Second part

Difúze, Ambipolární difúze,

Drift

Difúze a drift v přítomnosti magnetického pole

$$\overline{\vec{v}} = -D\frac{\nabla_r n}{n}$$

$$\overline{\vec{v}} = \pm\mu\vec{E}$$

Úvod do fyziky plazmatu

ČSAV, Academia Praha 1984

Francis F. Chen

Jak řešit magnetické pole

- Difúze

$$m\frac{d\mathbf{v}}{dt} = q\mathbf{v}\times\mathbf{B}.$$

cyklotronní frekvence

$$\omega_c \equiv \frac{|q|B}{m}$$

Položíme-li $\hat{\mathbf{z}}$ * do směru **B** ($\mathbf{B} = B\hat{\mathbf{z}}$), dostáváme

$$m\dot{v}_x = qBv_y, \qquad m\dot{v}_y = -qBv_x, \qquad m\dot{v}_z = 0,$$

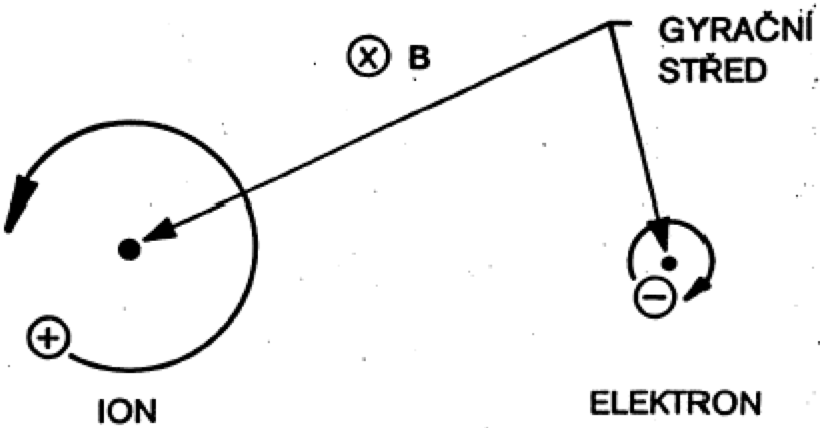
$$\ddot{v}_x = \frac{qB}{m}\dot{v}_y = -\left(\frac{qB}{m}\right)^2 v_x, \qquad \ddot{v}_y = -\frac{qB}{m}\dot{v}_x = -\left(\frac{qB}{m}\right)^2 v_y.$$

$$\varpi_c = \frac{eB}{m} = \frac{1.6x10^{-19}}{9.1x10^{-31}}B = 1.8x10^{11}B$$

$$at \quad B = 1T \quad \varpi_c = 1.8x10^{11} \sim 180\ GHz$$

$$at \quad B = 0.1T \quad \varpi_c = 1.8x10^{10} \sim 18\ GHz$$

$$r_L \equiv \frac{v_\perp}{\omega_c} = \frac{mv_\perp}{|q|B} \qquad [2\text{-}6]$$



Larmorovy orbity v magnetickém poli.

$$r_L \sim \frac{m\sqrt{kT/m}}{B}$$

$$at \quad T = 300K \quad \& \quad B = 1T \quad r_l = 1\mu m$$

$$at \quad T = 3K \quad \& \quad B = 1T \quad r_l = 0.1\mu m$$

$$at \quad T = 300K \quad \& \quad B = 0.1T \quad r_l = 10\mu m$$

Pro termální plazmu

$$r_L \sim \frac{m\sqrt{kT/m}}{B}$$

Jak řešit magnetické pole

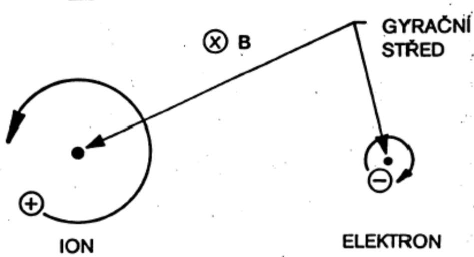
$$m\frac{d\mathbf{v}}{dt} = q\mathbf{v} \times \mathbf{B}\,.$$

Položíme-li $\hat{\mathbf{z}}$ * do směru $\mathbf{B}$ $(\mathbf{B} = B\hat{\mathbf{z}})$, dostáváme

$$m\dot{v}_x = qBv_y\,, \qquad m\dot{v}_y = -qBv_x\,, \qquad m\dot{v}_z = 0\,,$$

$$\ddot{v}_x = \frac{qB}{m}\dot{v}_y = -\left(\frac{qB}{m}\right)^2 v_x\,, \qquad \ddot{v}_y = -\frac{qB}{m}\dot{v}_x = -\left(\frac{qB}{m}\right)^2 v_y\,.$$

$$r_L \equiv \frac{v_\perp}{\omega_c} = \frac{mv_\perp}{|q|\,B} \qquad [2\text{-}6]$$



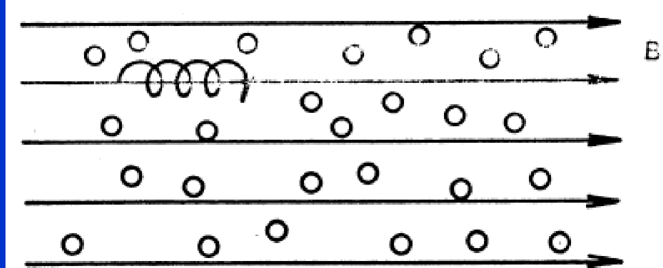
Larmorovy orbity v magnetickém poli.

- Difúze

$$\omega_c \equiv \frac{|q|\,B}{m}$$

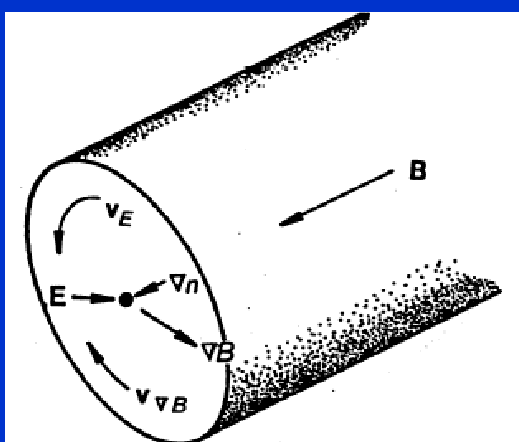
$$r_L \sim \frac{m\sqrt{kT/m}}{B}$$

Ve směru z $\qquad \Gamma_z = \pm\mu n E_z - D\frac{\partial n}{\partial z}.$



Nabitá částice bude v magnetickém poli rotovat okolo jedné siločáry, dokud se nesrazí.

Žádny ztráty ???!!!!



Drifty částic v cylindricky symetrickém sloupci plazmatu nevedou ke ztrátám.

Transportní rovnice pro plazma – vodivost a magnetické pole

a rovnice pro proudovou hustotu $\boldsymbol{j}$ (zobecněný Ohmův zákon) má nyní tvar

$$\frac{\partial \boldsymbol{j}}{\partial t} - \frac{e^2 n_e}{m_e}(\boldsymbol{E} + \boldsymbol{v}_0 \times \boldsymbol{B}) + \frac{e}{m_e}\boldsymbol{j} \times \boldsymbol{B} - \frac{e}{m_e}\nabla_r p_e - - \sum_{(k)} \frac{Z_k^2 e^2 n_k}{m_k}\boldsymbol{F}_k = -\frac{\boldsymbol{j}}{\tau_e}. \tag{3.216}$$

Zde je ovšem nutno poznamenat, že zobecněný Ohmův zákon (3.216) je použitelný pro plně ionizované plázma opět pouze v prvním přiblížení. Obecně totiž může pravá strana této rovnice záviset na magnetickém poli uvnitř plazmatu a gradientu teploty elektronů. Jestliže zanedbáme $\nabla_r T_e$, pak je pravá strana (3.216) stále ještě složena ze dvou členů; jestliže systém není daleko rovnováhy, pak přesnější tvar pravé strany rovnice (3.216) je

$$-\frac{1}{\tau_e}\left(\frac{\boldsymbol{j}_\parallel}{1{,}96} + \boldsymbol{j}_\perp\right), \tag{3.217}$$

kde $\boldsymbol{j}_\parallel$ je složka proudové hustoty ve směru magnetického pole a $\boldsymbol{j}_\perp$ je složka proudové hustoty ve směru kolmém na magnetické pole.

