SV UFP 2A 2024

Debye shielding

Coulomb logarithm

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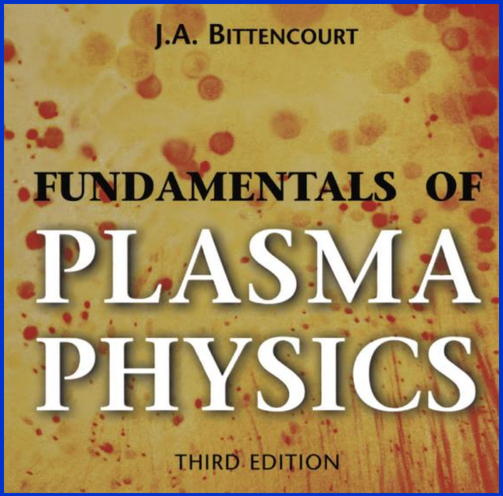

Introduction to Plasma Physics

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June 10, 2008

acknowledgements: Many slides borrowed from Prof. Fisch, Prof. Goldston, others

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Lecture Notes on

PRINCIPLES OF PLASMA PROCESSING

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pdf

INTRODUCTION TO
PLASMA PHYSICS
AND CONTROLLED
FUSION

SECOND EDITION

Volume 1: Plasma Physics

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Fundamentals of Plasma Physics

Paul M. Bellan

PLASMA

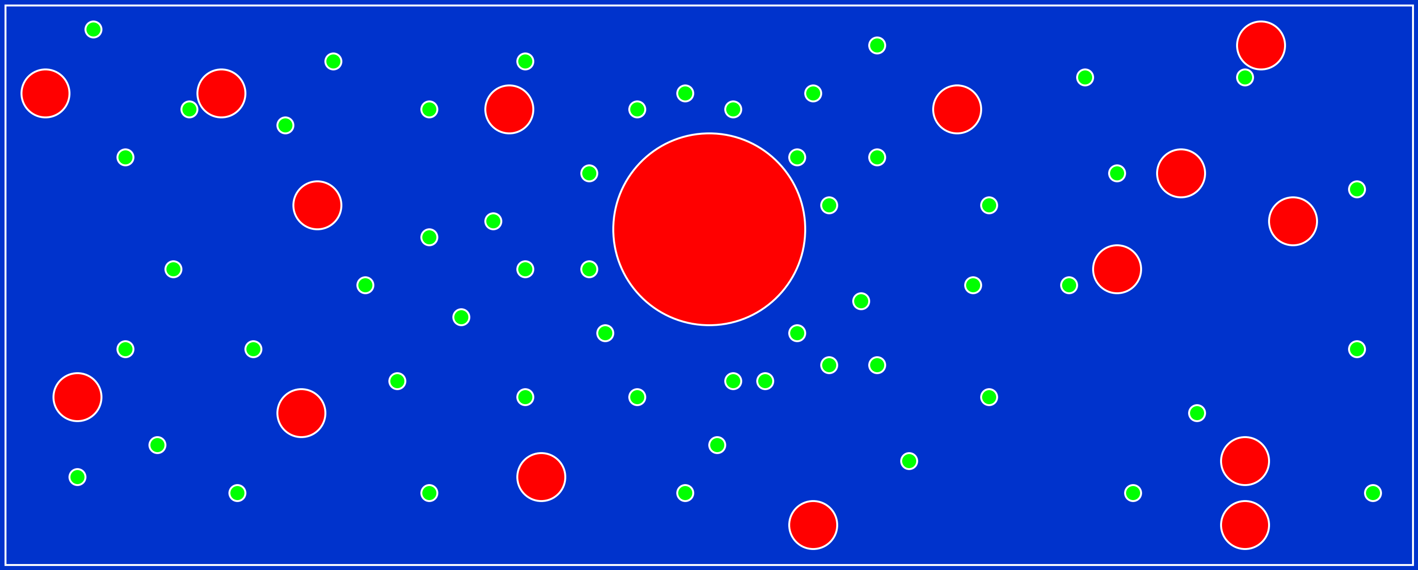
<u>Simple definition</u>:
a quasi-neutral gas of charged particles showing collective behaviour.

<u>Quasi-neutrality</u>: number densities of electrons, n_e, and ions, n_i , with charge state Z are locally balanced: $n_e \approx Zn_i$

<u>Collective behaviour</u>: long range of Coulomb potential (1/r) leads to nonlocal influence of disturbances in equilibrium.

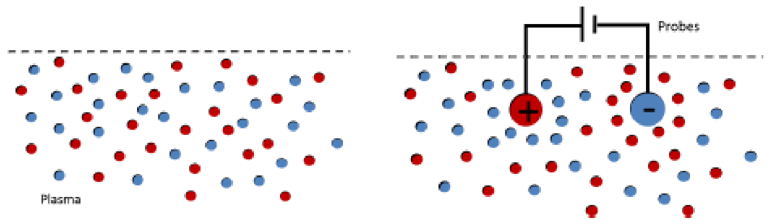
Macroscopic fields usually dominate over microscopic fluctuations, e.g.:

$$\rho = e(Zn_i - n_e) \Rightarrow \nabla.\boldsymbol{E} = \rho/\varepsilon_0$$



Plasma Shielding

Debye shielding



What is the potential $\phi(r)$ of an ion (or positively charged sphere) immersed in a plasma?

