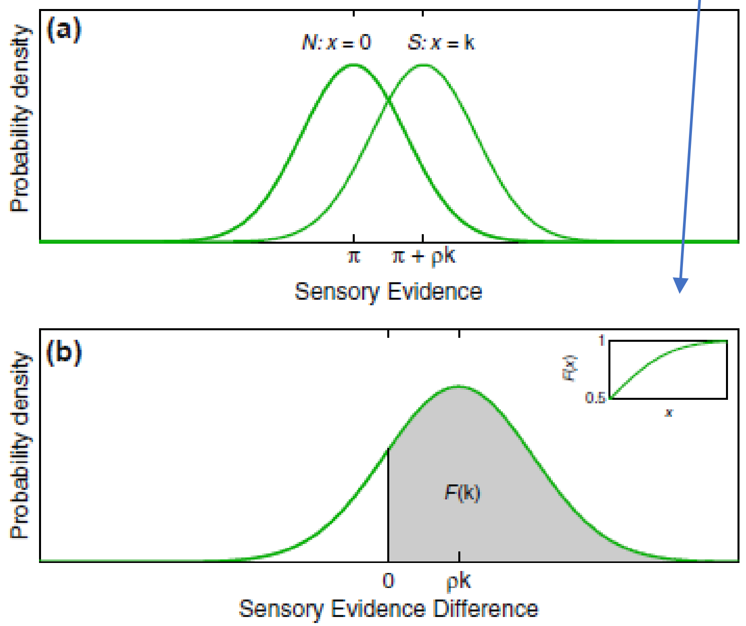


FIGURE 4.9 The relation between SDT and the PF. (a) Probability density functions for the sensory evidence obtained in the noise (*N*) and stimulus (*S*) intervals and (b) probability density function for the difference in sensory evidence obtained between the *N* and *S* intervals.

PF according SDT

- Observer never guess!

$$\psi(x;\alpha,\beta,\gamma,\lambda) = \gamma + (1-\gamma-\lambda)F(x;\alpha,\beta)$$

- Shape of function makes sense for SDT, but γ has different meaning
- Nowadays SDT is commonly used, but we use guessing rate and threshold from HTT
- Correct way would be: *The amount of sensory evidence accumulated while sampling from the signal presentation happened to exceed the amount of sensory evidence accumulated while sampling from the noise presentation*
- But we say: Observer guessed an answer.

What functions can we use?

- We can use several functions for F(x; α,β)
  - Cumulative normal distribution function
  - Logistic function
  - Weibull
  - Gumbel
  - Quick/Log-Quick
  - Hyperbolic Secant $F_{HS}(x;\alpha,\beta) = \frac{2}{\pi}\tan^{-1}\exp\left(\frac{\pi}{2}\beta(x-\alpha)\right)$
  - Our own..

Cumulative normal

$x \in (-\infty, +\infty)$, $\alpha \in (-\infty, +\infty)$, and $\beta \in (0, +\infty)$

$$F_N(x; \alpha, \beta) = \frac{\beta}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{\beta^2 (x-\alpha)^2}{2}\right)$$

No closed form

- Makes sense according to theory

- α – corresponds to thresholdu ($F_N(x = \alpha; \alpha, \beta) = 0.5$)
- β – corresponds to slope

- If x=0 means absence of signal, this can be used x is log-transformed

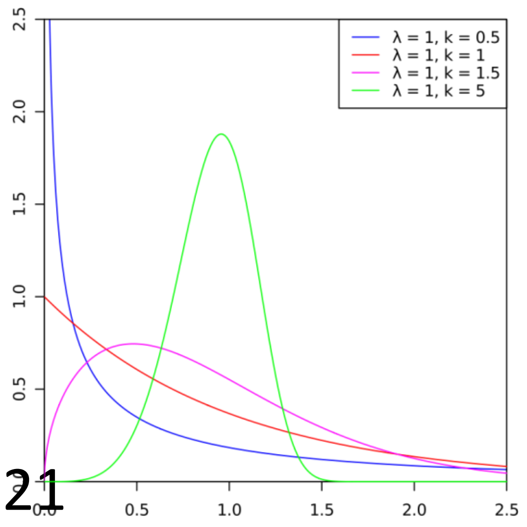
Logistic function

- α – corresponds to threshold ($F_N(x = \alpha; \alpha, \beta) = 0.5$
- β – corresponds to threshold
- It is an approximation of CNF ($\beta_L \approx 1.7/\beta_N$)
- Advantage over CNF is existence of closed form
- Again problem with x=0

$$F_L(x; \alpha, \beta) = \frac{1}{1 + \exp(-\beta(x - \alpha))}$$

$x \in (-\infty, +\infty)$, $\alpha \in (-\infty, +\infty)$, and $\beta \in (0, +\infty)$