Rozdíl mezi (3.217) a (3.215) resp. pravou stranou (3.216), si ukážeme názorně na příkladu. Předpokládejme stacionární stav, tj. $\partial \boldsymbol{j}/\partial t = 0$ a nechť $\boldsymbol{F}_k = 0$. Z rovnice (3.216) pak dostaneme

$$\boldsymbol{E}' = \frac{\boldsymbol{j}}{\sigma} + \frac{1}{n_e e}\boldsymbol{j} \times \boldsymbol{B}, \tag{3.218}$$

kde

$$\boldsymbol{E}' = \boldsymbol{E} + (\boldsymbol{v}_0 \times \boldsymbol{B}) + \frac{1}{n_e e}\nabla_r p_e \tag{3.219}$$

a

$$\sigma = \frac{e^2 n_e \tau_e}{m_e} \tag{3.220}$$

VODIVOST

je vodivost plazmatu. Vyjádříme-li z (3.218) $\boldsymbol{j}$, dostaneme po menších úpravách, že

$$\boldsymbol{j} = \sigma \boldsymbol{E}'_\parallel + \frac{\sigma}{1 + \omega_{ce}^2 \tau_e^2}\{\boldsymbol{E}'_\perp + \omega_{ce}\tau_e(\boldsymbol{h} \times \boldsymbol{E}')\}, \tag{3.221}$$

kde

$$\boldsymbol{h} = \frac{\boldsymbol{B}}{B} \quad \text{a} \quad \omega_{ce} = \frac{eB}{m_e} \tag{3.222}$$

je elektronová cyklotronová frekvence.

$$\varrho_i \frac{\partial \bar{\boldsymbol{v}}_i}{\partial t} + \nabla_r p_i - en(\boldsymbol{E} + \bar{\boldsymbol{v}}_i \times \boldsymbol{B}) - n_i \boldsymbol{F}_i = \frac{n_e m_e}{\tau_e}(\bar{\boldsymbol{v}}_e - \bar{\boldsymbol{v}}_i) \tag{3.205'}$$

$$\varrho_e \frac{\partial \bar{\boldsymbol{v}}_e}{\partial t} + \nabla_r p_e + en(\boldsymbol{E} + \bar{\boldsymbol{v}}_e \times \boldsymbol{B}) - n_e \boldsymbol{F}_e = -\frac{n_e m_e}{\tau_e}(\bar{\boldsymbol{v}}_e - \bar{\boldsymbol{v}}_i) \tag{3.206'}$$

vodivost

$$\sigma = \frac{e^2 n_e \tau_e}{m_e}$$

Jestliže nyní použijeme na pravé straně (3.216) výrazu (3.217), dostaneme, že

$$\boldsymbol{E}' = \frac{\boldsymbol{j}_\parallel}{\sigma_\parallel} + \frac{\boldsymbol{j}_\perp}{\sigma_\perp} + \frac{1}{n_e e}(\boldsymbol{j} \times \boldsymbol{B}), \tag{3.223}$$

kde

$$\sigma_\parallel = 1{,}96\sigma \quad \text{a} \quad \sigma_\perp = \sigma; \tag{3.224}$$

vyloučením $\boldsymbol{j}$ pak dostaneme

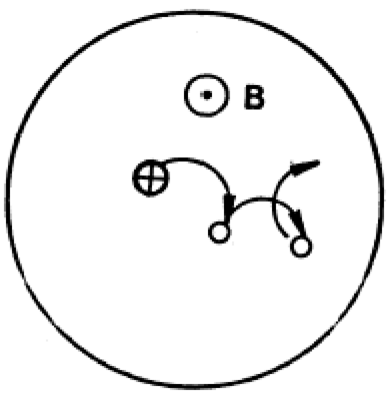
$$\boldsymbol{j} = \sigma_\parallel \boldsymbol{E}'_\parallel + \frac{\sigma_\perp}{1 + \omega_{ce}^2 \tau_e^2}\{\boldsymbol{E}'_\perp + \omega_{ce}\tau_e(\boldsymbol{h} \times \boldsymbol{E}')\}. \tag{3.225}$$

Odtud již vidíme, že i v tak jednoduchém případě, kdy $\nabla_r T_e = 0$ a systém je blízko rovnováhy, je vodivost plazmatu závislá na směru magnetického pole.

V obecném případě má vodivost plazmatu tenzorový charakter.*)

Je to de facto drift

Srážky a magnetické pole



Difúze rotujících částic při srážkách s neutrálními atomy.

- Vzpomínka na minulost

Jestliže nyní použijeme na pravé straně (3.216) výrazu (3.217), dostaneme, že

(3.223) $$\boldsymbol{E}' = \frac{\boldsymbol{j}_\parallel}{\sigma_\parallel} + \frac{\boldsymbol{j}_\perp}{\sigma_\perp} + \frac{1}{n_e e}(\boldsymbol{j} \times \boldsymbol{B}),$$

kde

(3.224) $$\sigma_\parallel = 1{,}96\sigma \quad \text{a} \quad \sigma_\perp = \sigma ;$$

vyloučením $\boldsymbol{j}$ pak dostaneme

(3.225) $$\boldsymbol{j} = \sigma_\parallel \boldsymbol{E}'_\parallel + \frac{\sigma_\perp}{1 + \omega_{ce}^2 \tau_e^2} \{\boldsymbol{E}'_\perp + \omega_{ce}\tau_e(\boldsymbol{h} \times \boldsymbol{E}')\} .$$

Odtud již vidíme, že i v tak jednoduchém případě, kdy $\nabla_r T_e = 0$ a systém je blízko rovnováhy, je vodivost plazmatu závislá na směru magnetického pole.

V obecném případě má vodivost plazmatu tenzorový charakter.*)

Přidáme magnetické pole

Rovnice B

Nenulové magnetické pole

$$\frac{\partial f_1}{\partial t} + \nabla_r f_0 + \frac{\Gamma}{v}\frac{\partial f_0}{\partial v} - (\omega_c \times f_1) = -\nu_1 f_1$$

$\frac{\partial}{\partial t} = 0$ $\nabla_r = 0$ $\vec{E} \neq 0$ $B \neq 0$

$$\frac{\Gamma}{v}\frac{\partial f_0}{\partial v} - (\varpi_c x \vec{f}_1) = -\nu_1 \vec{f}_1$$

$$\vec{f}_1 = -\frac{\Gamma}{\nu_1 v}\frac{\partial f_0}{\partial v} + \frac{1}{\nu_1}(\varpi_c x \vec{f}_1)$$

$$\overline{\varphi} = \frac{4\pi}{n}\int_0^\infty v^2 f_0 \varphi \, dv$$

$$\overline{\vec{\varphi}} = \frac{4\pi}{3}\frac{1}{n}\int_0^\infty v^4 \vec{f}_1 \varphi' dv$$

$$\vec{\varphi} = \vec{v}.\varphi(v, \vec{r}, t)$$

$$\varphi = 1$$

Per-partes
+ conditions of limits of *f*

$$\overline{\vec{v}} = \frac{4\pi}{3}\frac{1}{n}\int v^4 \vec{f}_1 dv = -\frac{4\pi}{3}\frac{1}{n}\int v^4 \frac{1}{\nu_1}\frac{\vec{\Gamma}}{v}\frac{\partial f_0}{\partial v} dv + \frac{4\pi}{3}\frac{1}{n}\int v^4 \frac{1}{\nu_1}(\varpi_c x \vec{f}_1) dv$$