For equal ion and electron temperatures ($T_e = T_i$), we have:

$$\frac{1}{2} m_e v_e^2 = \frac{1}{2} m_i v_i^2 = \frac{3}{2} k_B T_e \qquad (2)$$

Therefore,

$$\frac{v_i}{v_e} = \left(\frac{m_e}{m_i}\right)^{1/2} = \left(\frac{m_e}{A m_p}\right)^{1/2} = \frac{1}{43} \qquad \text{(hydrogen, Z=A=1)}$$

Ions are almost stationary on electron timescale!
To a good approximation, we can often write:

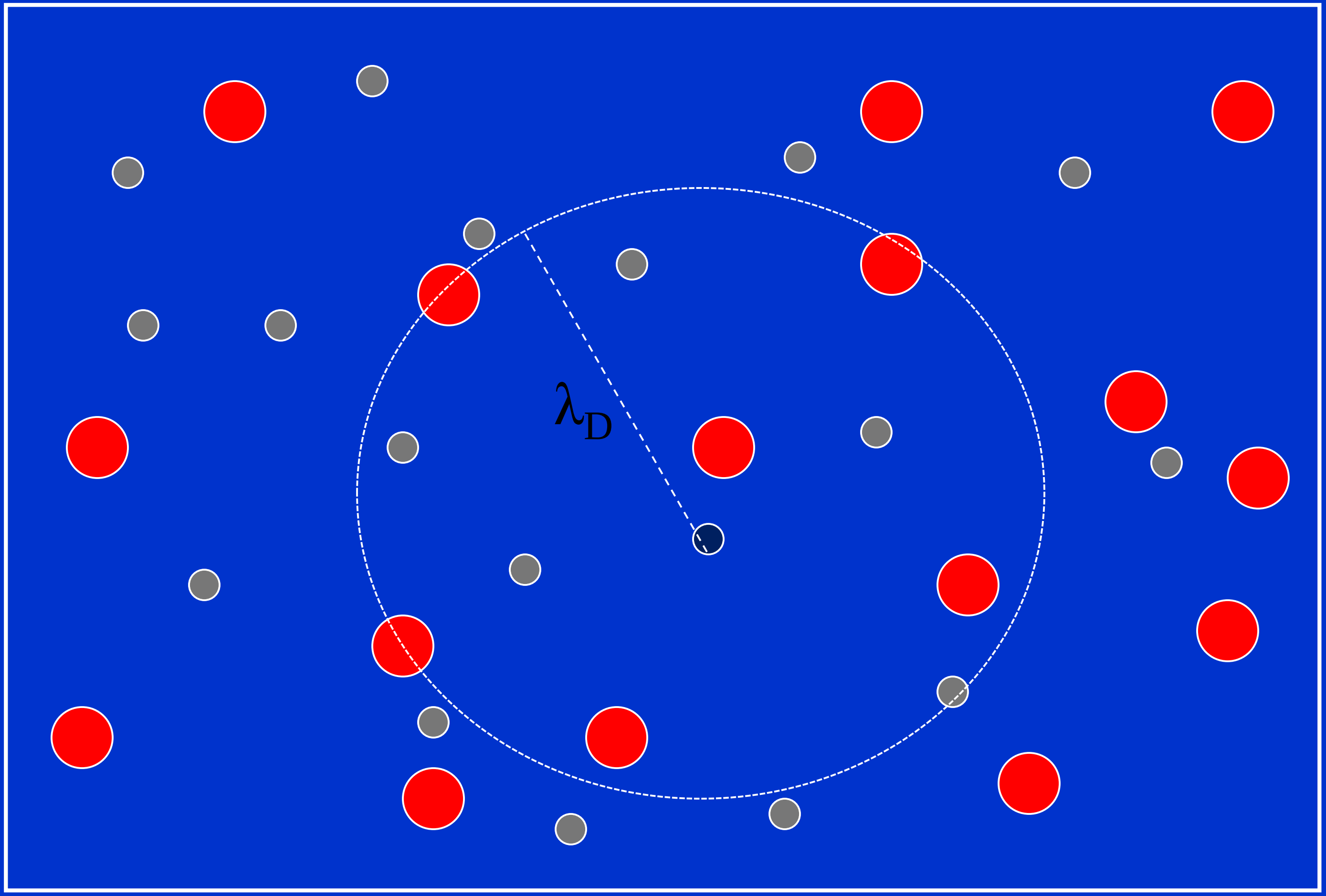
$$n_i \simeq n_0,$$

where the material (eg gas) number density, $n_0 = N_A \rho_m / A$.