Weibull and Gumbel function



- Weibull
  - α – corresponds to threshold ($F_N(x = \alpha; \alpha, \beta) = 1\text{-}\exp(-1) \approx 0.6321$
  - β – corresponds to threshold (but also depends on α)
    - change in α leads to change in slope, even when β is constant

$$F_W(x; \alpha, \beta) = 1 - \exp\left(-\left(\frac{x}{\alpha}\right)^{\beta}\right)$$

$x \in [0, +\infty)$, $\alpha \in (0, +\infty)$, and $\beta \in (0, +\infty)$.

  - Not for log scale, as x ≥ 0
- Gumbel (log-Weibull)
  - Like Weibull, but on logarithmic scale

$$F_G(x; \alpha, \beta) = 1 - \exp\left(-10^{(\beta(x-\alpha))}\right)$$

$x \in (-\infty, +\infty)$, $\alpha \in (-\infty, +\infty)$, and $\beta \in (0, +\infty)$

Quick a Log-Quick

- Quick
  - Here is threshold 0.5

$$F_Q(x;\alpha,\beta) = 1 - 2^{\left(-\left(\frac{x}{\alpha}\right)^{\beta}\right)}$$

$$F_{lQ}(x;\alpha,\beta) = 1 - 2^{\left(-10^{(\beta(x-\alpha))}\right)}$$

- Log-Quick
  - Again log version of Quick

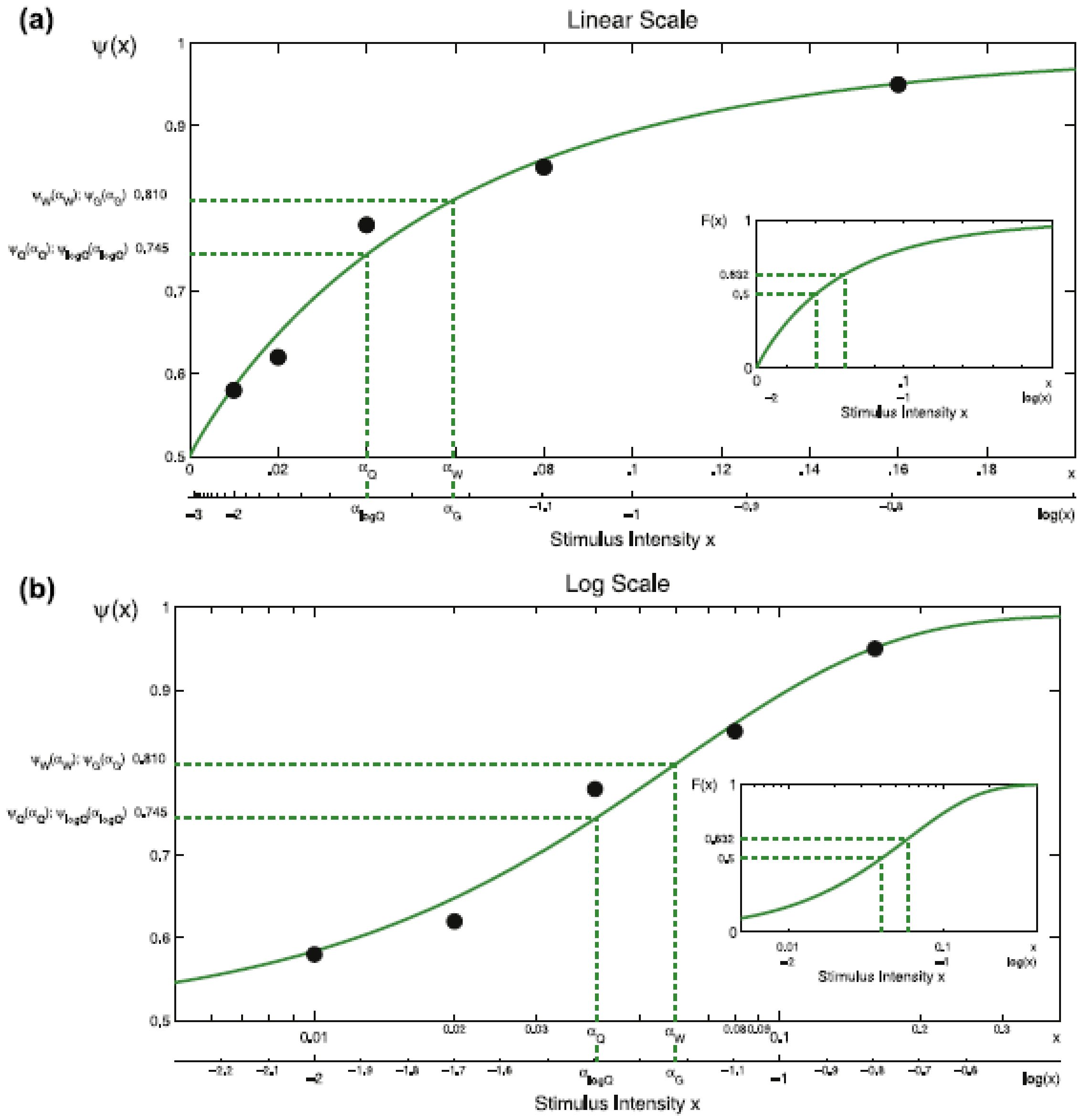
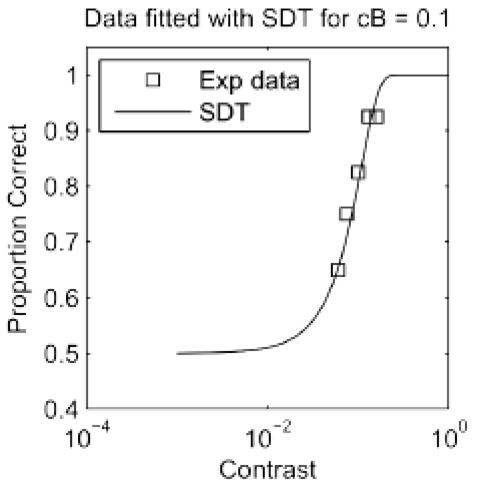