$$= -\frac{4\pi}{3}\frac{1}{nm\nu_1} e\vec{E}\int v^3 \frac{\partial f_0}{\partial v} dv + \frac{4\pi}{3}\frac{1}{n}\varpi_c x \int v^4 \frac{1}{\nu_1}\vec{f}_1 dv$$

$$= -\frac{4\pi}{3}\frac{3}{nm\nu_1} e\vec{E}\int v^2 f_0 dv + \varpi_c x\left(\frac{4\pi}{3}\frac{1}{n}\int v^4 \frac{1}{\nu_1}\vec{f}_1 dv\right)$$

$$== -\frac{e}{m\nu_1}\vec{E}.1 + \varpi_c x \overline{\left(\frac{\vec{v}}{\nu_1}\right)}$$

$$\overline{\vec{v}} = \mu\vec{E} + \varpi_c x \overline{\left(\frac{\vec{v}}{\nu_1}\right)} = \mu\vec{E} + \frac{e}{m}\vec{B}x\overline{\left(\frac{\vec{v}}{\nu_1}\right)} = \mu\vec{E} + \frac{e}{m\nu_1}\vec{B}x\overline{\vec{v}}$$

$$\overline{\vec{v}} = \mu\vec{E} + \frac{e}{m\nu_1}\vec{B}x\overline{\vec{v}}$$

Odvození rovnice pro drift a difuzi z B. rovnice

Rovnice B

Drift

$$\overline{\vec{\varphi}} = \frac{4\pi}{3}\frac{1}{n}\int_0^\infty v^4 \vec{f}_1 \varphi' dv \qquad \overline{\varphi} = \frac{4\pi}{n}\int_0^\infty v^2 f_0 \varphi\, dv$$

$$\frac{\partial f_1}{\partial t} + \nabla_r f_0 + \frac{\Gamma}{v}\frac{\partial f_0}{\partial v} - (\omega_c \times f_1) = -\nu_1 f_1$$

$$\frac{\partial}{\partial t} = 0 \qquad \nabla_r \neq 0 \qquad \vec{E} \neq 0 \qquad B \neq 0$$

$$\nabla_r f_0 + \frac{\Gamma}{v}\frac{\partial f_0}{\partial v} - (\varpi_c x \vec{f}_1) = -\nu_1 \vec{f}_1$$

$$\vec{f}_1 = -\frac{\Gamma}{\nu_1 v}\frac{\partial f_0}{\partial v} + \frac{1}{\nu_1}(\varpi_c x \vec{f}_1) - \frac{1}{\nu_1}\nabla_r f_0$$

$$\overline{\vec{v}} = \frac{4\pi}{3}\frac{1}{n}\int v^4 \vec{f}_1 dv = -\frac{4\pi}{3}\frac{1}{n}\int v^4 \frac{1}{\nu_1}\frac{\Gamma}{v}\frac{\partial f_0}{\partial v} dv + \frac{4\pi}{3}\frac{1}{n}\int v^4 \frac{1}{\nu_1}(\varpi_c x \vec{f}_1) dv - \frac{4\pi}{3}\frac{1}{n}\int v^4 \frac{\nabla f_0}{\nu_1} dv$$

$$= -\frac{4\pi}{3}\frac{1}{nm\nu_1} e\vec{E}\int v^3 \frac{\partial f_0}{\partial v} dv + \frac{4\pi}{3}\frac{1}{n}\varpi_c x\int v^4 \frac{1}{\nu_1}\vec{f}_1 dv - \frac{1}{3}\frac{4\pi}{n}\frac{1}{n}\int \frac{v^2}{\nu_1} v^2 \nabla f_0 dv$$

$$= -\frac{4\pi}{3}\frac{3}{nm\nu_1} e\vec{E}\int v^2 f_0 dv + \varpi_c x\left(\frac{4\pi}{3}\frac{1}{n}\int v^4 \frac{1}{\nu_1}\vec{f}_1 dv\right) - \frac{1}{3}\frac{4\pi}{n}\frac{1}{n}\int \frac{v^2}{\nu_1} v^2 \nabla f_0 dv$$

$$= -\frac{e}{m\nu_1}\vec{E}.1 + \varpi_c x\overline{\left(\frac{\vec{v}}{\nu_1}\right)} - \frac{1}{3}\nabla\left(\left(\overline{\frac{v^2}{\nu_1}}\right).n\right)$$

$$\overline{\vec{v}} = \mu\vec{E} + \varpi_c x\overline{\left(\frac{\vec{v}}{\nu_1}\right)} - \frac{1}{3}\nabla\left(\left(\overline{\frac{v^2}{\nu_1}}\right).n\right) = \mu\vec{E} + \frac{e}{m}\vec{B}x\overline{\left(\frac{\vec{v}}{\nu_1}\right)} - \frac{1}{3}\nabla\left(\left(\overline{\frac{v^2}{\nu_1}}\right).n\right) = \mu\vec{E} + \frac{e}{m\nu_1}\vec{B}x\overline{\vec{v}} - D\frac{\nabla n}{n}$$

Difúze a Drift v magnetickém poli

- B

$$\vec{E} \neq 0 \quad , \quad \vec{B} \neq 0 \quad , \quad \frac{\nabla n}{n} \neq 0$$

$$\overline{\vec{v}} = \mu\vec{E} + \varpi_c x \overline{\left(\frac{\vec{v}}{\nu_1}\right)} - \frac{1}{3}\nabla((\frac{\overline{v^2}}{\nu_1}).n) = \mu\vec{E} + \frac{e}{m}\vec{B}x\overline{\left(\frac{\vec{v}}{\nu_1}\right)} - \frac{1}{3}\nabla((\frac{\overline{v^2}}{\nu_1}).n) = \mu\vec{E} + \frac{e}{m\nu_1}\vec{B}x\overline{\vec{v}} - D\frac{\nabla n}{n}$$

Zjednodušeně

$$\overline{\vec{v}} = \mu\vec{E} + \frac{e}{m\nu_1}\vec{B}x\overline{\vec{v}} - D\frac{\nabla n}{n}$$

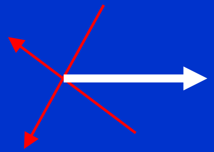
A co dál ????…… budeme řešit pro jednoduché konfigurace

Difúze napříč magnetickým polem

$$\vec{v} = \mu\vec{E} + \frac{e}{m\nu_1}\vec{B}x\vec{v} - D\frac{\nabla n}{n}$$

- Chen

$$mn\frac{d\mathbf{v}_\perp}{dt} = \pm en(\mathbf{E} + \mathbf{v}_\perp \times \mathbf{B}) - KT\,\nabla n - mn\nu\mathbf{v} = 0\,.$$



$$mn\nu v_x = \pm enE_x - KT\frac{\partial n}{\partial x} \pm env_yB\,,$$

$$mn\nu v_y = \pm enE_y - KT\frac{\partial n}{\partial x} \pm env_xB\,.$$

$$v_x = \pm\mu E_x - \frac{D}{n}\frac{\partial n}{\partial x} \pm \frac{\omega_c}{\nu}v_y$$

$$v_y = \pm\mu E_y - \frac{D}{n}\frac{\partial n}{\partial x} \mp \frac{\omega_c}{\nu}v_x$$

$$\tau = \nu^{-1}$$

$$v_y(1 + \omega_c^2\tau^2) = \pm\mu E_y - \frac{D}{n}\frac{\partial n}{\partial y} - \omega_c^2\tau^2\frac{E_x}{B} \pm \omega_c^2\tau^2\frac{KT}{eB}\frac{1}{n}\frac{\partial n}{\partial x}$$