$M_p/m_e \sim 1836$

Debye Length

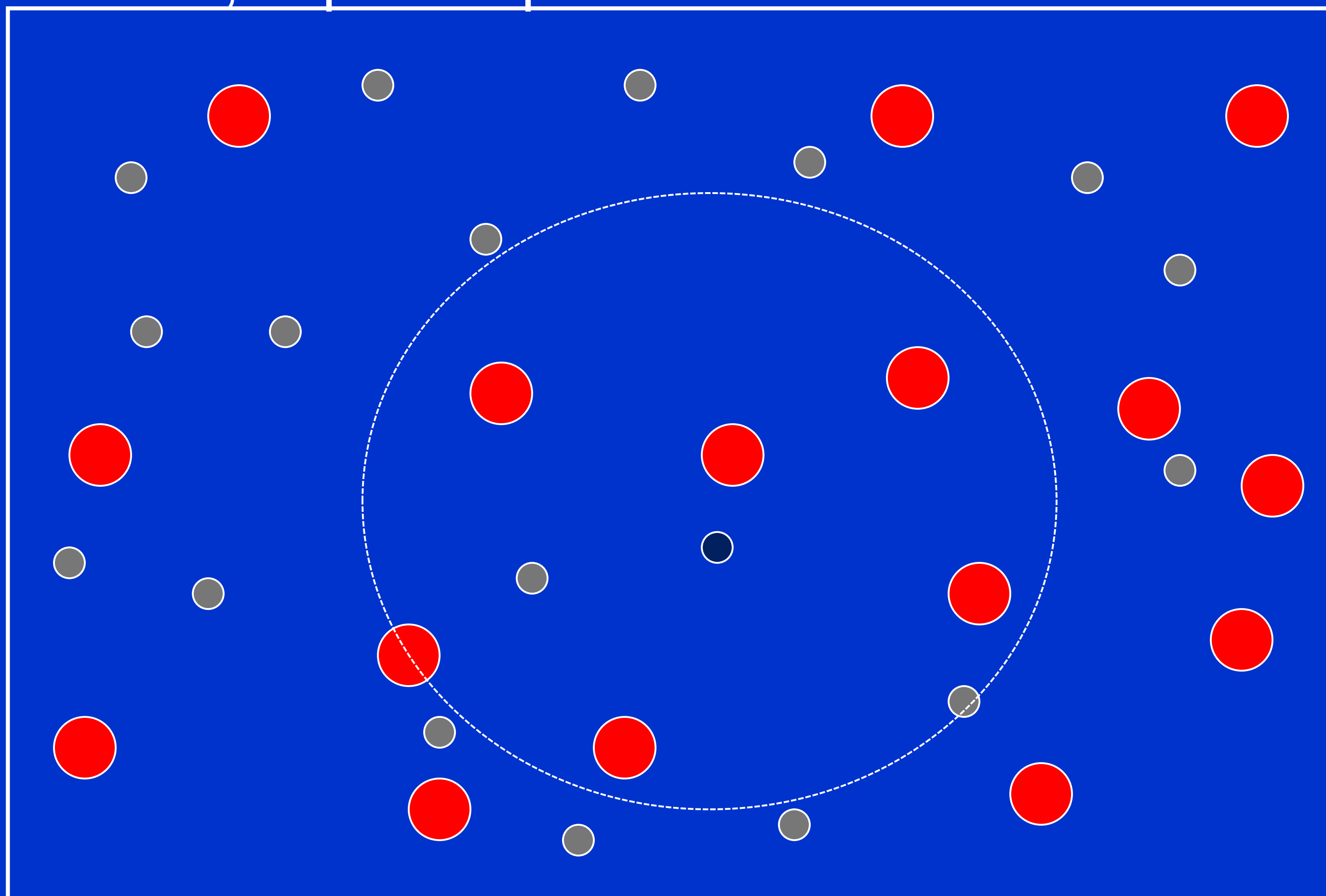
- Debye length = range of influence, e.g., for single electron



Electrons: e^- Ions: H^+

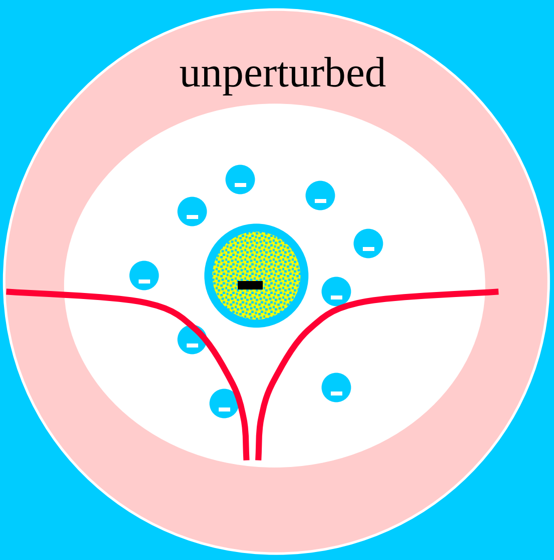
Debye Length

- In neighborhood of an electron there is deficit of other electrons, suplus of positive ions