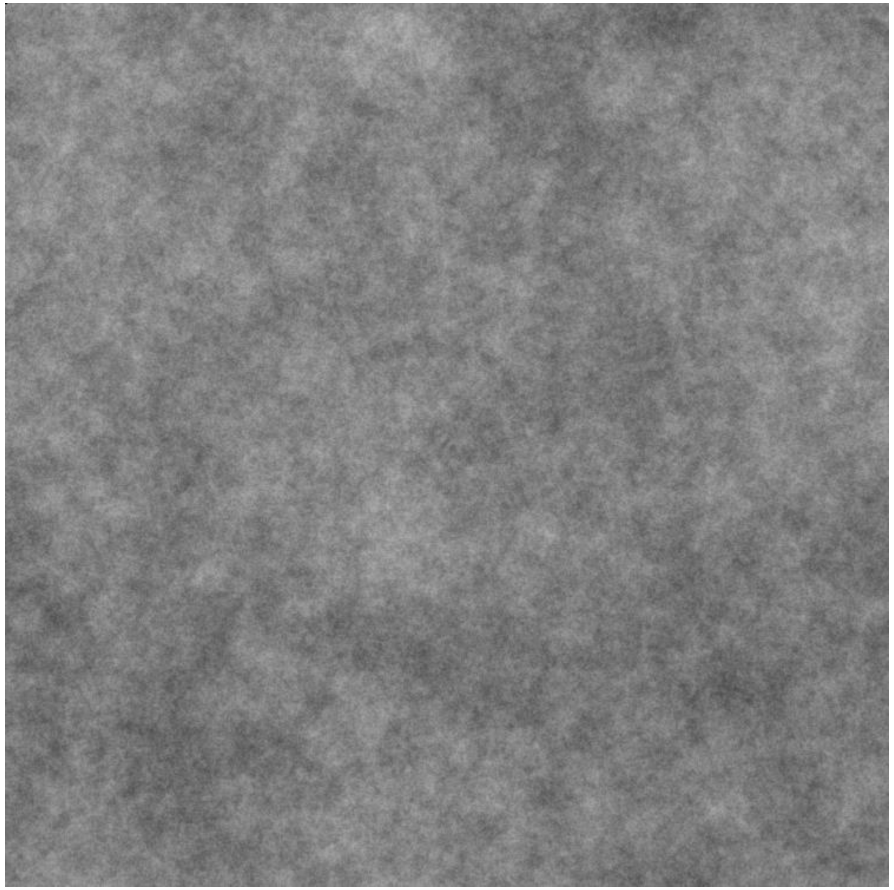
FIGURE B4.5.1 The relationships among the PFs in the Weibull family: Weibull, Gumbel, Quick, and log-Quick functions. Functions are plotted on linear scale (a) as well as logarithmic scale (b).