$$v_x(1 + \omega_c^2\tau^2) = \pm\mu E_x - \frac{D}{n}\frac{\partial n}{\partial x} - \omega_c^2\tau^2\frac{E_y}{B} \mp \omega_c^2\tau^2\frac{KT}{eB}\frac{1}{n}\frac{\partial n}{\partial y}$$

$$\mu_\perp = \frac{\mu}{1 + \omega_c^2\tau^2}$$

$$D_\perp = \frac{D}{1 + \omega_c^2\tau^2}$$

$$v_\perp = \pm\mu_\perp E - D_\perp\frac{\nabla n}{n} +$$

Pozor bude to inak

Difúze napříč magnetickým polem

$$\vec{v} = \mu\vec{E} + \frac{e}{m\nu_1}\vec{B}x\vec{v} - D\frac{\nabla n}{n}$$

- Chen

$$\tau = \nu^{-1}$$

$$v_y(1+\omega_c^2\tau^2) = \pm\mu E_y - \frac{D}{n}\frac{\partial n}{\partial y} - \omega_c^2\tau^2\frac{E_x}{B} \pm \omega_c^2\tau^2\frac{KT}{eB}\frac{1}{n}\frac{\partial n}{\partial x}$$

$$v_x(1+\omega_c^2\tau^2) = \pm\mu E_x - \frac{D}{n}\frac{\partial n}{\partial x} - \omega_c^2\tau^2\frac{E_y}{B} \mp \omega_c^2\tau^2\frac{KT}{eB}\frac{1}{n}\frac{\partial n}{\partial y}$$

$$v_{Ex} = \frac{E_y}{B}, \qquad v_{Ey} = -\frac{E_x}{B},$$

$$v_{Dx} = \mp\frac{KT}{eB}\frac{1}{n}\frac{\partial n}{\partial y}, \qquad v_{Dy} = \pm\frac{KT}{eB}\frac{1}{n}\frac{\partial n}{\partial x}$$

$$\mu_\perp = \frac{\mu}{1+\omega_c^2\tau^2}$$

$$D_\perp = \frac{D}{1+\omega_c^2\tau^2}$$

$$v_\perp = \pm\mu_\perp E - D_\perp\frac{\nabla n}{n} + \frac{v_E + v_D}{1+(\nu^2/\omega_c^2)}$$

…použijeme na pravé straně (3.216) výrazu (3.217), dostaneme, že

$$E' = \frac{j_\parallel}{\sigma_\parallel} + \frac{j_\perp}{\sigma_\perp} + \frac{1}{n_e e}(j \times B),$$

$$\sigma_\parallel = 1{,}96\sigma \quad \text{a} \quad \sigma_\perp = \sigma\,;$$

vyloučením j pak dostaneme

(3.225) $$j = \sigma_\parallel E'_\parallel + \frac{\sigma_\perp}{1+\omega_{ce}^2\tau_e^2}\{E'_\perp + \omega_{ce}\tau_e(h \times E')\}.$$

Odtud již vidíme, že i v tak jednoduchém případě, kdy $\nabla_r T_e = 0$ a systém je blízko rovnováhy, je vodivost plazmatu závislá na směru magnetického pole.

V obecném případě má vodivost plazmatu tenzorový charakter.*)

Difúze napříč magnetickým polem

- Interpretace

$\tau = \nu^{-1}$

$$D_\perp = \frac{D}{1+\omega_c^2\tau^2} \qquad \mu_\perp = \frac{\mu}{1+\omega_c^2\tau^2}$$

$$v_\perp = \pm\mu_\perp E - D_\perp \frac{\nabla n}{n} + \frac{v_E + v_D}{1 + (\nu^2/\omega_c^2)}$$

$$v_{Ex} = \frac{E_y}{B}, \qquad v_{Ey} = -\frac{E_x}{B},$$

$$v_{Dx} = \mp \frac{KT}{eB}\frac{1}{n}\frac{\partial n}{\partial y}, \qquad v_{Dy} = \pm \frac{KT}{eB}\frac{1}{n}\frac{\partial n}{\partial x}$$

Rovnobežné s gradienty Kolmé na gradienty

Z tohoto výrazu je zřejmé, že kolmá rychlost toho či onoho druhu částic se skládá ze dvou částí. Za prvé jsou to obyčejné driftové rychlosti $\mathbf{v}_E$ a $\mathbf{v}_D$ *kolmé* na gradienty potenciálu a hustoty. Tyto drifty jsou zpomaleny srážkami s neutrálními částicemi; brzdící faktor $1 + (\nu^2/\omega_c^2)$ se v limitě $\nu \to 0$ rovná jedničce. Za druhé jsou to drifty způsobené pohyblivostí a difúzí, *rovnoběžné* s gradienty potenciálu a hustoty. Tyto drifty jsou vyjádřeny stejným způsobem jako v případě $B = 0$, ale koeficienty μ a D jsou zmenšeny faktorem $1 + \omega_c^2\tau^2$.

Rovnoběžné s gradienty

$$\mu_\perp = \mu \frac{1}{1+(\varpi_c/\nu_1)^2} \qquad D_\perp = D\frac{1}{1+(\varpi_c/\nu_1)^2}$$

Kolmé na gradienty

$$\sim \frac{1}{1+(\nu_1/\varpi_c)^2}$$

- A

Difúze napříč magnetickým polem

- Interpretace

$$\tau = \nu^{-1}$$

$$v_\perp = \pm\mu_\perp E - D_\perp \frac{\nabla n}{n} + \frac{v_E + v_D}{1 + (\nu^2/\omega_c^2)}$$

$$v_{Ex} = \frac{E_y}{B}, \qquad v_{Ey} = -\frac{E_x}{B},$$

$$v_{Dx} = \mp \frac{KT}{eB}\frac{1}{n}\frac{\partial n}{\partial y}, \qquad v_{Dy} = \pm \frac{KT}{eB}\frac{1}{n}\frac{\partial n}{\partial x}$$

$$\omega_c\tau = \omega_c/\nu = \mu B \cong \lambda_s/r_L$$

$$\mu_\perp = \mu \frac{1}{1+(\varpi_c/v_1)^2} \qquad D_\perp = D\frac{1}{1+(\varpi_c/v_1)^2}$$

Rovnoběžné s gradienty

Kolmé na gradienty

$$\sim \frac{1}{1+.(v_1/\varpi_c)^2}$$

$$(\varpi_c/v_1)^2 >> 1$$

Snižuje difúzi napříč B

$$D_\perp = \frac{KT}{m\nu}\frac{1}{\omega_c^2\tau^2} = \frac{KT\nu}{m\omega_c^2}$$

Difúze bez B

$$D = \frac{kT}{mv_1}$$

$$(\varpi_c/v_1)^2 << 1$$

Malý vliv na difúzi

Difúze napříč magnetickým polem

- Interpretace

$$\tau = \nu^{-1}$$

$$v_\perp = \pm\mu_\perp E - D_\perp \frac{\nabla n}{n} + \frac{v_E + v_D}{1 + (\nu^2/\omega_c^2)}$$

$$v_{Ex} = \frac{E_y}{B}, \qquad v_{Ey} = -\frac{E_x}{B},$$

$$v_{Dx} = \mp\frac{KT}{eB}\frac{1}{n}\frac{\partial n}{\partial y}, \qquad v_{Dy} = \pm\frac{KT}{eB}\frac{1}{n}\frac{\partial n}{\partial x}$$

$$\omega_c\tau = \omega_c/\nu = \mu B \cong \lambda_s/r_L$$

$$\mu_\perp = \mu\frac{1}{1+(\varpi_c/\nu_1)^2} \qquad D_\perp = D\frac{1}{1+(\varpi_c/\nu_1)^2}$$

