Measuring similarity

Scaling

- Sometimes we wonder how we can scale the intensity of a stimulus to the perceptual level. For one particular threshold level, we can do this with PF
- Steven's law $\psi = aS^{\tau}$

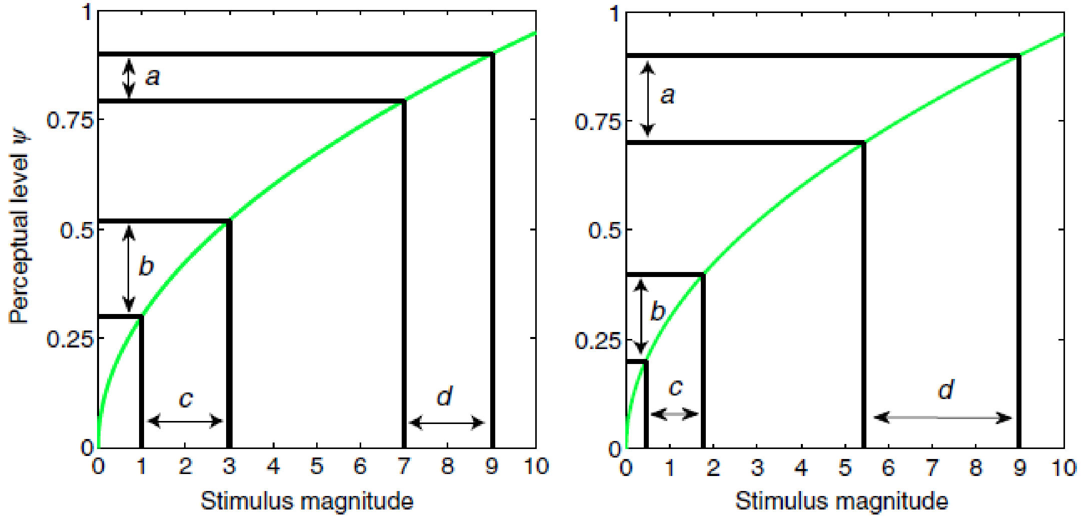


FIGURE 8.1 Hypothetical perceptual scale. Left: two pairs of stimuli (*c*, *d*) with the same physical difference produce different values of perceived difference (*b*, *a*). Right: two pairs of stimuli with different physical difference (*c*, *d*) produce equal perceptual differences (*b*, *a*).

Discrimination scaling

- We start at some baseline S1, find the JND at level S2, and this is how we get the whole relationship

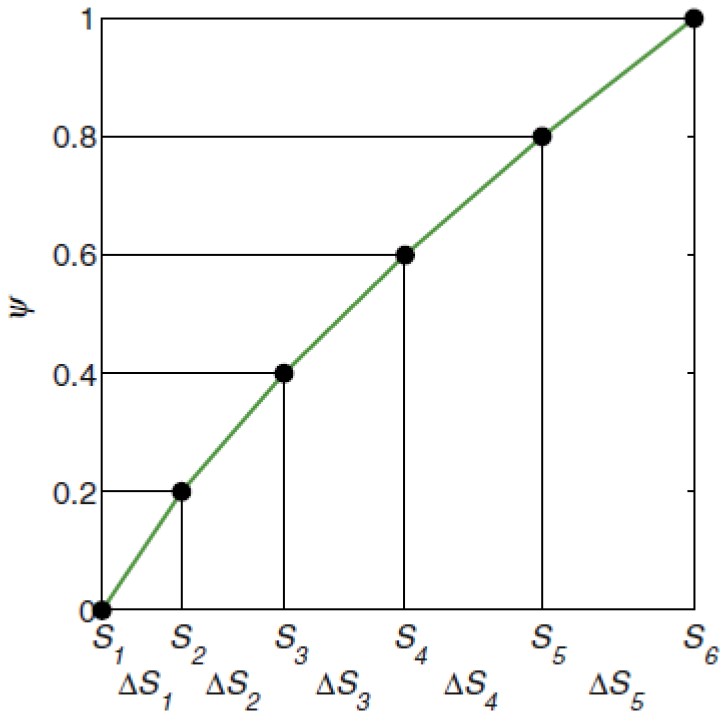


FIGURE 8.2 Constructing a discrimination scale by summing JNDs. $S_{1\ldots6}$ are baselines and $\Delta S_{1\ldots5}$ are discrimination thresholds.

Weber’s law

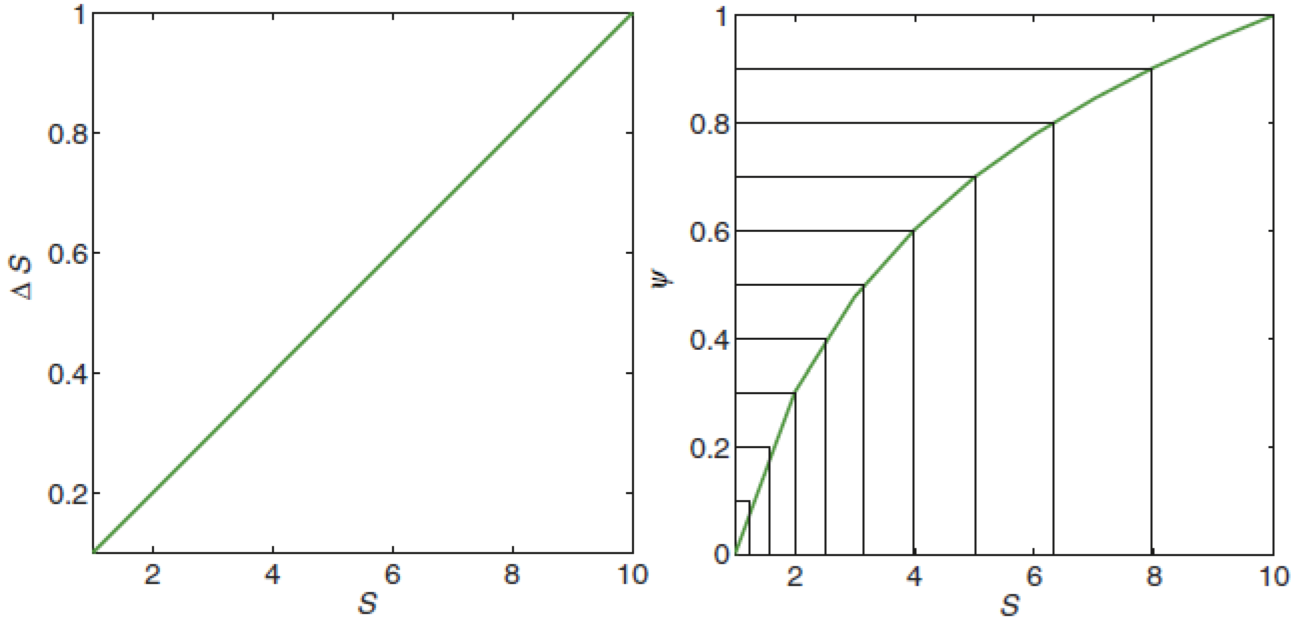
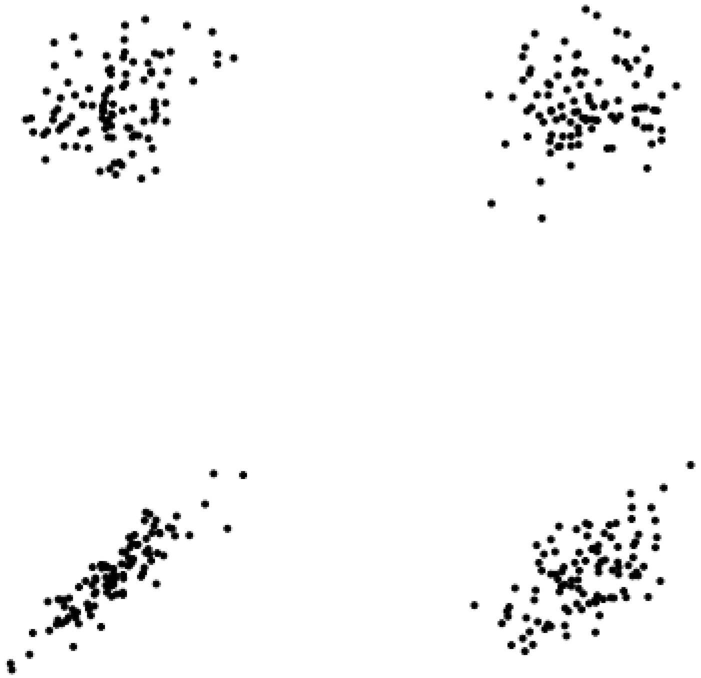


FIGURE 8.3 Left: Weber’s Law: ΔS is proportional to S. Right: Fechner’s Law: ψ is proportional to the logarithm of S. On the right, the intervals on the abscissa between Ss or ΔSs increase proportionately with S. When these are mapped onto equal perceptual intervals via the horizontal lines the function mapped out is logarithmic.