Custom function

- If we have correct theory, we can create PF based on that



$$P_{corr}(c; c_T, \beta) = \Phi(0.5 \cdot (\frac{c}{c_T})^{\beta})$$



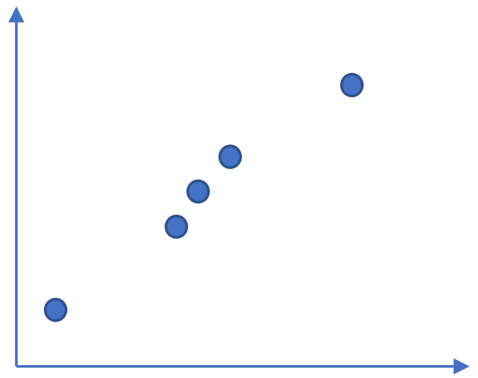
PF and variance

- Because β is not transferable between functions (β=2 in CNF means steeper slope than β=2 in logistic)
- Ve take some predefined range of data

$$\sigma = \psi^{-1}(1-\lambda-\delta;\alpha,\beta,\gamma,\lambda) - \psi^{-1}(\gamma+\delta;\alpha,\beta,\gamma,\lambda).$$

How to find the best curve?

- Maximum likelihood estimate (MLE)
- Likelihood: P(data|θ), where θ are free parameters)
  - Dat are fixed – compare with probability, where we try to predict the data
- Used as a common tools to estimate parameters



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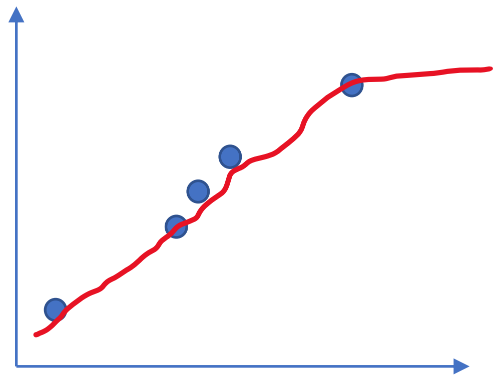