Rovnoběžné s gradienty

Kolmé na gradienty

$$\sim\frac{1}{1+.(\nu_1/\varpi_c)^2}$$

Difúze bez B

$$D = KT/m\nu \sim v_t^2\tau \sim \lambda_s^2/\tau.$$

$$(\varpi_c/\nu_1)^2 >> 1$$

Snižuje difúzi napříč B

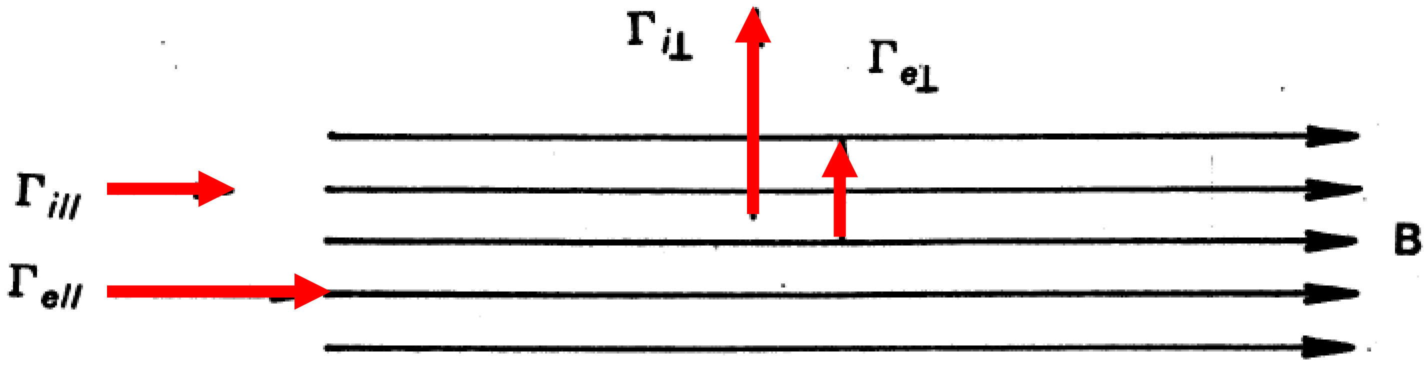
$$D_\perp = \frac{KT\nu}{m\omega_c^2} \sim v_t\frac{r_L^2}{v_t^2}\nu \sim \frac{r_L^2}{\tau}$$

„Difúzní krok“

$$(\varpi_c/\nu_1)^2 << 1$$

Malý vliv na difúzi

Ambipolární difúze napříč B



OBR. 5-13 Toky částic rovnoběžné s magnetickým polem a kolmé na něj.

Srovnávají se divergence…

$$\nabla \cdot \Gamma_i = \nabla \cdot \Gamma_e$$

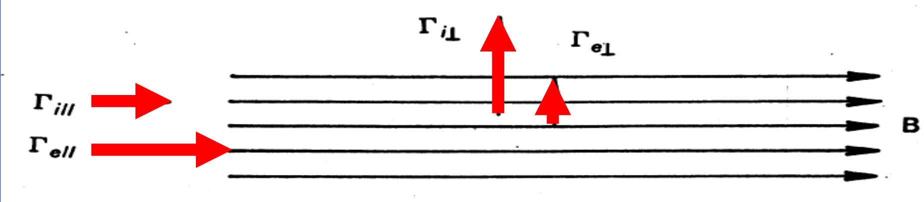
Je to složité

$$\nabla \cdot \Gamma_i = \nabla_\perp \cdot (\mu_{i\perp} n \mathbf{E}_\perp - D_{i\perp} \nabla n) + \frac{\partial}{\partial z}\left(\mu_i n E_z - D_i \frac{\partial n}{\partial z}\right)$$

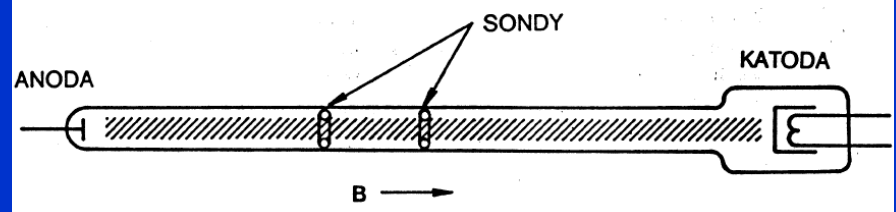
$$\nabla \cdot \Gamma_e = \nabla_\perp \cdot (-\mu_{e\perp} n \mathbf{E}_\perp - D_{e\perp} \nabla n) + \frac{\partial}{\partial z}\left(-\mu_e n E_z - D_e \frac{\partial n}{\partial z}\right)$$

Ambipolární difúze napříč B, experiment

- Experiment



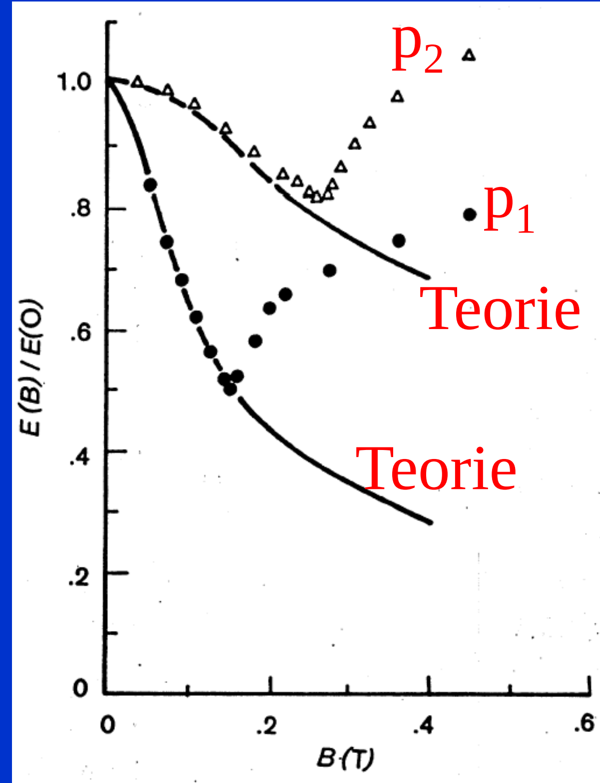
OBR. 5-13 Toky částic rovnoběžné s magnetickým polem a kolmé na něj.



OBR. 5-14 Lehnertův–Hohův experiment ověřující vliv magnetického pole na difúzi v slabě ionizovaném plynu.

Otázka, zda magnetické pole zmenšuje příčnou difúzi v souhlase s rov. [5-51], se stala předmětem četných zkoumání. Prvý experiment uskutečněný v dostatečně dlouhé trubici, takže difúzi ke koncům bylo možno zanedbat, provedli Lehnert a Hoh ve Švédsku. Kladný sloupec héliového výboje měl průměr 1 cm a byl dlouhý 3,5 m (obr. 5-14). V takovém plazmatu elektrony plynule unikají radiální difúzí ke stěnám a jsou nahrazovány ionizací neutrálního plynu elektrony z konce rychlostního rozdělení. Tyto rychlé elektrony jsou zase nahrazovány urychlením v podélném elektrickém poli.

Ionty unikající difúzi jsou nahrazovaný ionizaci
➔ E_z bude úměrné difúzi.