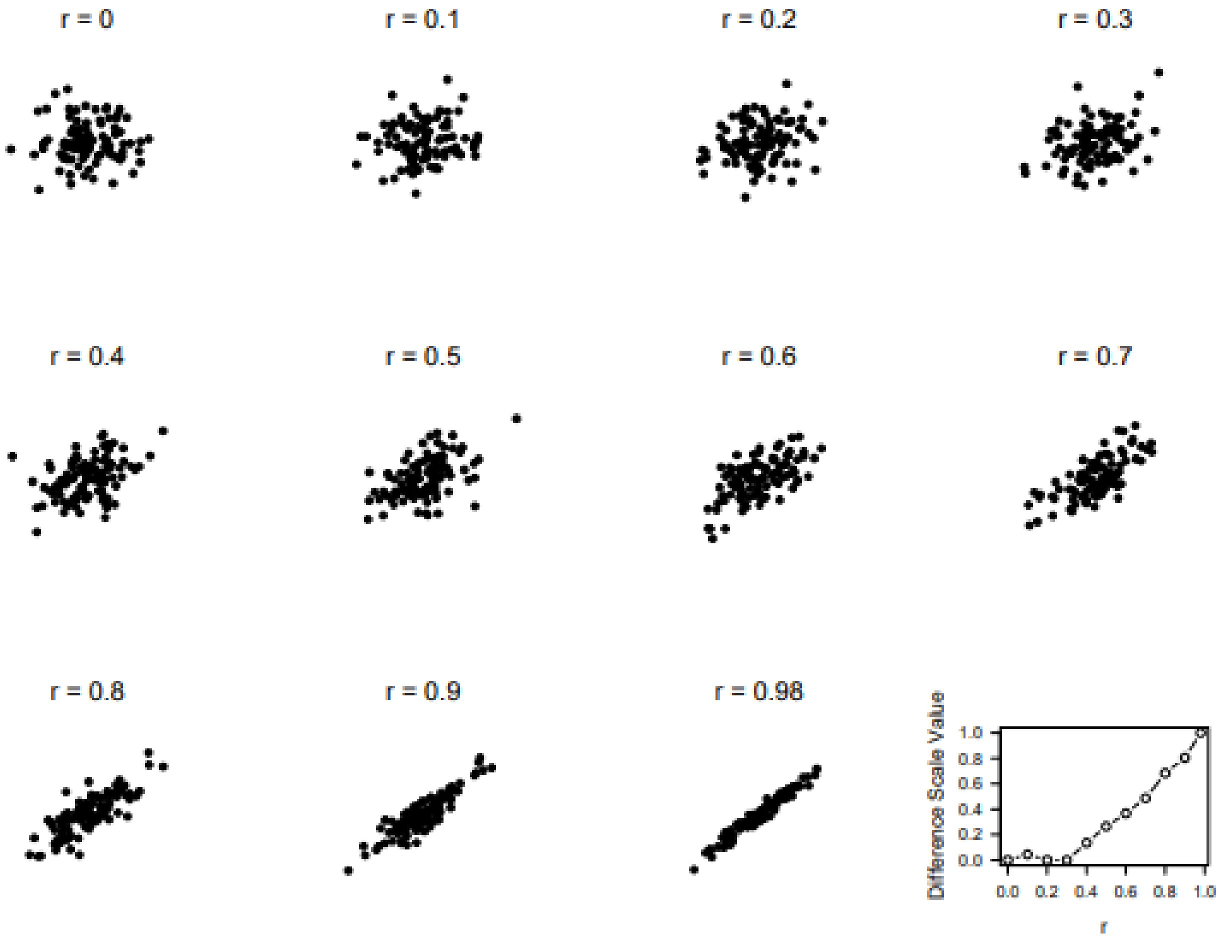
Maximum Likelihood Difference Scaling (MLDS)

- Complex method for scaling
- Eg - how do we recognize correlation coefficients?
- Always display two pairs of stimuli and compare which pair has the greater perceptual distance



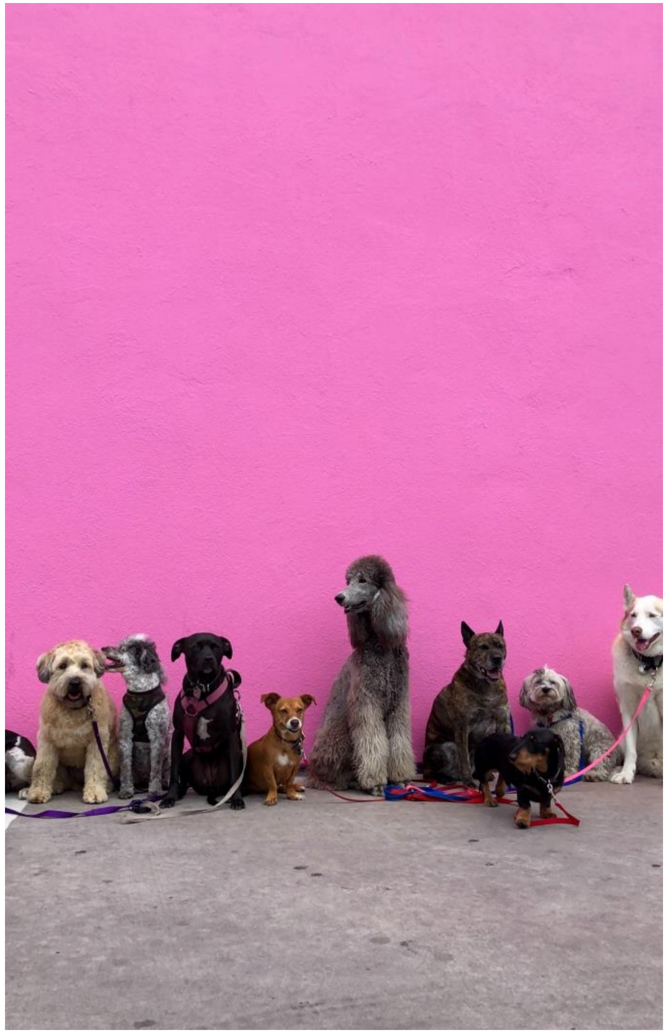
MLDS

- The number of pairwise comparisons increases rapidly (even if only non-overlapping pairs are used, in which case 330 comparisons need to be made - about 10-12 minutes)



Scaling for complex stimuli

vs



Which pair is more similar?

How would you measure similarity?

How would you measure similarity?

- „subjective“ – ask about similarity
- „objective“ – quantify somehow each stimuli and compute similarity between them

How would you measure similarity?

How similar are these images? (these dogs)

1 - very disimilar

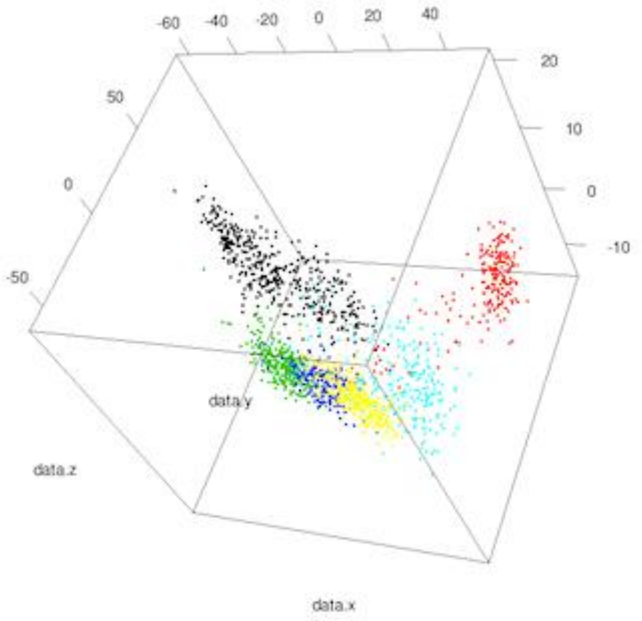
7 - very similar

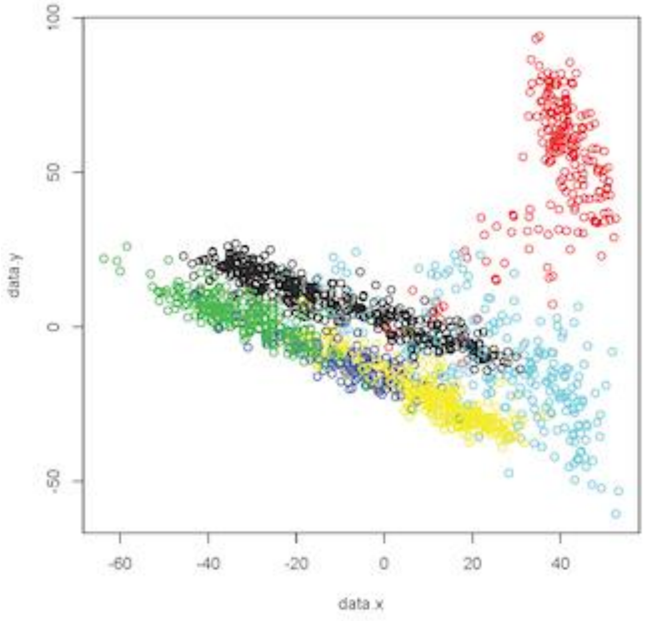
How would you measure similarity?

	Description	Pros	Cons
(1) Pairwise similarity judgment	Each pair of items is presented in isolation and the subject rates the dissimilarity on a scale	• Each pair is independently rated (this is a pro, if set context is thought to distort judgments or a con, if set context is thought to anchor and inform judgments)	• Slow: $(n^2 - n)/2$ separate judgments* required, thus only feasible for small item sets • Interpretation of the dissimilarity scale may drift as previous judgments are not visible for comparison

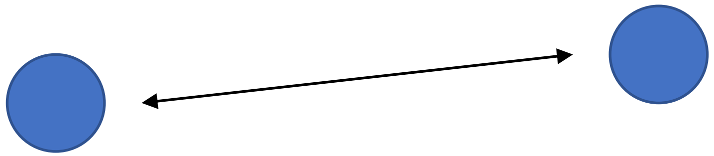
Sidenote - MDS

- We assume that similarity is multidimensional, but it is difficult to visualize
- We can reduce the complexity for visualization and multidimensional scaling technique



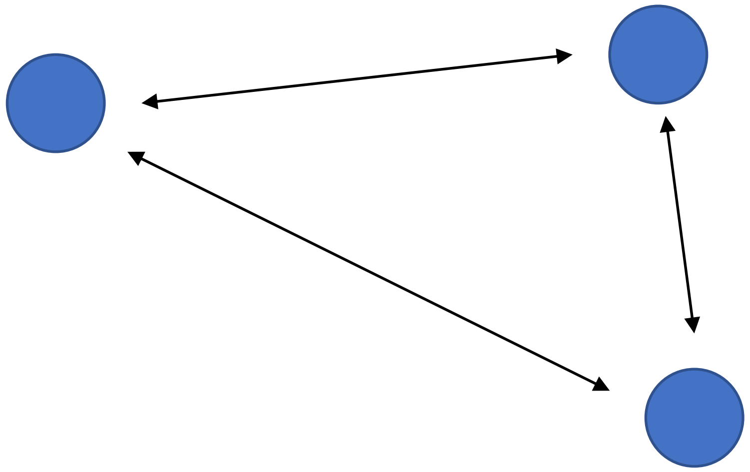


How would you measure similarity?