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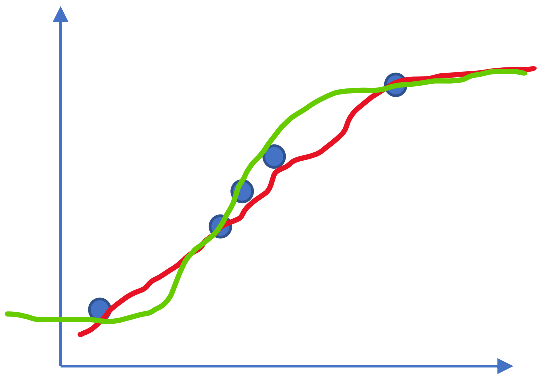
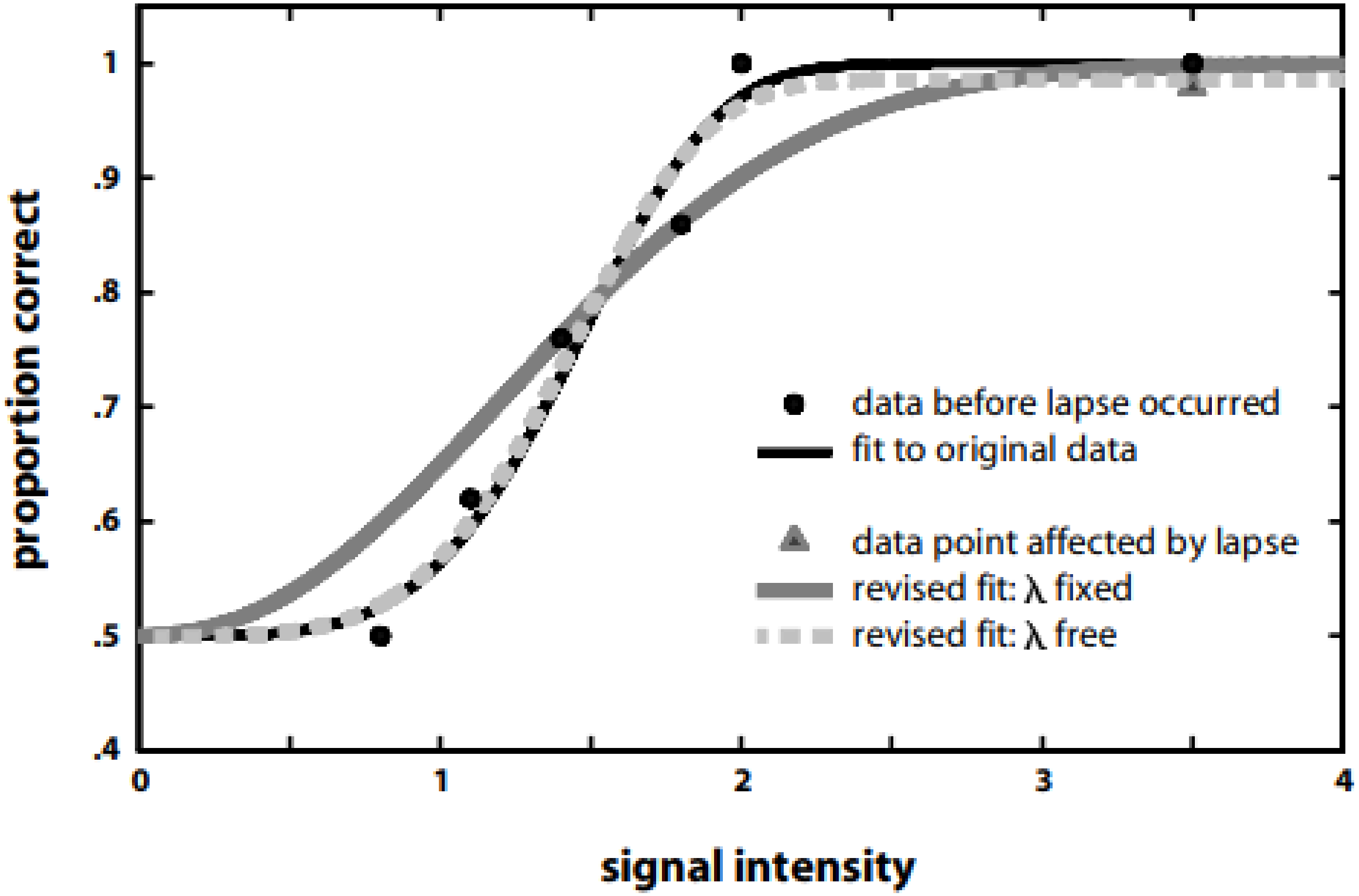


Lapse rate can change the fit

- Lapse rate is a problem for upper asymptote
- Alternative:

$$\psi(x) = \gamma + (1 - \gamma - \lambda)F(x\,;\alpha,\beta) \quad \text{when} \quad x < \text{APL}$$
$$\psi(x) = 1 - \lambda \quad \text{when} \quad x = \text{APL},$$



When fit is not good

- It is important to have good stimuli levels

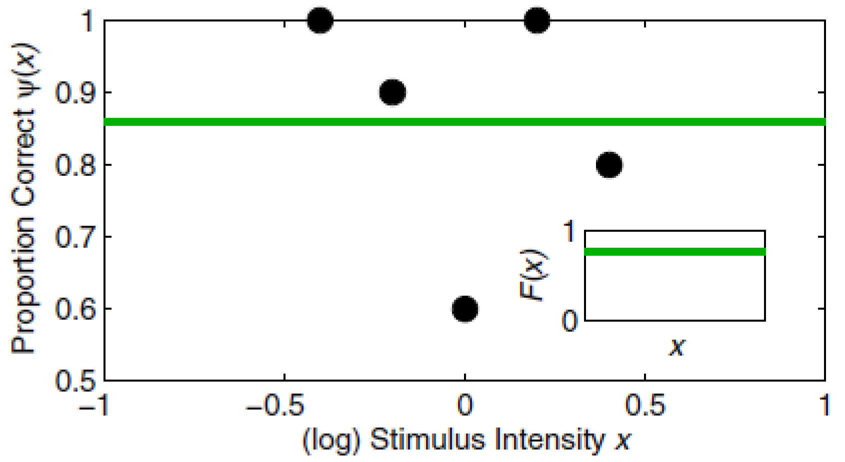


FIGURE B4.7.4 The green line shows a Gumbel function with threshold equal to −4550.9613 and slope equal to 0.000035559, guess rate equal to 0.5, and lapse rate equal to 0.03. Within the stimulus range shown, it is asymptotically near a horizontal line at 0.86, the best fit to these data that can be obtained under the constraint that a Gumbel function is to be fit. See text for details.

Example dataset