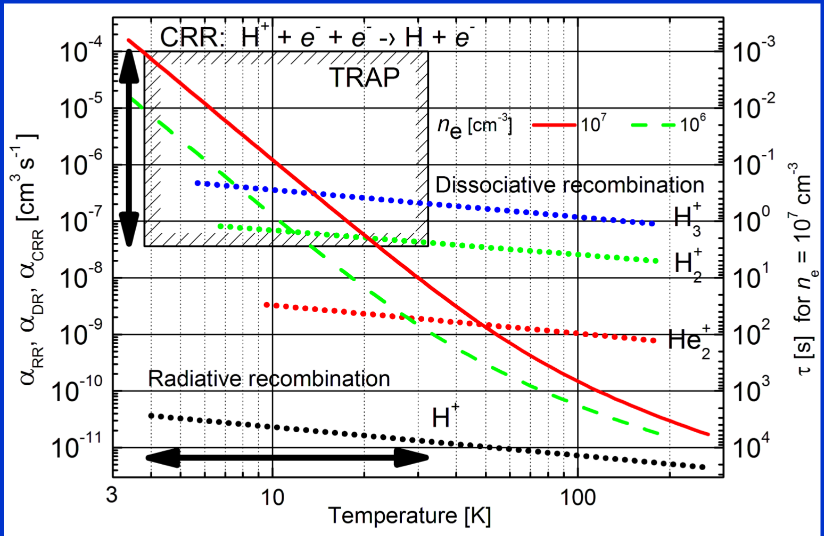
Můžeme proto očekávat, že E_z bude zhruba úměrné velikosti příčné difúze. Dvěma sondami u stěn výbojové trubice měřili E_z při různém B. Na obr. 5-15 je vynesen poměr $E_z(B)$ ku $E_z(0)$ jako funkce B. Pro malé B experimentální body velmi dobře sledují předpovězenou křivku vypočtenou na základě rovnice [5-52]. Při určitém kritickém poli B_k okolo 0,1 T se však experimentální body vzdalují od teorie a vykazují ve skutečnosti *vzrůst* difúze

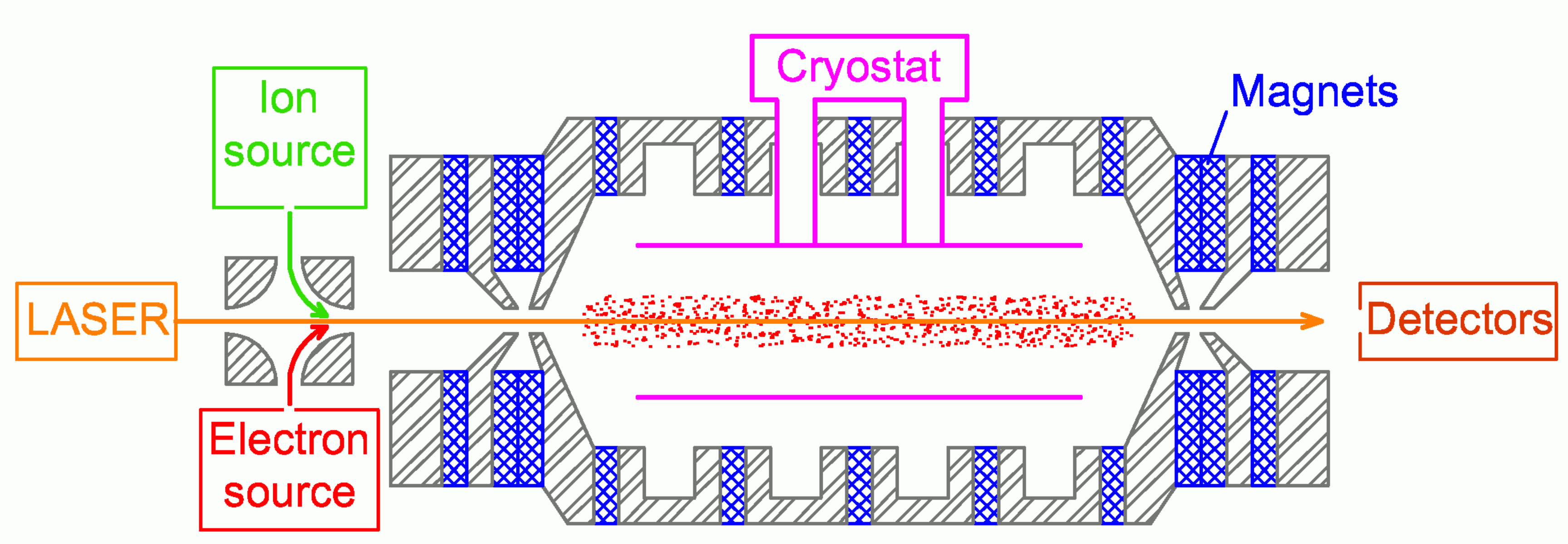
←→ Nestability v plazmatu

Zákony zachování – řešení B.