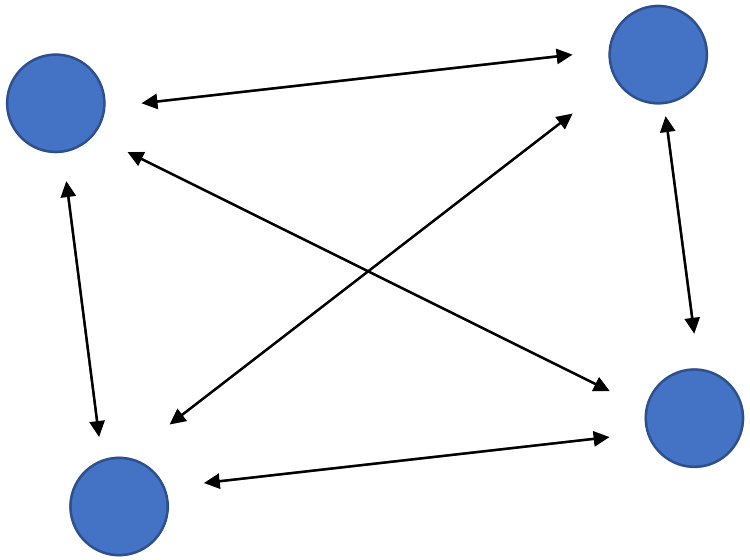
1 comparison

How would you measure similarity?



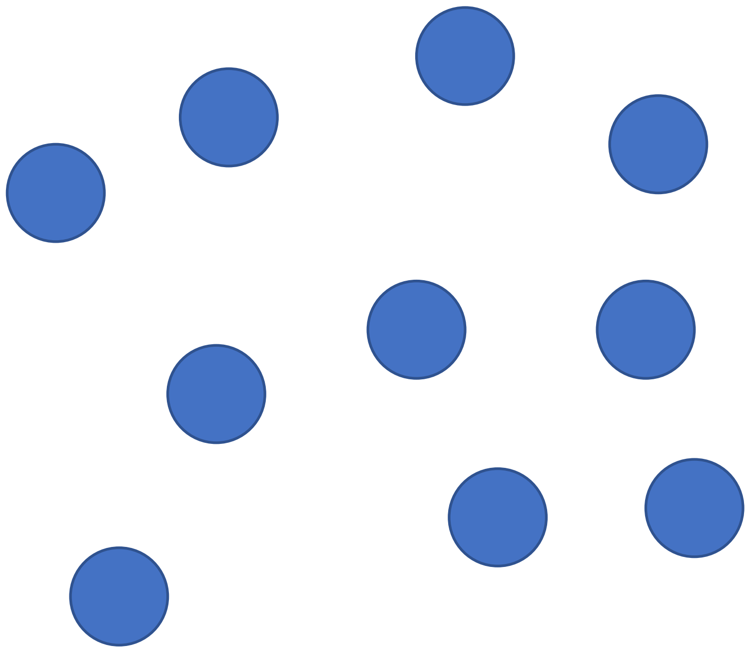
3 comparisons

How would you measure similarity?



6 comparisons

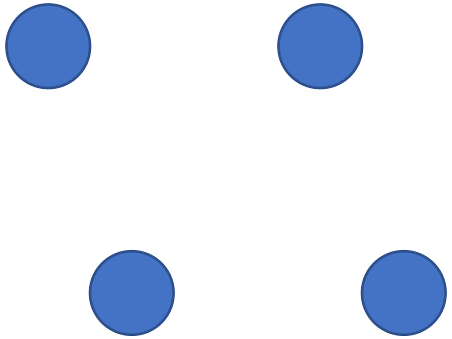
How would you measure similarity?

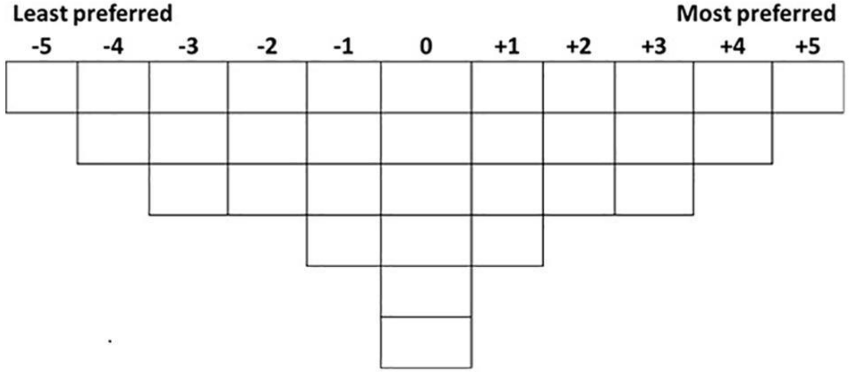


10 items - 45 comparisons
100 items – 4950 comparisons

Sorting methods

- Subjects sort stimuli into piles based on given criteria
  - Q sorting
  - Similar/dissimilar piles
  - Hierarchical ranking





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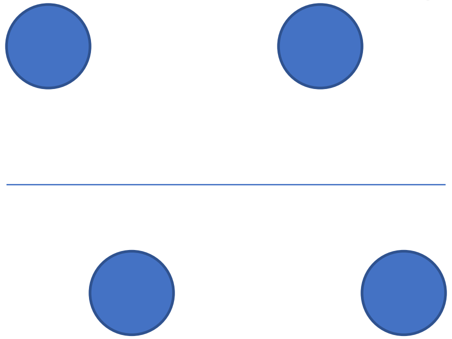
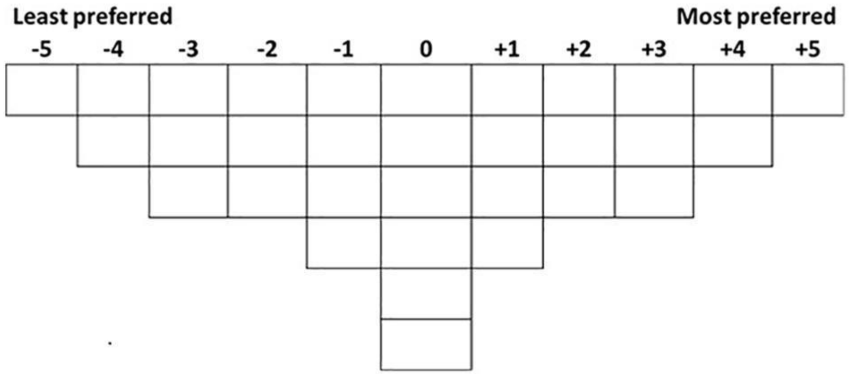


better

worse



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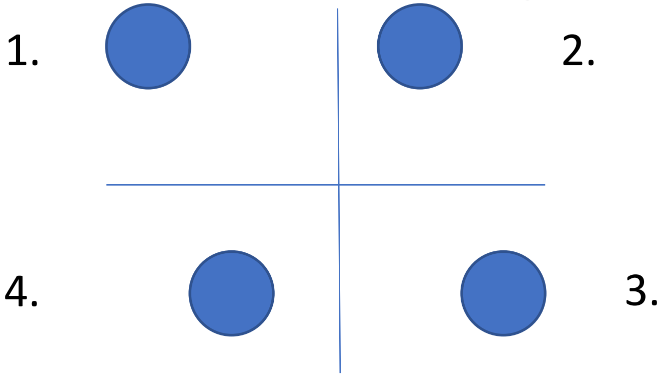
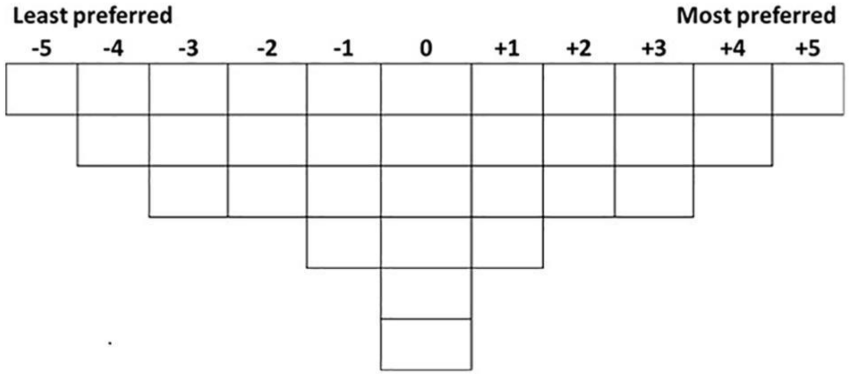




Sorting methods

- Subjects sort stimuli into piles based on given criteria
  - Q sorting
  - Similar/dissimilar piles
  - Hierarchical ranking

	Description	**Pros**	**Cons**
(2) Free sorting	The subject sorts the items into a freely chosen number of piles (i.e., categories)	• Quick: requires only *n* placements*, thus has essentially linear time complexity (neglecting the time taken to decide the categories), thus feasible for large item sets	• Gives only binary dissimilarities (same pile, different pile) for a single-subject • Category definition might be dominated by the first items and might drift if piles are perceived to be represented by the item on top

Single arrangement

- Spatial Arrangement Method (SpAM)