Electrons: e^- Ions: H^+

Debye shielding 1



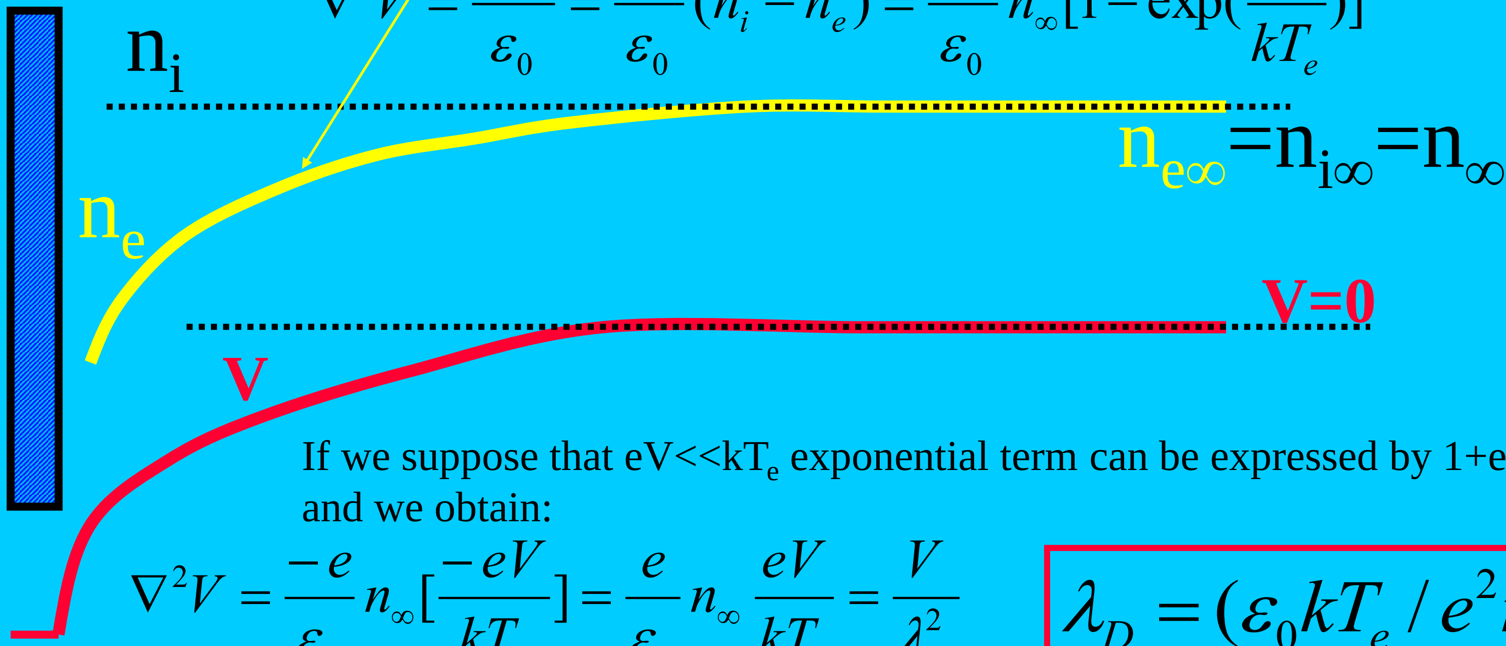
The effect of potential – perturbing charge in a plasma are generally much shorter- range than in vacuum because the charges in plasma tend to distribute themselves so as to shield the plasma from the electric field the perturbing charge generates.

The effect can be deduced readily from Poisson's equation by assuming, for example, that ions do not move but that electrons adopt a thermal equilibrium distribution in which electron distribution is determined by the Boltzmann factor:

$$n_e = n_\infty \exp(eV / kT_e)$$

n_∞ is electron density far from the perturbing charge where potential V is taken as a zero. Poisson's equation is:

$$\nabla^2 V = \frac{-\rho}{\varepsilon_0} = \frac{-e}{\varepsilon_0}(n_i - n_e) = \frac{-e}{\varepsilon_0} n_\infty [1 - \exp(\frac{eV}{kT_e})]$$



If we suppose that $eV \ll kT_e$ exponential term can be expressed by $1+eV/kT_e$ and we obtain:

$$\nabla^2 V = \frac{-e}{\varepsilon_0} n_\infty [\frac{-eV}{kT_e}] = \frac{e}{\varepsilon_0} n_\infty \frac{eV}{kT_e} = \frac{V}{\lambda_D^2}$$

$$\lambda_D = (\varepsilon_0 k T_e / e^2 n_\infty)^{1/2}$$

Linear approximation just to understand problem, signs are roughly OK

Debye shielding 2

$$n_e = n_\infty \exp(eV/kT_e)$$

$$\nabla^2 V = \frac{-\rho}{\varepsilon_0} = \frac{-e}{\varepsilon_0}(n_i - n_e) = \frac{-e}{\varepsilon_0} n_\infty [1 - \exp(\frac{eV}{kT_e})]$$