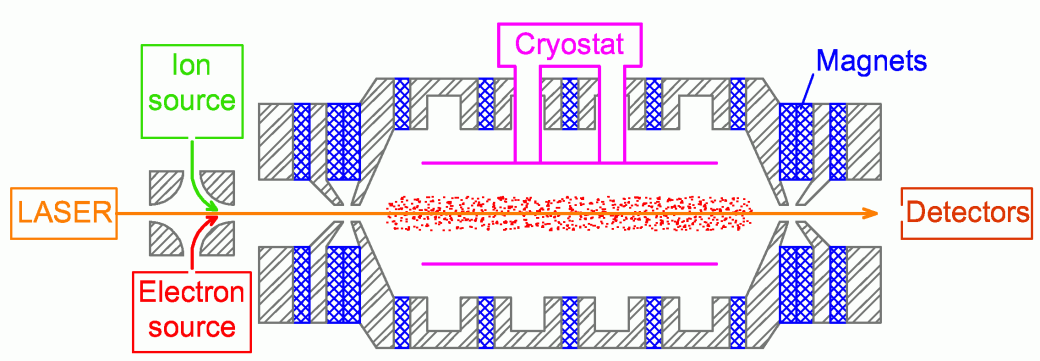
- A

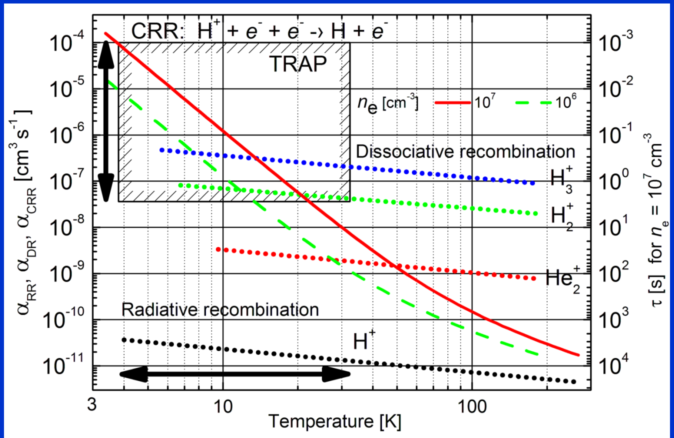
In magnetic and electric field example

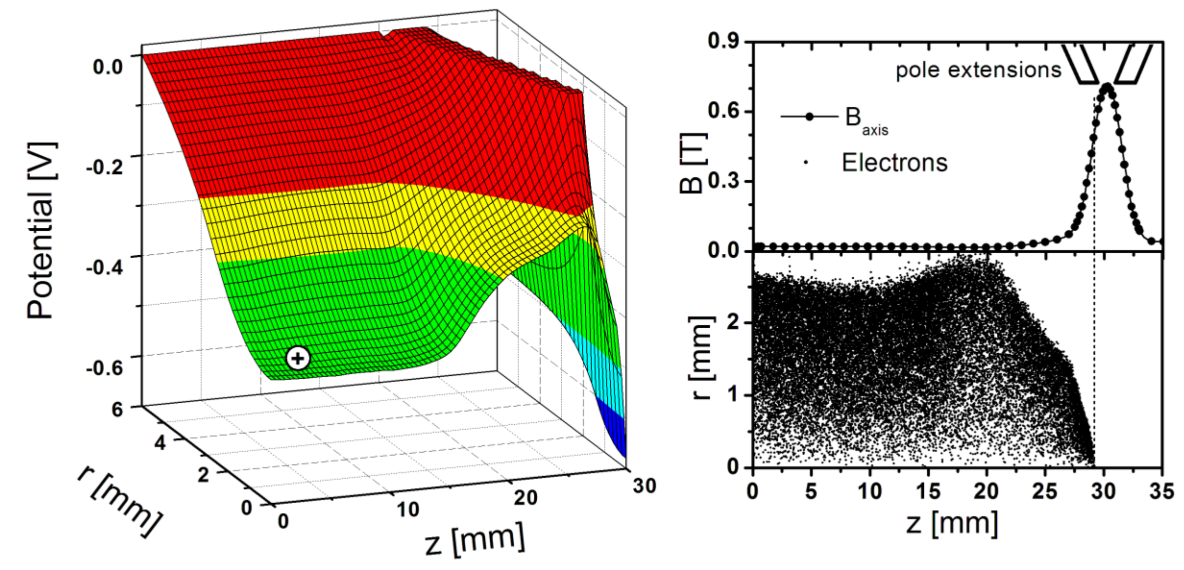




In magnetic and electric field example

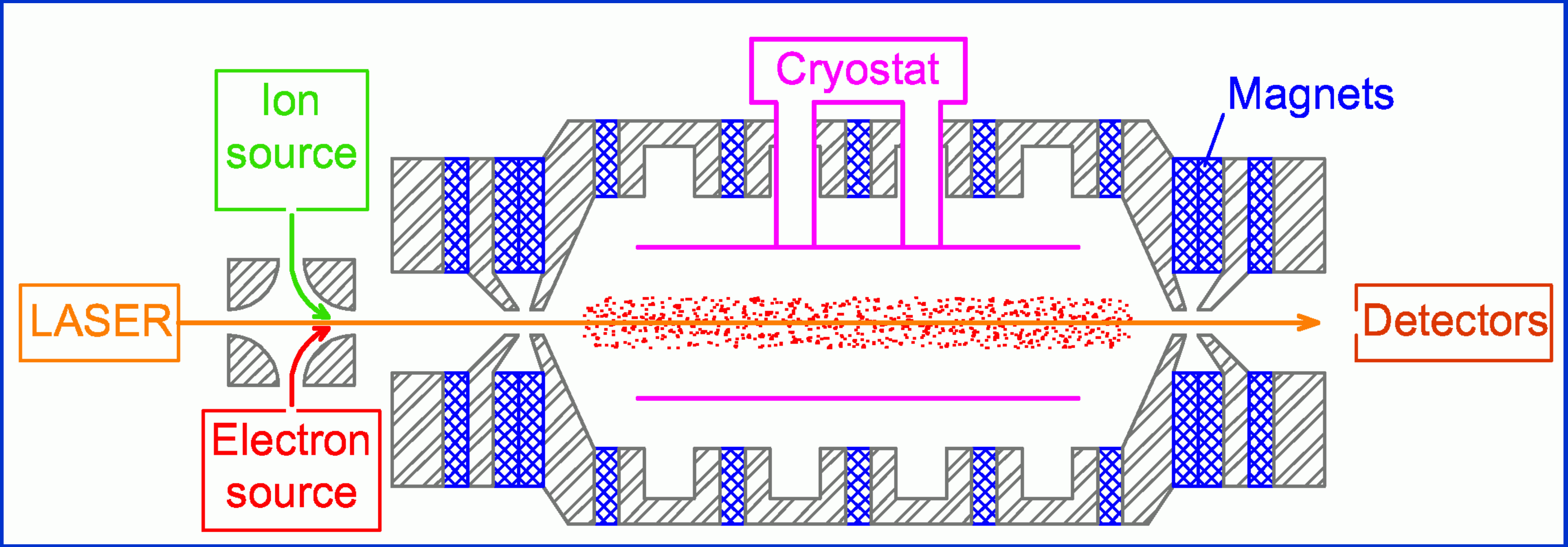






In magnetic and electric field

example



Zákony zachování – řešení B.

- A

<u>Ternary electron assisted recombination</u>

Ternary electron assisted recombination

$$Ar^{+} + e + e \rightarrow Ar + e$$

$$\frac{dn_e}{dt} = \frac{d[Ar^{+}]}{dt} = -K_e[Ar^{+}]n_e^{\;2} = -\alpha_{eff}[Ar^{+}]n_e$$

Collisional Radiative Recombination

CRR

K_{CRR} **[cm⁶s⁻¹]**

$$\alpha_{eff} = K_e n_e$$

Ternary neutral assisted recombination

$$Ar^{+} + e + He \rightarrow Ar + He$$

$$\frac{dn_e}{dt} = \frac{d[Ar^{+}]}{dt} = -K_M[Ar^{+}]n_e[He] = -\alpha_{eff}[Ar^{+}]n_e$$

K_M **[cm⁶s⁻¹]**

$$\alpha_{eff} = K_M[He]$$

Stevefelt [*Stevefelt et al.*, 1975] derived analytical formula for apparent binary rate coefficient of CRR:

$$\alpha_{\mathrm{CRR}} = 3.8\times10^{-9} T_{\mathrm{e}}^{-4.5} n_{\mathrm{e}} + 1.55\times10^{-10} T_{\mathrm{e}}^{-0.63} + 6\times10^{-9} T_{\mathrm{e}}^{-2.18} n_{\mathrm{e}}^{0.37}\ [\mathrm{cm^3 s^{-1}}], \quad (4)$$

where T_{e} is electron temperature given in K and n_{e} is electron number density in $\mathrm{cm^{-3}}$. The first term in

$$\alpha_{\mathrm{CRR}} = K_{\mathrm{CRR}} n_{\mathrm{e}}$$

For quasineutral plasma the differential equation describing the overall losses of charged particles in plasma due to above mentioned processes described by equations (1), (2) and (3) is:

$$\frac{\mathrm{d}n_{\mathrm{e}}}{\mathrm{d}t} = \frac{\mathrm{d}[\mathrm{A}^+]}{\mathrm{d}t} = -\alpha_{\mathrm{BIN}}[\mathrm{A}^+]n_{\mathrm{e}} - K_{\mathrm{M}}[\mathrm{M}][\mathrm{A}^+]n_{\mathrm{e}} - K_{\mathrm{CRR}}[\mathrm{A}^+]n_{\mathrm{e}}^2 - \frac{n_{\mathrm{e}}}{\tau_{\mathrm{D}}} = -\alpha_{\mathrm{eff}} n_{\mathrm{e}}^2 - \frac{n_{\mathrm{e}}}{\tau_{\mathrm{D}}} \quad (6)$$

where $[\mathrm{A}^+] = n_{\mathrm{e}}$ is the number density of ions, [M] is the number density of particles of buffer gas and τ_{D} describes the diffusion losses. We introduced the effective recombination rate coefficient α_{eff}:

$$\alpha_{\mathrm{eff}} = \alpha_{\mathrm{BIN}} + K_{\mathrm{CRR}} n_{\mathrm{e}} + K_{\mathrm{He}}[\mathrm{He}] \quad (7)$$

Theoretical calculations of rate coefficient of electron – ion ternary recombination (assisted by the particle of buffer gas) [*Thomson*, 1924; *Pitaevskii*, 1962; *Bates et al.*, 1965; *Bates*, 1980; *Flannery*, 1991] propose less pronounced temperature dependence of this process ($\alpha \approx T^{-2.5}$) than in case of CRR and also that the recombination coefficient should be lower for heavy ions than for light ions. Flannery [*Flannery*, 1991] derived following formula for helium assisted ternary recombination:

$$K_{\mathrm{He}} = 2.3\times10^{-27}(300/T_e)^{2.5}\ \mathrm{cm^6 s^{-1}}. \quad (8)$$