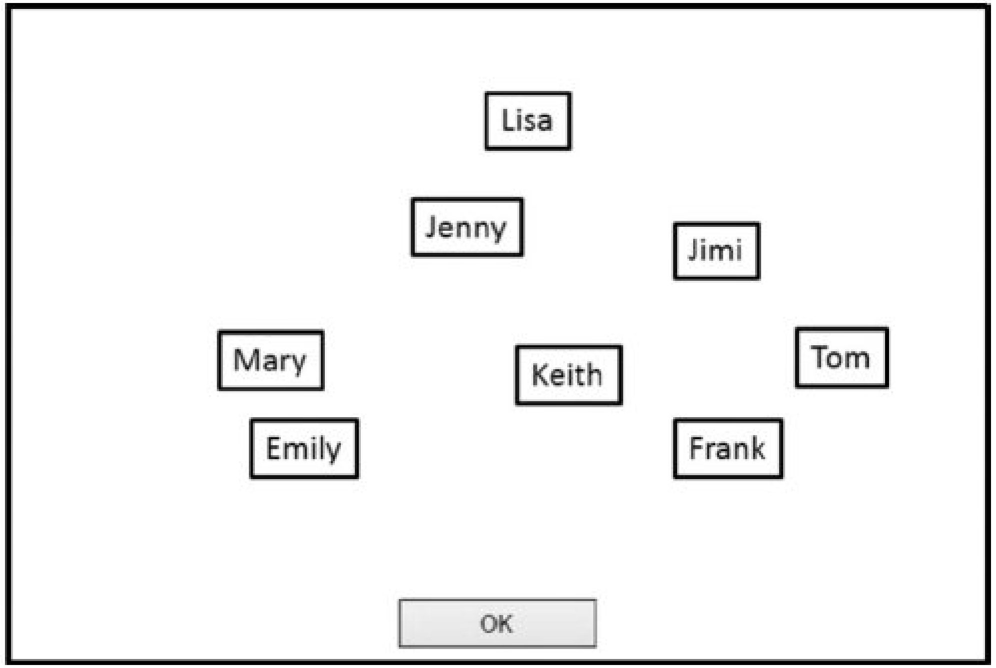
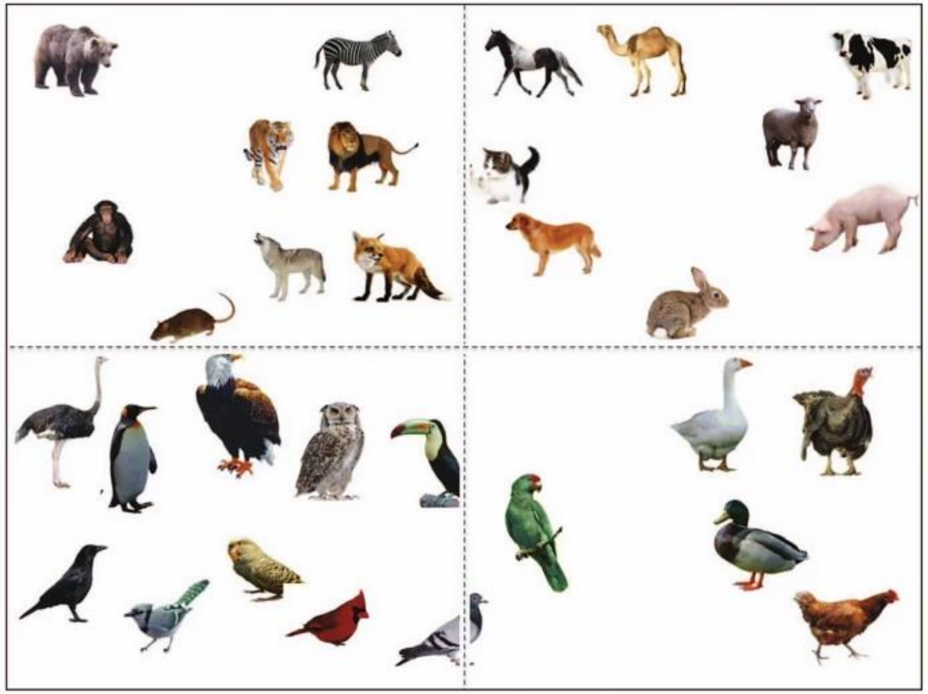


Fig. 7.



A SpAM trial of animals that conveys information about four dimensions. The main two dimensions distinguish mammals (*top*) from birds (*bottom*) and wild (*left*) from domestic (*right*). Within each quadrant, animals are also arranged according to size (y-axis) and whether they are commonly eaten by people (x-axis)

Alves, H., Koch, A., & Unkelbach, C. (2016). My friends are all alike—the relation between liking and perceived similarity in person perception. *Journal of Experimental Social Psychology*, *62*, 103-117.

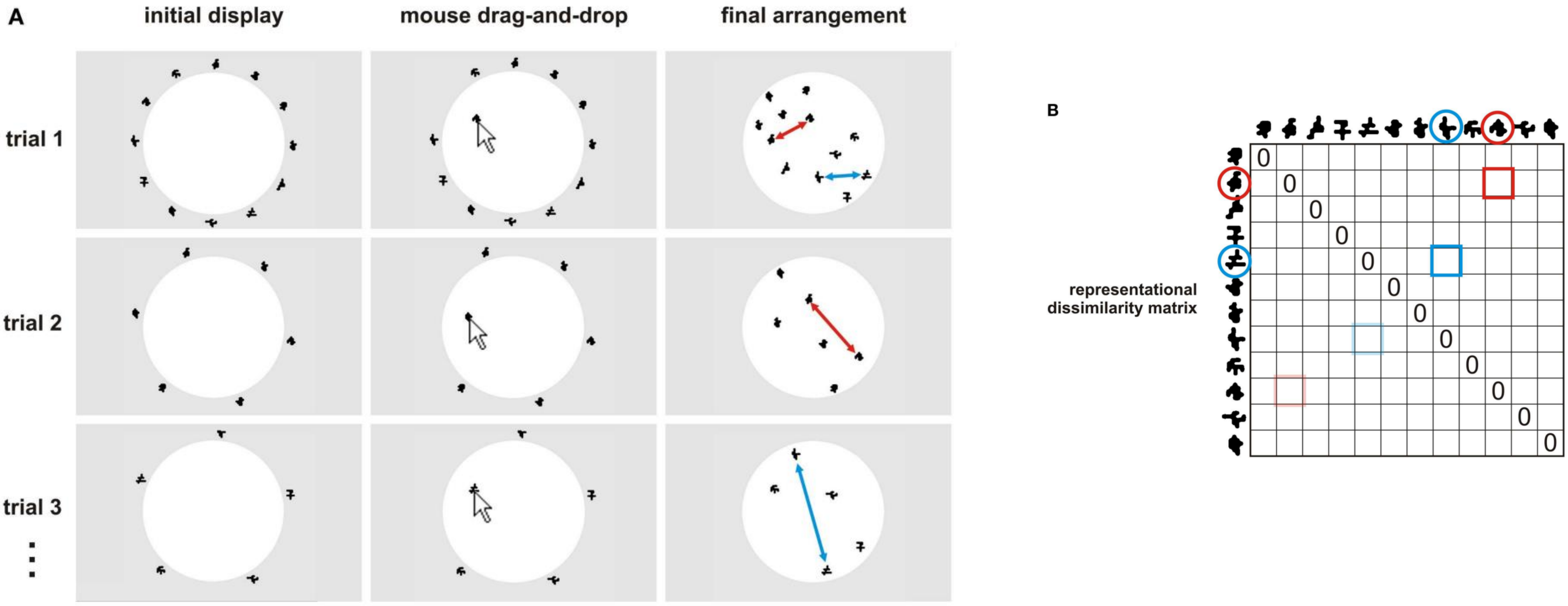
Richie, R., White, B., Bhatia, S., & Hout, M. C. (2020). The spatial arrangement method of measuring similarity can capture high-dimensional semantic structures. *Behavior research methods*, *52*, 1906-1928.

Single arrangement

- Spatial Arrangement Method (SpAM)

	Description	Pros	Cons
(3) Single arrangement	The subject arranges the items in 2D with the distances taken to reflect the dissimilarities	• Relatively quick: each placement of an item communicates multiple dissimilarity judgments (superlinear, but subquadratic time complexity) • The relationships of multiple pairs are considered in context	• Restriction to 2D prevents communication of higher-dimensional dissimilarity structures