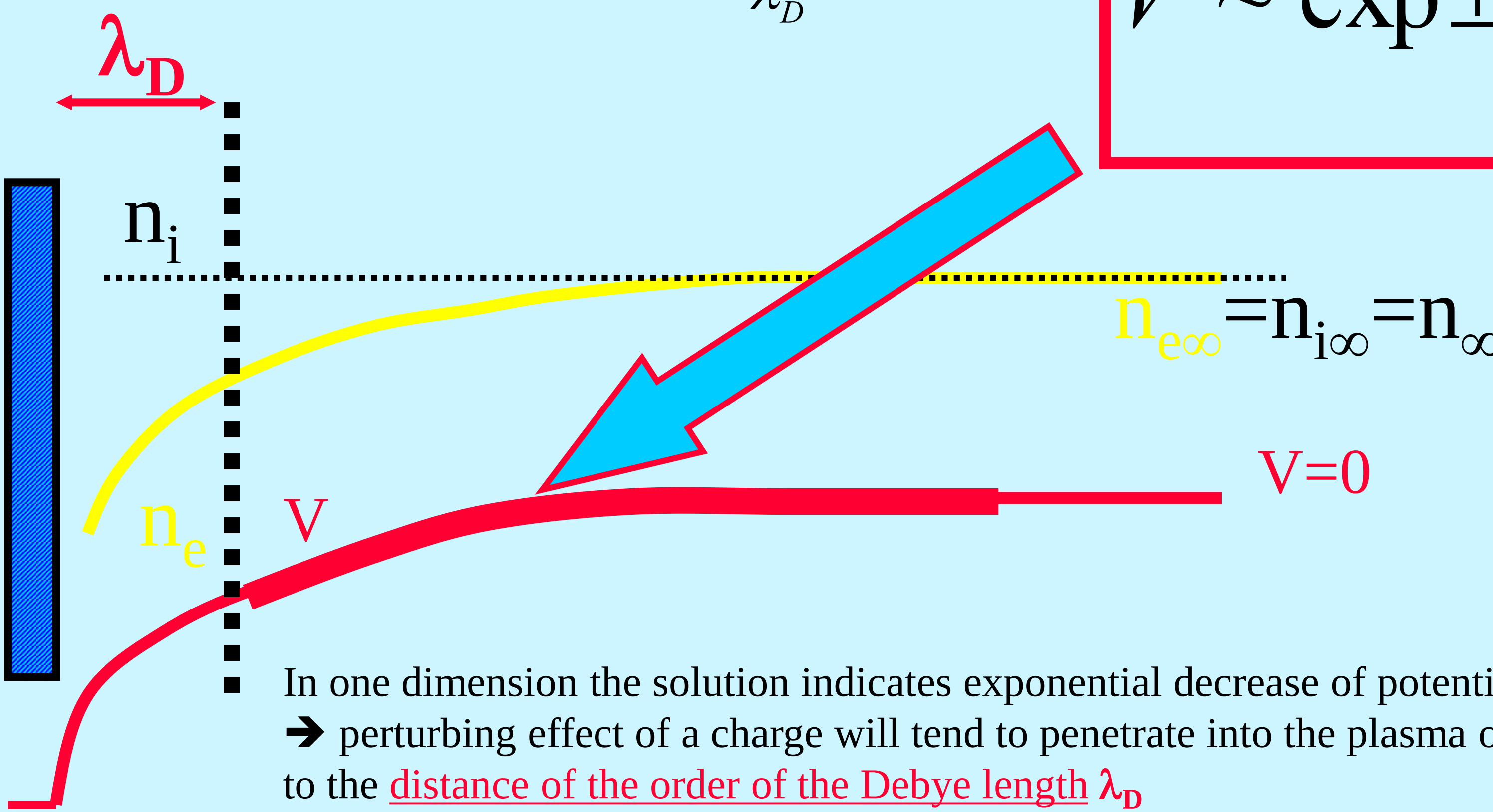
$eV \ll kT_e$, exponential can be approximated by linear term ➔

$$\nabla^2 V = \frac{-e}{\varepsilon_0} n_\infty [\frac{-eV}{kT_e}] = \frac{e}{\varepsilon_0} n_\infty \frac{eV}{kT_e} = \frac{V}{\lambda_D^2}$$

$$\lambda_D = (\varepsilon_0 kT_e / e^2 n_\infty)^{1/2}$$

$$\nabla^2 V = \frac{V}{\lambda_D^2}$$

$$V \sim \exp \pm \frac{x}{\lambda_D}$$



In one dimension the solution indicates exponential decrease of potential ➔ perturbing effect of a charge will tend to penetrate into the plasma only to the distance of the order of the Debye length λ_D

Linear approximation just to understand problem

Debye shielding 3

$$n_e = n_\infty \exp(eV / kT_e)$$

eV<<kT$_e$

$$\nabla^2 V = \frac{V}{\lambda_D^2} \Rightarrow V \sim \exp \pm \frac{x}{\lambda_D}$$

$$\lambda_D = (\varepsilon_0 k T_e / e^2 n_\infty)^{1/2}$$

$$l_d = 69\sqrt{\frac{T}{n}}, \quad T\ in\ K, n\ in\ m^{-3}$$

For laboratory plasma with T_e=1eV and $n_e=10^{11}cm^{-3}$ ➔ **λ_D=23μm**

$n_e=10^{11}cm^{-3}$ ➔ “distance” between particles ~ **2μm**;