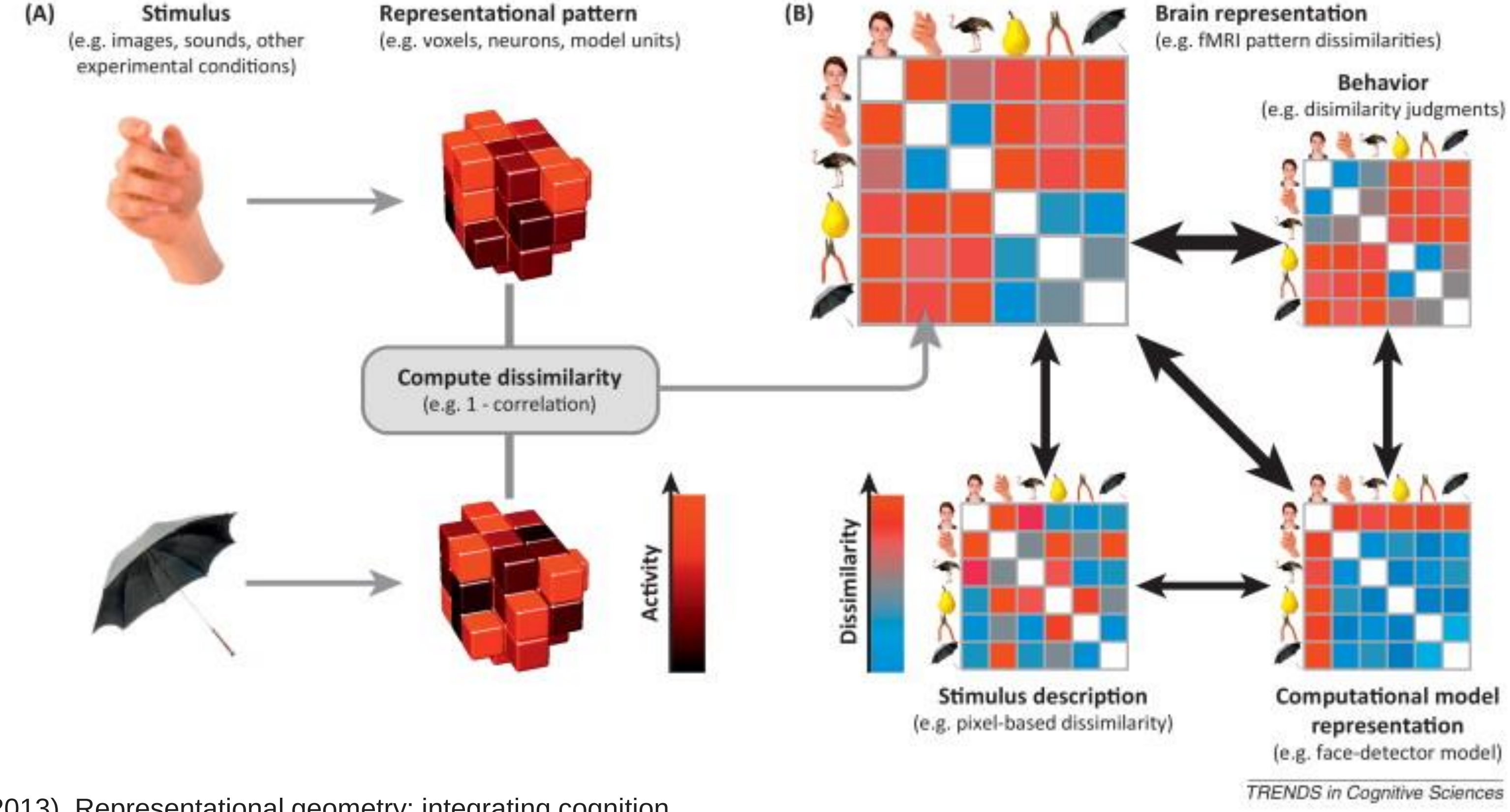
Multiple arrangement



Kriegeskorte, N., & Mur, M. (2012). Inverse MDS: Inferring dissimilarity structure from multiple item arrangements. *Frontiers in psychology*, *3*, 245.

Representational dissimilarity matrix (RDM)

- Commonly used to compare behavior and neural response



Kriegeskorte, N., & Kievit, R. A. (2013). Representational geometry: integrating cognition, computation, and the brain. *Trends in cognitive sciences*, *17*(8), 401-412.

Multiple arrangement

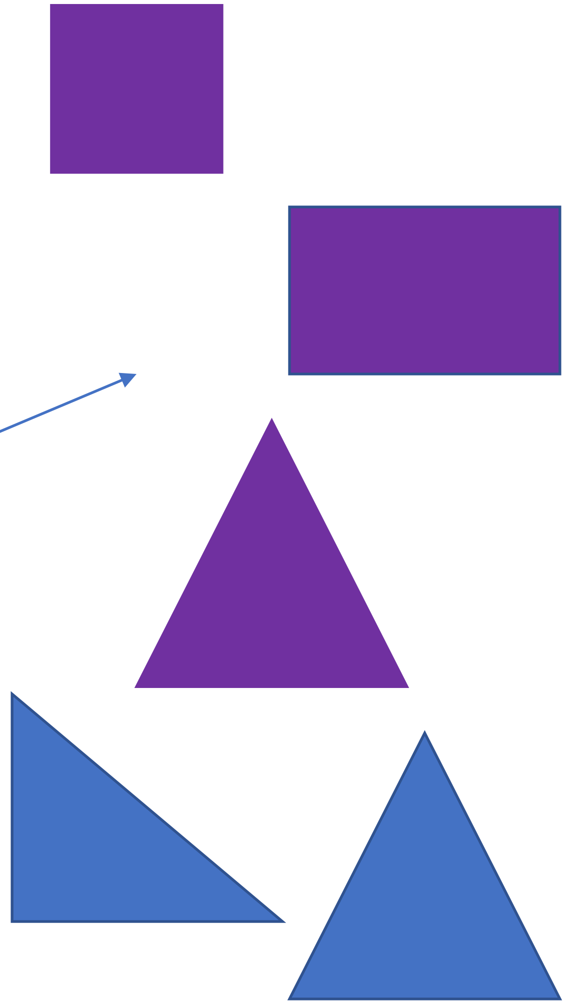
	Description	Pros	Cons
(4) Multi-arrangement (proposed method)	A generalization of (1), (2), and (3), in which multiple item subsets are arranged in a low-dimensional (e.g., 2D) space and the dissimilarity structure is inferred from the redundant distance information	• Includes methods (1)–(3) as special cases, so cannot do worse • Enables us to quickly acquire judgments reflecting higher-dimensional dissimilarity structures • Anytime behavior: process can be terminated anytime after a first trial containing all items (=single arrangement) • Addresses the cons of methods (1), (2), and (3)	• Requires a method for constructing subsets (which may involve assumptions that affect the results) • Requires a method for estimating the dissimilarity structure from multiple item-subset arrangements (which may involve assumptions that affect the results)

Stimuli in similarity space

- Although we have measured similarity, there are some features that could describe each stimulus

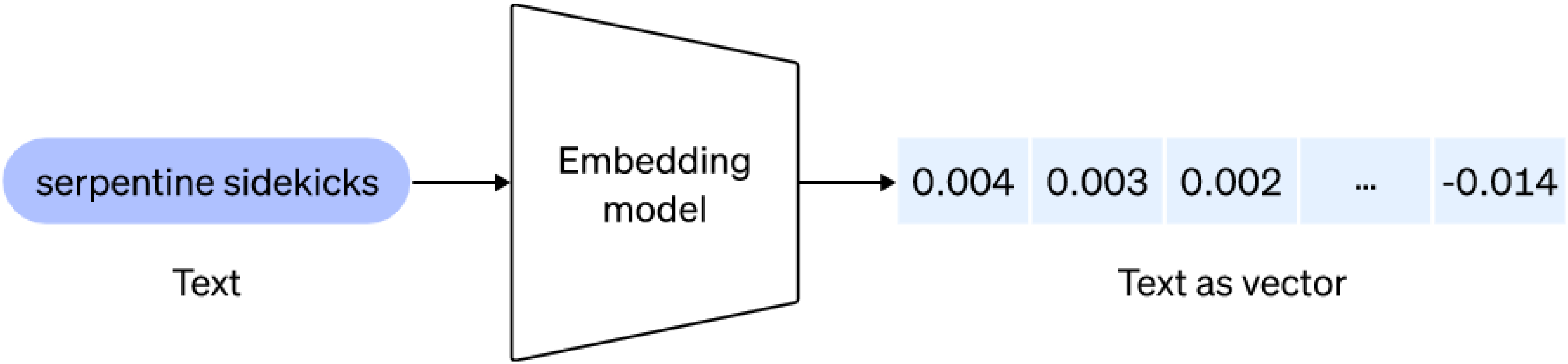
- E.g.: number of sides, colour

- But what should we use for complex stimuli?



Representational embeddings

- Reduce complex stimuli into several continuous variables



Embeddings of similarity space

- Pairwise comparisons are “expensive”
- Idea: measure subset of data and fill the missing values