For plasma with T_e=0.001eV (~10K) and $n_e=10^{11}cm^{-3}$ ➔ **λ_D=0.7μm**

$n_e=10^{11}cm^{-3}$ ➔ “distance” between particles ~ **2μm**;

For plasma with T_e=0.001eV (~10K) and $n_e=10^{7}cm^{-3}$ ➔ **λ_D=74μm**

Linear approximation just to understand problem

Potential of a Uniform Sphere of Charge

$$\nabla^2 V = \frac{\partial^2 V}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} + \frac{2}{r}\frac{\partial V}{\partial r} + \frac{\cot\theta}{r^2}\frac{\partial V}{\partial \theta} = \frac{-\rho}{\varepsilon_0}$$

$$\frac{\partial^2 V}{\partial r^2} + \frac{2}{r}\frac{\partial V}{\partial r} = \frac{-\rho}{\varepsilon_0}$$

Debye shielding 4

(not in SI!):

In spherical symmetry Poisson's equation gives *(not in SI!)*:

$$\frac{1}{r}\frac{d^2}{dr^2}\{Vr\} = \frac{4\pi e^2 n_{e\infty}}{kT_e} V(r) \quad \Rightarrow \quad V = \frac{A}{r}\exp(-r/\lambda_{DX}) \quad + \quad \frac{B}{r}\exp(r/\lambda_{DX})$$

$$\lambda_{DX} = (kT_e / 4\pi e^2 n_\infty)^{1/2}$$

Applying the boundary condition that as r tends to infinity V must tend to zero gives B=0,
V must tend to e/r as r tend to zero ➔

$$V = \frac{e}{r}\exp(-r/\lambda_{DX})$$

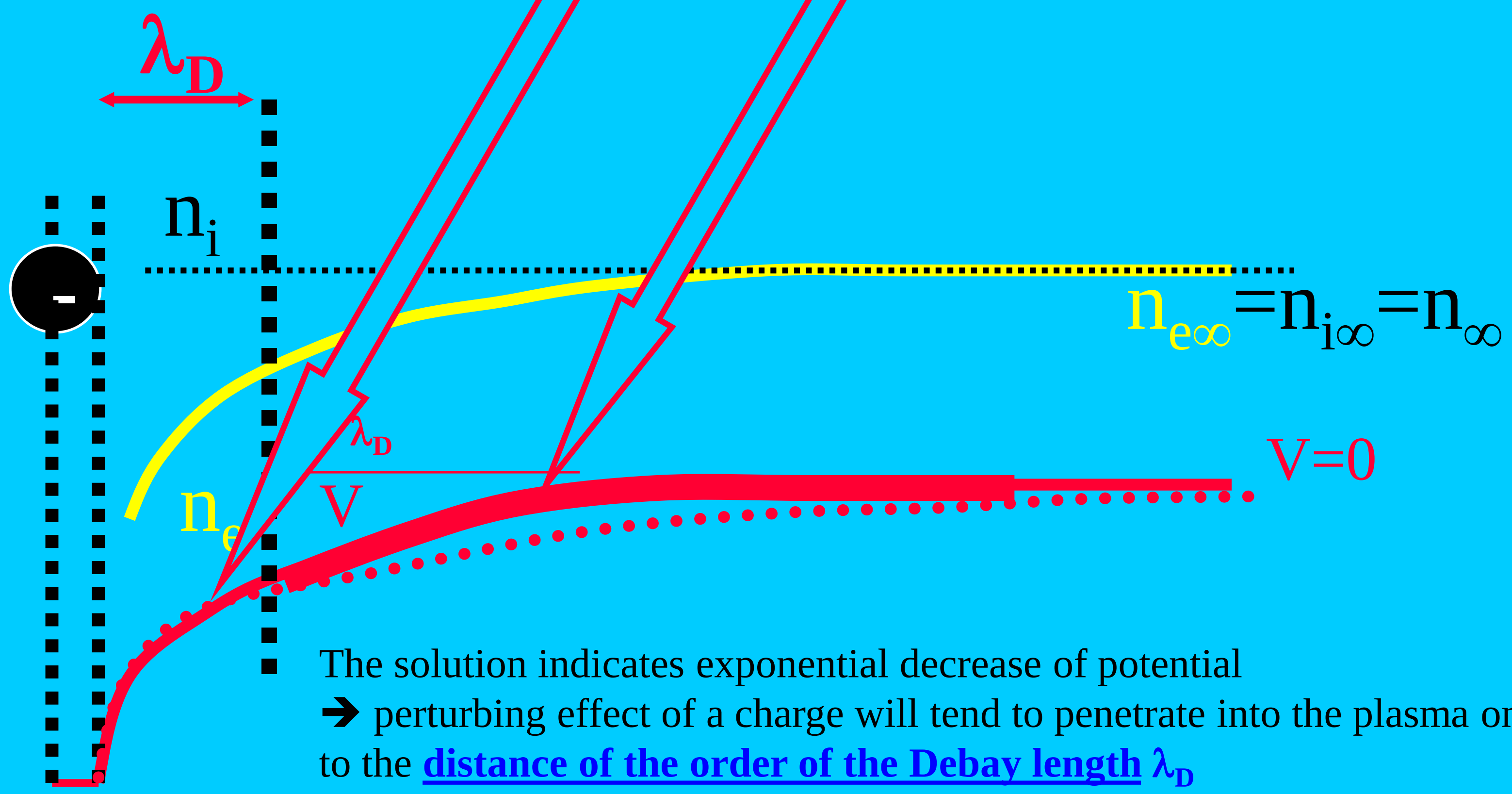
(not in SI!):