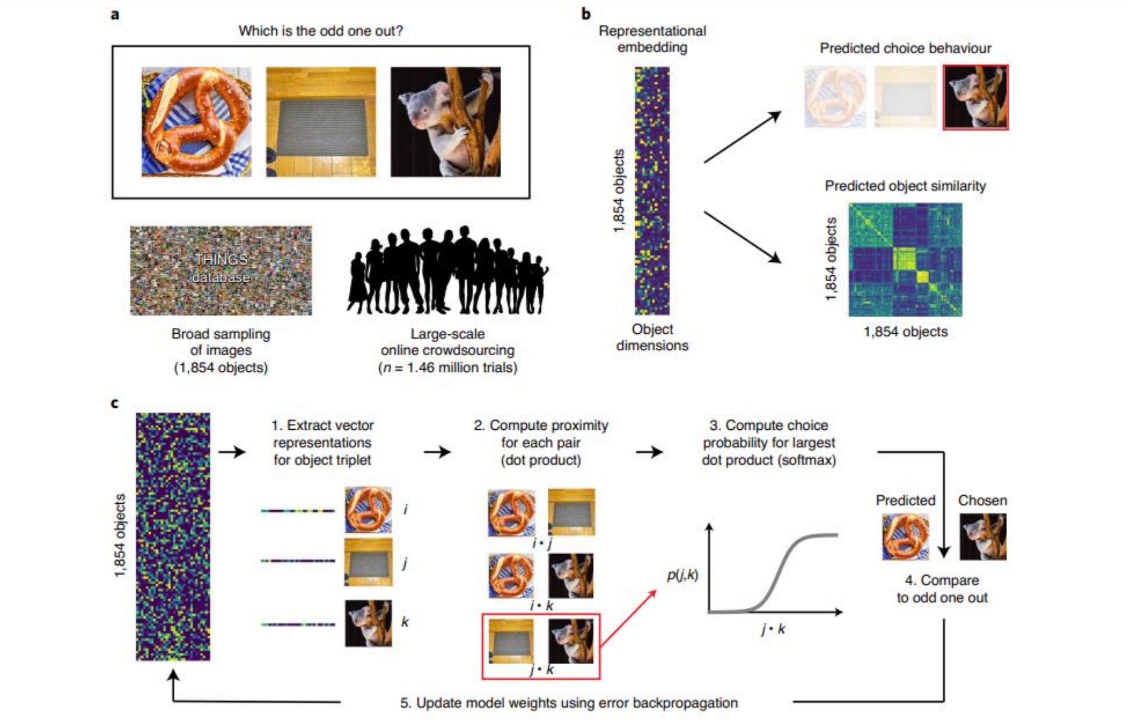
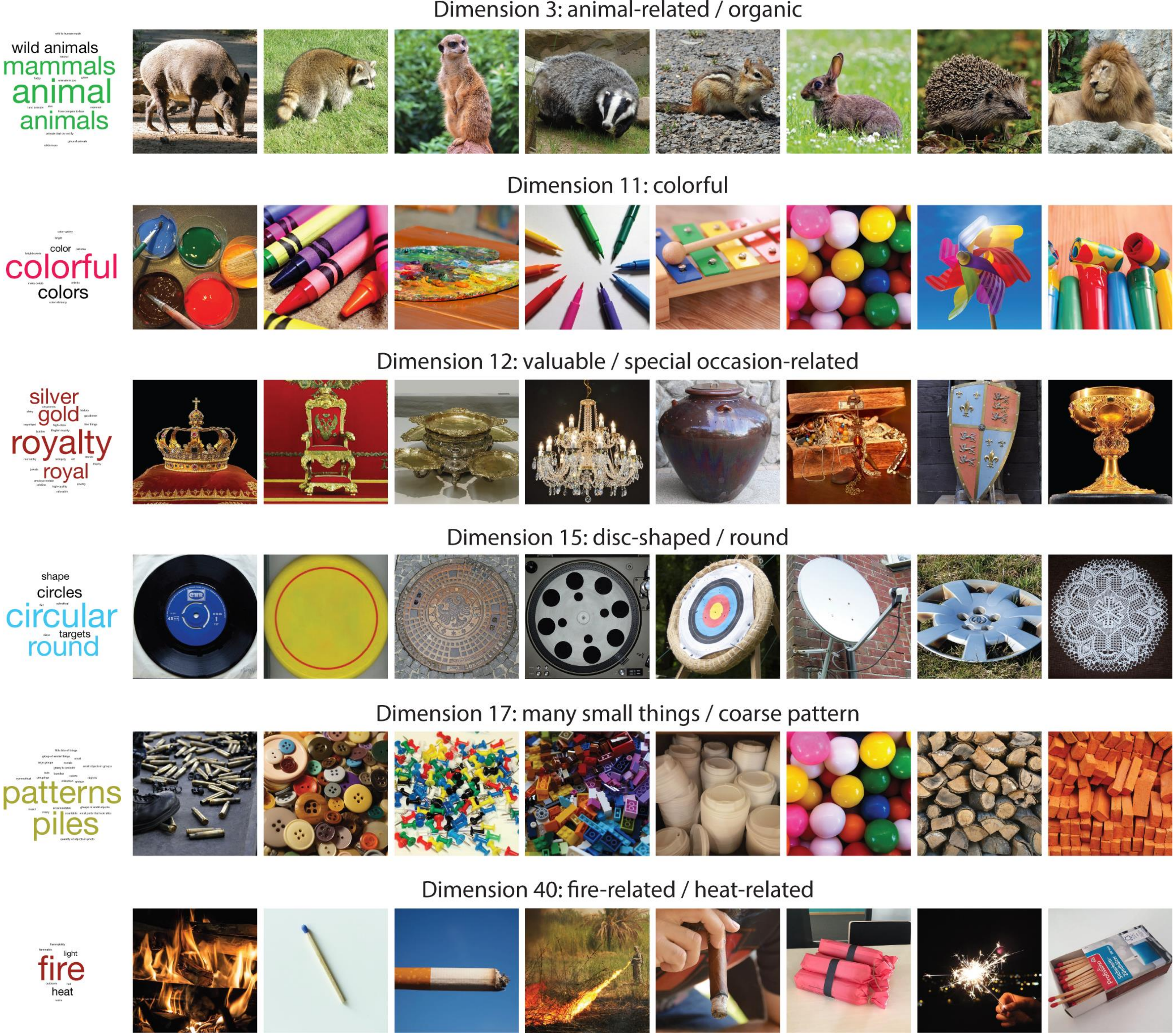
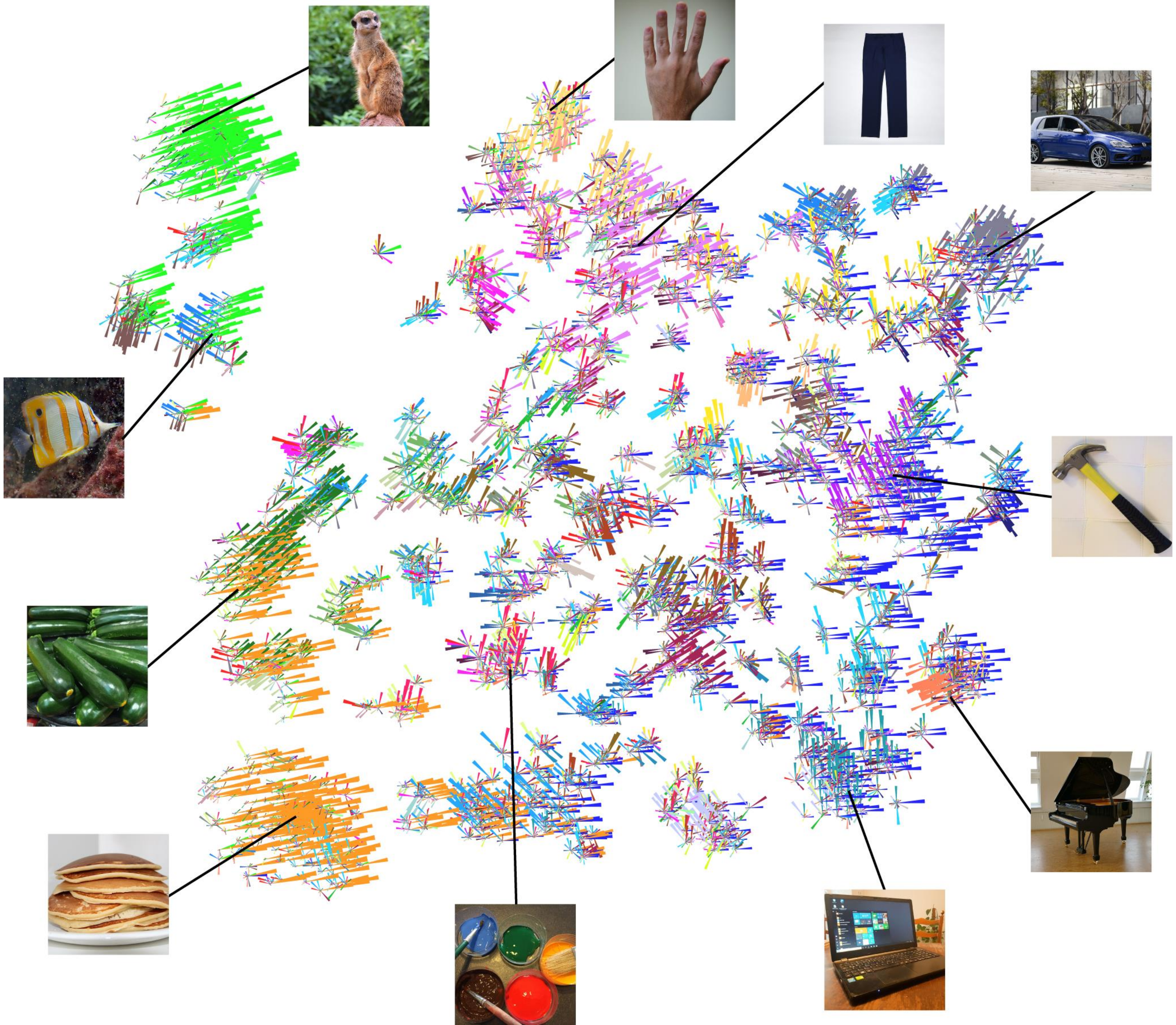
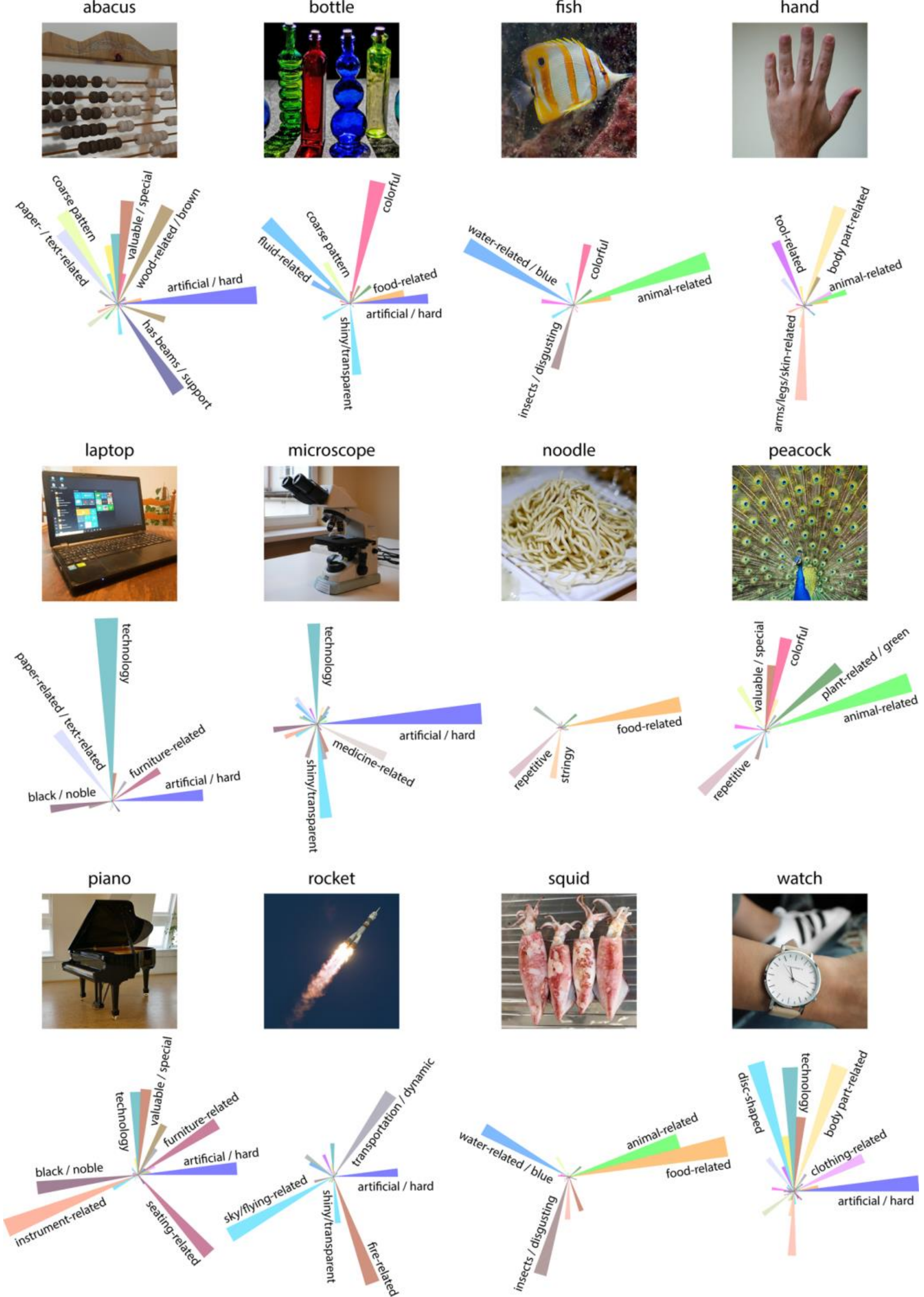