$$\lambda_D = (\varepsilon_0 kT_e / e^2 n_\infty)^{1/2}$$

Debye shielding 5

$$V = \frac{e}{r}\exp(-r/\lambda_{DX})$$

$$\lambda_{DX} = (kT_e / 4\pi e^2 n_\infty)^{1/2}$$



The solution indicates exponential decrease of potential
➔ perturbing effect of a charge will tend to penetrate into the plasma only to the **distance of the order of the Debay length** λ_D

Debye shielding 6

$$V = \frac{e}{r}\exp(-r/\lambda_{DX})$$

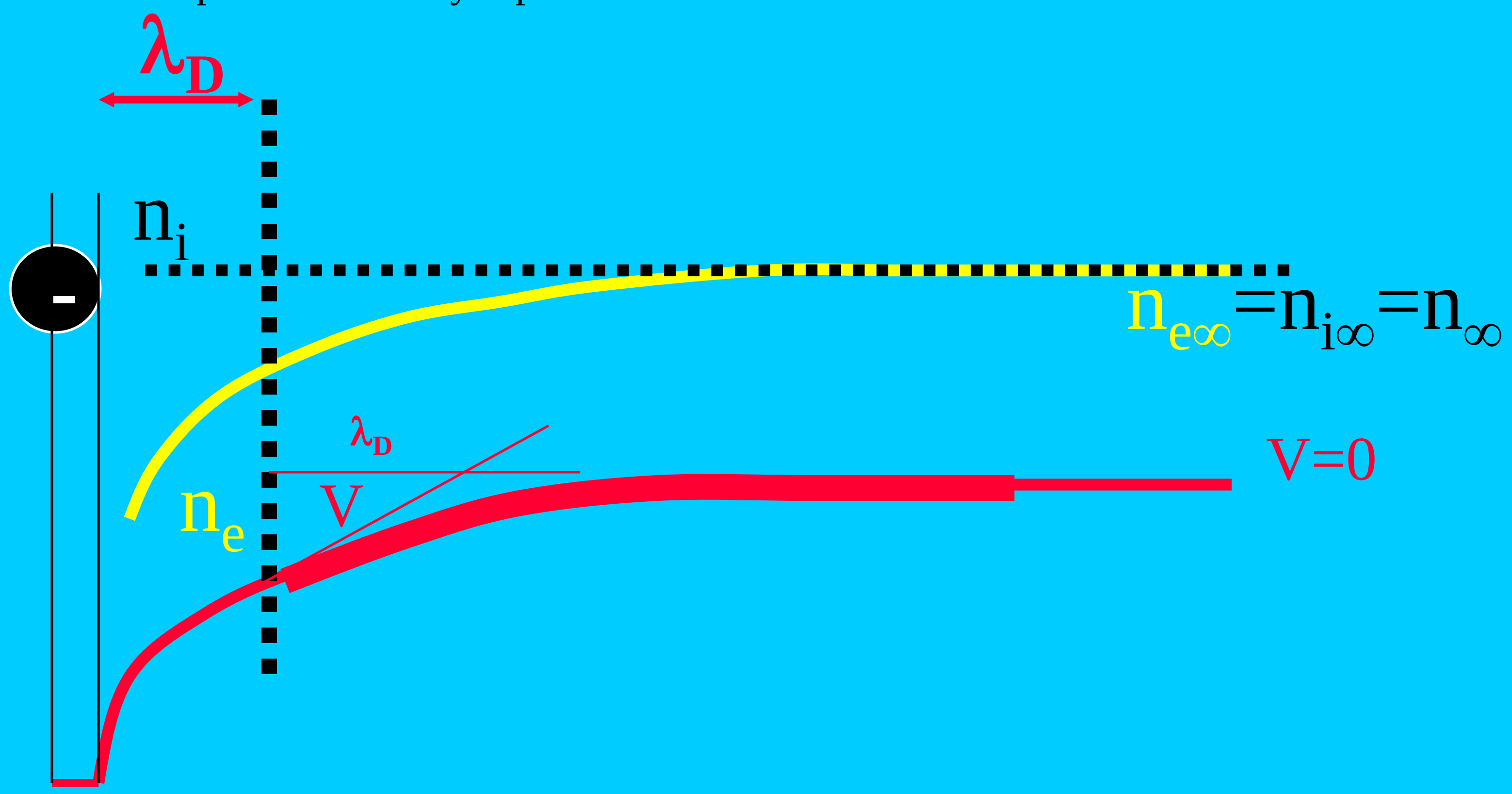
$$\lambda_{DX} = (kT_e / 4\pi e^2 n_\infty)^{1/2}$$

Just rewritten

$$(n_\infty \lambda_{DX}^3 4/3\pi)3e^2/\lambda_{DX} = kT_e$$

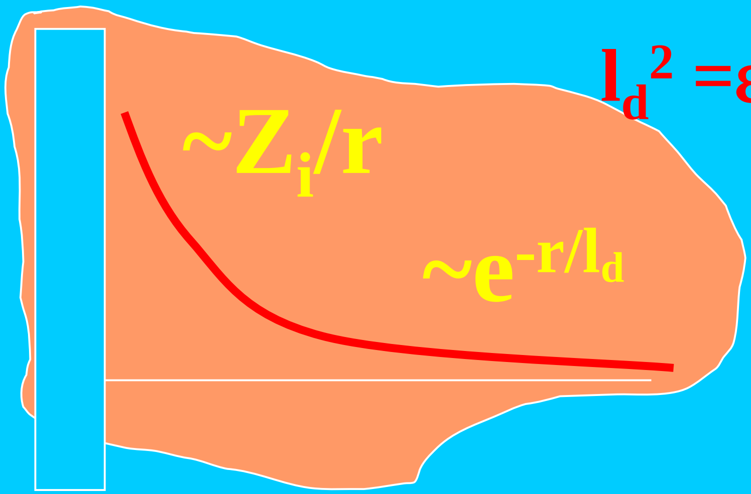
$$N(e^2/\lambda_{DX}) \approx Ne(e/\lambda_{DX}) \approx Ne\varphi \approx \text{potencial energy} \approx kT_e$$

N – number of particles in debye sphere



Again….

- Stínění v plazmě
- Ustanovení debyovského stínění



$$l_d^2 = \varepsilon_0 kT/ne^2$$

$$\phi(r) = (Z_i e/4\pi\varepsilon_0)/r \ast e^{-r/l_d}$$

$$\sigma_c(v) = 2\pi \int b \, db$$

Problém srážek na velkou vzdálenost

Výpočet:

$$l_d = 69\sqrt{\frac{T}{n}}, \quad T \, in \, K, n \, in \, m^{-3}$$

at 1000K, $n=4.8\times10^{12} m^{-3} = 4.8\times10^{6} cm^{-3}$

l_d=1 mm = 0.001m

at 10K, $n=1\times10^{10} m^{-3} = 1\times10^{4} cm^{-3}$

l_d~2 mm ~ 0.002m

Další kroky

$$\lambda_{De} \equiv \sqrt{\frac{\epsilon_0 T_e}{n_e e^2}} \simeq 7434 \sqrt{\frac{T_e(\mathrm{eV})}{n_e(\mathrm{m}^{-3})}}\ \mathrm{m}, \quad \text{electron Debye length.}$$

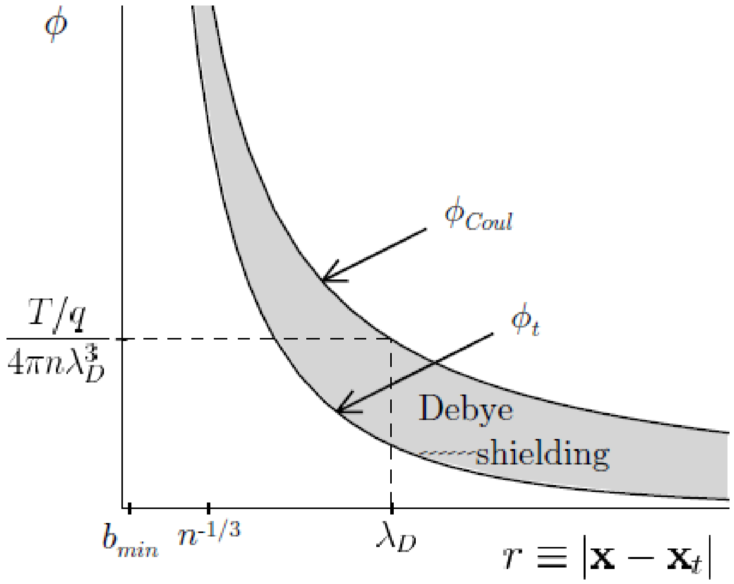


Figure 1.1: Potential ϕ_t around a test particle of charge q_t in a plasma and Coulomb potential ϕ_{Coul}, both as a function of radial distance from the test particle. The shaded region represents the Debye shielding effect. The characteristic distances are: λ_D, Debye shielding distance; $n_e^{-1/3}$, mean electron separation distance; $b_{\mathrm{min}}^{\mathrm{cl}} = q^2/(\{4\pi\epsilon_0\}T)$, classical distance of "closest approach" where the $e\phi/T << 1$ approximation breaks down.