Fig. 1 | Task and modelling procedure for the large-scale identification of mental object representations. a, We applied a triplet odd-one-out similarity task to images of the 1,854 objects in the THINGS database[17] and collected a large number of ratings (1.46 million) using online crowdsourcing. The triplet odd-one-out task measures object similarity as the probability of choosing two objects together. This task evokes different minimal contexts as a basis for grouping objects together, which in turn emphasizes the relevant dimensions. **b**, The goal of the modelling procedure was to learn an interpretable representational embedding that captures choice behaviour in the odd-one-out task and predicts object similarity across all pairs of objects. Since only a subset of all possible triplets had been sampled (0.14% of 1.06 billion possible combinations), this model additionally served to complete the sparsely sampled similarity matrix. **c**, The model reflects the assumed cognitive process underlying the odd-one-out task. The embedding was initialized with random weights and would carry out predictions for which object pair was the most similar, on the basis of the dot product. Predicting the most similar pair is equivalent to predicting the remaining object as the odd one out. The model predictions were initially at chance (see the example for a prediction that deviates from the choice) but learned gradually to predict behavioural choices. To allow for error backpropagation to the weights, the model was implemented as a shallow neural network. For this figure, all images were replaced by images with similar appearance from the public domain. Images used under a CC0 license, from Pixabay: monika1607, OpenClipart-Vectors; Wikimedia: Vita Vilcina.

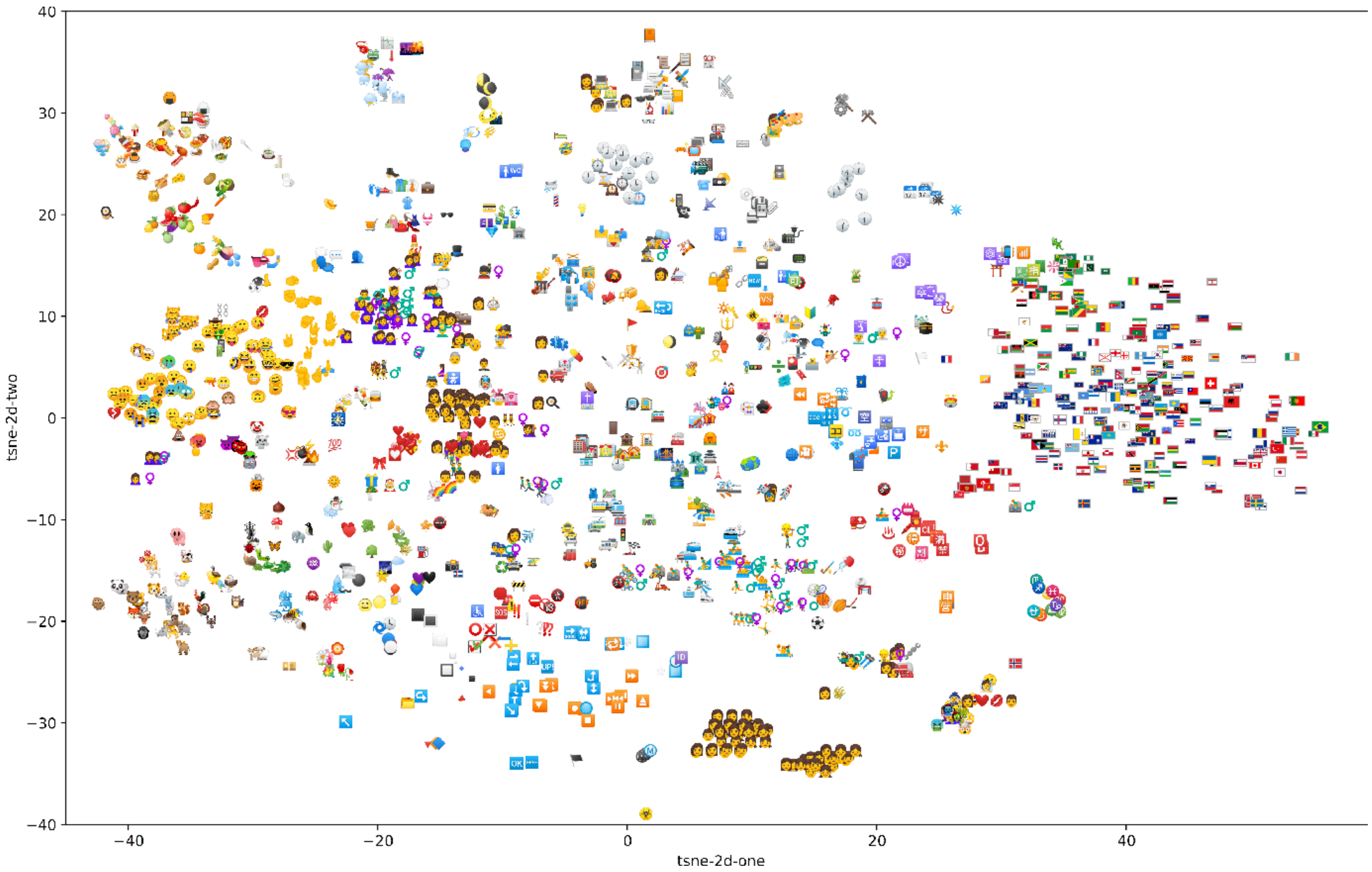
Hebart, M. N., Zheng, C. Y., Pereira, F., & Baker, C. I. (2020). Revealing the multidimensional mental representations of natural objects underlying human similarity judgements. *Nature human behaviour*, *4*(11), 1173-1185.


Dimension 3: animal-related / organic
wild animals
mammals
animal
animals
Dimension 11: colorful
color
colorful
colors
Dimension 12: valuable / special occasion-related
silver
gold
royalty
royal
Dimension 15: disc-shaped / round
shape
circles
circular
targets
round
Dimension 17: many small things / coarse pattern
patterns
piles
Dimension 40: fire-related / heat-related
light
fire
heat




abacus
paper- / text-related
coarse pattern
valuable / special
wood-related / brown
artificial / hard
has beams / support
bottle
colorful
coarse pattern
fluid-related
food-related
artificial / hard
shiny/transparent
fish
water-related / blue
colorful
animal-related
insects / disgusting
hand
tool-related
body part-related
animal-related
arms/legs/skin-related
laptop
technology
paper-related / text-related
furniture-related
artificial / hard
black / noble
microscope
technology
artificial / hard
medicine-related
shiny/transparent
noodle
food-related
repetitive
stringy
peacock
valuable / special
colorful
plant-related / green
animal-related
repetitive
piano
technology
valuable / special
furniture-related
artificial / hard
black / noble
instrument-related
seating-related
rocket
transportation / dynamic
artificial / hard
sky/flying-related
shiny/transparent
fire-related
squid
water-related / blue
animal-related
food-related
insects / disgusting
watch
disc-shaped
technology
body part-related
clothing-related
artificial / hard

Pairwise similarity



- How to compare similarity of individual items?

How to measure pairwise similarity

Stimuli 1

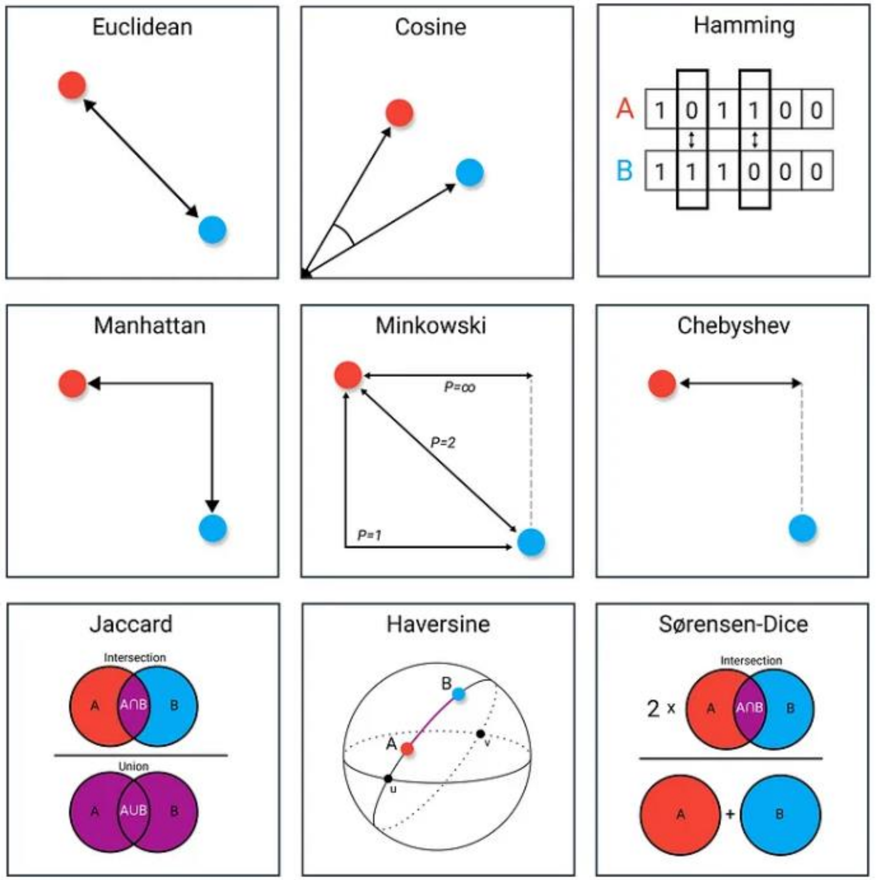
-0.027	-0.001	-0.020	...	-0.023

Stimuli 2

0.004	0.003	0.002	...	-0.014

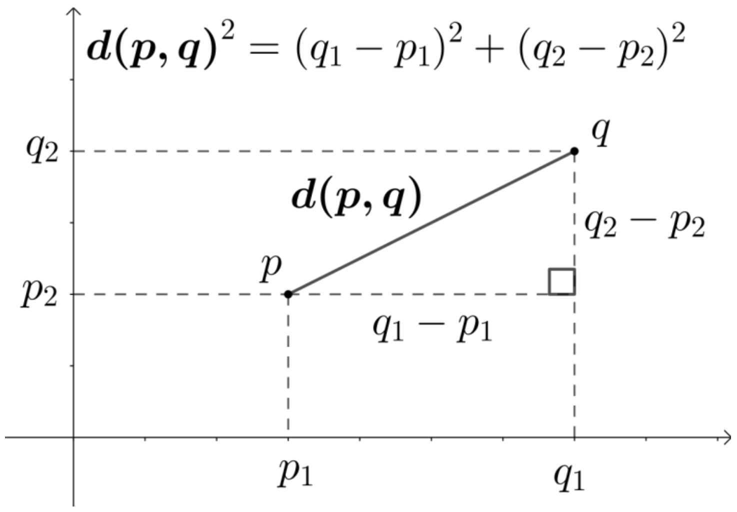
Any ideas?

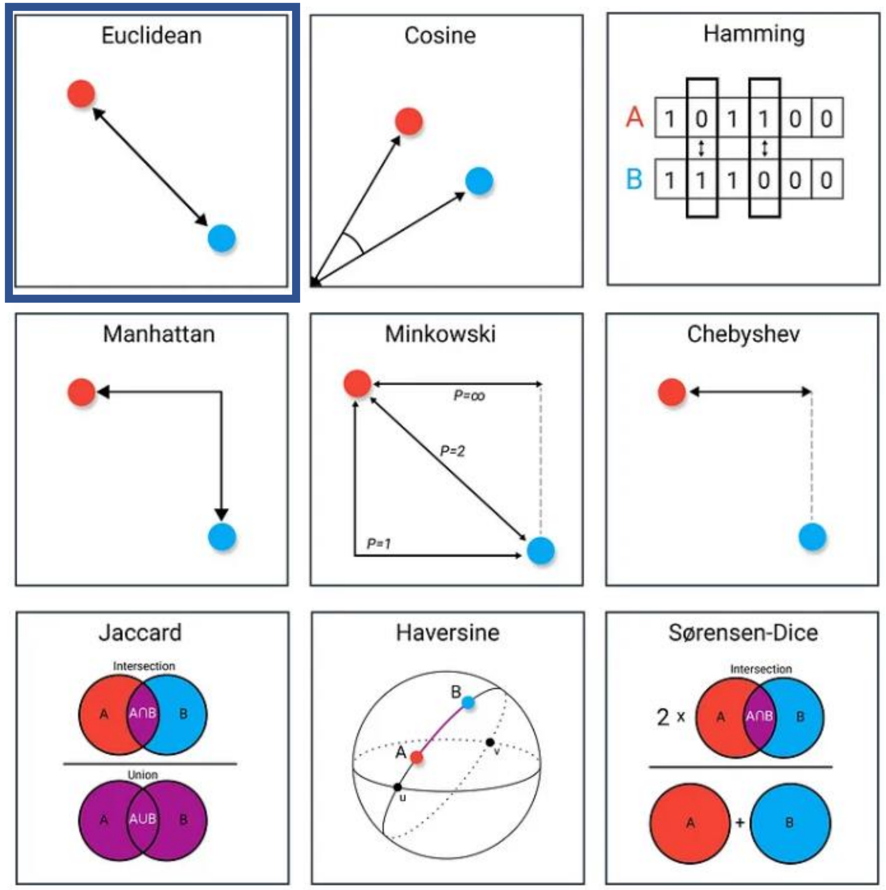
Pairwise similarity metrics



Distance Measures. Image by the author.

Pairwise similarity metrics

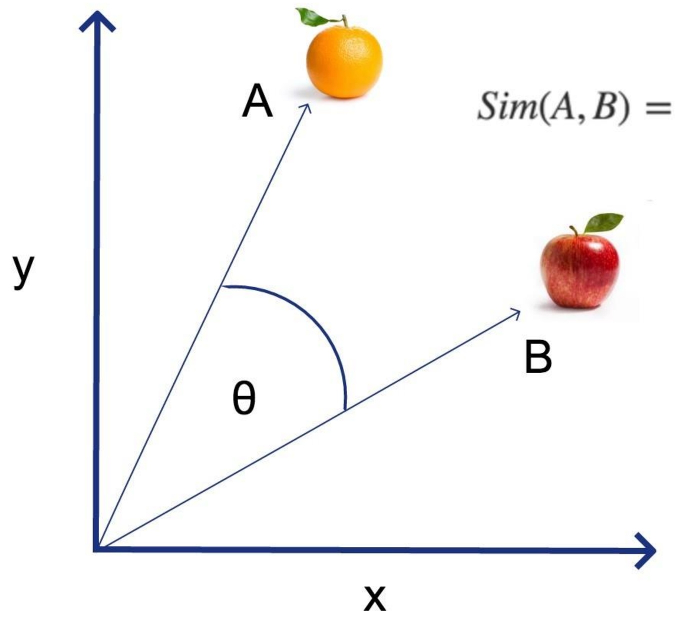




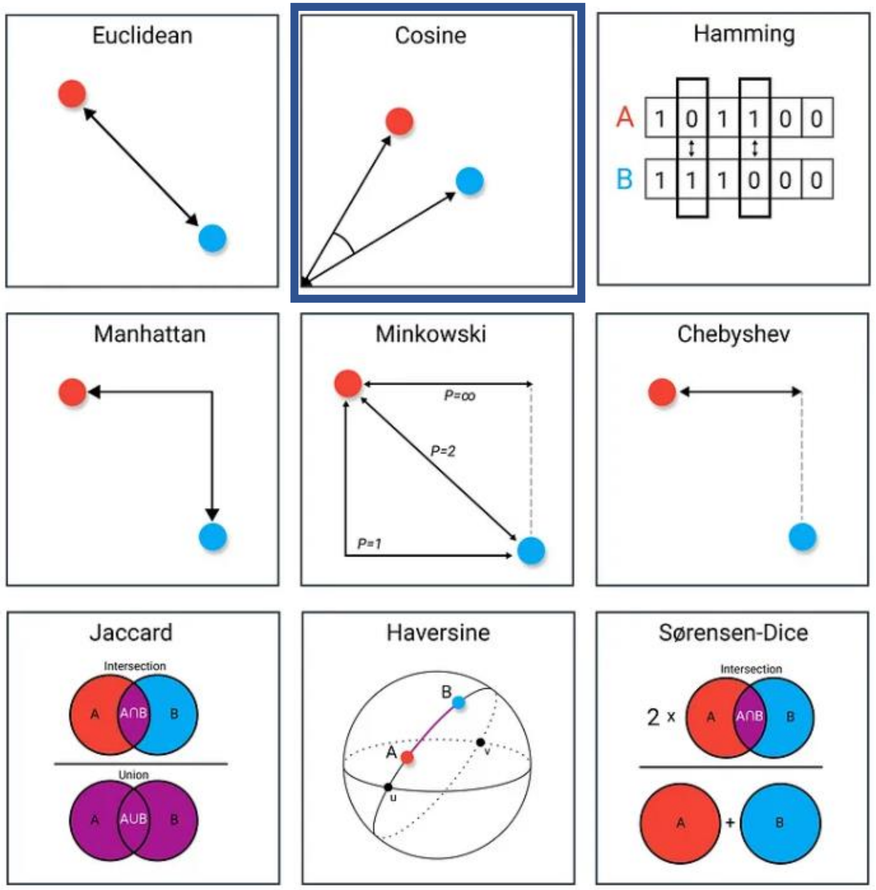
Distance Measures. Image by the author.

Pairwise similarity metrics

Cosine Similarity



$$Sim(A,B) = \cos(\theta) = \frac{A \cdot B}{\|A\| \|B\|}$$



Distance Measures. Image by the author.

- Often used with text

https://aitechtrend.com/how-cosine-similarity-can-improve-your-machine-learning-models/