$Ar^{+} + e^{-} + e^{-}$

$H^{+} + e^{-} + e^{-}$

Anti hydrogen formation

Colisional Radiative Recombination -CRR

$$\frac{dn_e}{dt} = -K_{CRR}\ [Ar^{+}]n_e^2 - \frac{n_e}{\tau_D} = -K_{CRR}\ \ n_e^3 - \frac{n_e}{\tau_D}$$

$$\alpha_{CRR} = K_{CRR}\ n_e$$

PRL **98,** 133201 (2007) PHYSICAL REVIEW LETTERS week ending 30 MARCH 2007

Three-Body Recombination of Atomic Ions with Slow Electrons

2007

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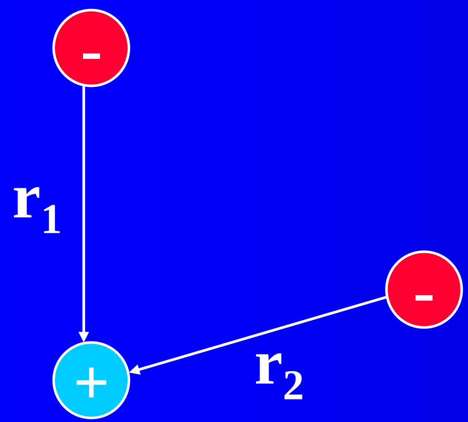
$H^+ + e^- + e^- \rightarrow H + e^-$



We consider the simplest TBR in the case of hydrogen formation, in which two free electrons interact with a proton. To investigate the three-body interaction dynamics, we numerically solve the six-dimensional (6D) time-dependent Schrödinger equation, which has the following form (atomic units are used throughout):

$$i\frac{\partial}{\partial t}\Phi(\mathbf{r}_1, \mathbf{r}_2, t) = \left[-\frac{1}{2}(\Delta_{\mathbf{r}_1} + \Delta_{\mathbf{r}_2}) - \frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|}\right]\Phi(\mathbf{r}_1, \mathbf{r}_2, t), \quad (1)$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the position vectors of each electron, with respect to the proton. We obtain a more tractable

with respect to the proton. We obtain a more tractable solution by using the close-coupling recipe [12]: expanding the 6D wave function $\Phi(\mathbf{r}_1, \mathbf{r}_2|t)$ in terms of bipolar spherical harmonics $Y^{LS}_{l_1 l_2}(\Omega_1, \Omega_2)$, $\Phi(\mathbf{r}_1, \mathbf{r}_2|t) = \sum_{LS}\sum_{l_1 l_2}[\Psi^{(LS)}_{l_1 l_2}(r_1, r_2|t)/r_1 r_2]Y^{LS}_{l_1 l_2}(\Omega_1, \Omega_2)$, for a specific symmetry (LS). We can also expand the Coulomb repulsion term $1/|\mathbf{r}_1 - \mathbf{r}_2|$ in terms of spherical harmonics. Substituting these expansions into the above Schrödinger Eq. (1) and integrating over the angles Ω_1 and Ω_2 yields a set of coupled partial differential equations with only two radial variables r_1 and r_2 left:

$$i\frac{\partial}{\partial t}\Psi_j(r_1, r_2|t) = [\hat{T}_1 + \hat{T}_2 + \hat{V}_c]\Psi_j(r_1, r_2|t) + \sum_k \hat{V}^I_{j,k}(r_1, r_2|t)\Psi_k(r_1, r_2|t), \quad (2)$$

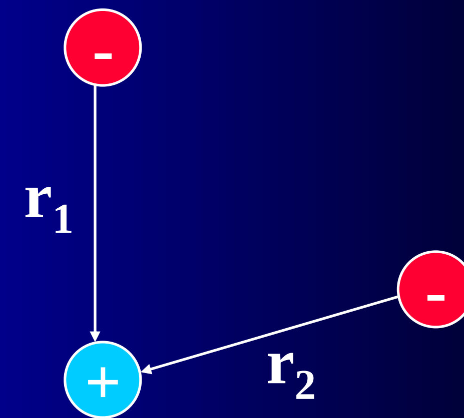
where the partial-wave index j runs from 1 to the total number N of partial waves used for expansion. In Eq. (2),

$H^+ + e^- + e^- \rightarrow H + e^-$

$$i\frac{\partial}{\partial t}\Psi_j(r_1, r_2|t) = [\hat{T}_1 + \hat{T}_2 + \hat{V}_c]\Psi_j(r_1, r_2|t) + \sum_k \hat{V}^I_{j,k}(r_1, r_2|t)\Psi_k(r_1, r_2|t), \quad (2)$$

$$P_{nl}(E_2) = 2\sum_{LS}\sum_{l_2}\left|\int dr_1 \int dr_2 \phi^*_{nl}(r_1)\phi^*_{k_2 l_2}(r_2)\Psi^{(LS)}_{ll_2}(r_1, r_2, t = t_f)\right|^2,$$

K_E=0.1 eV



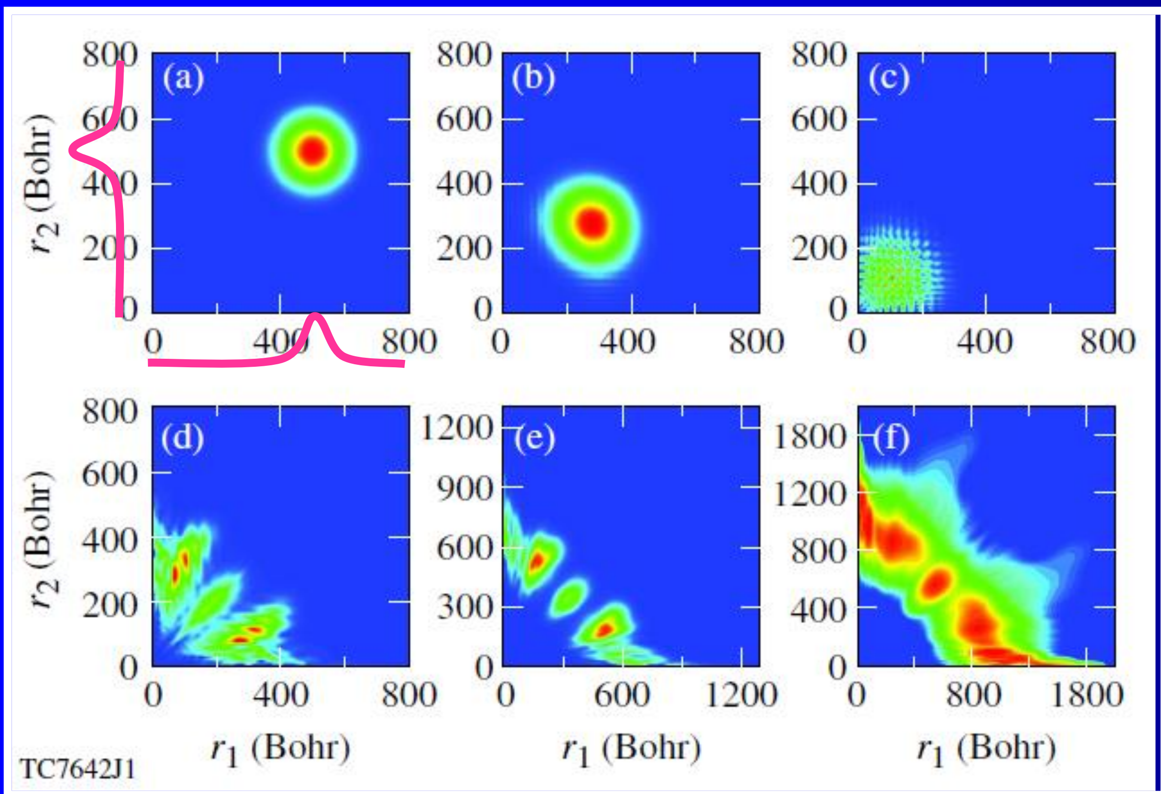
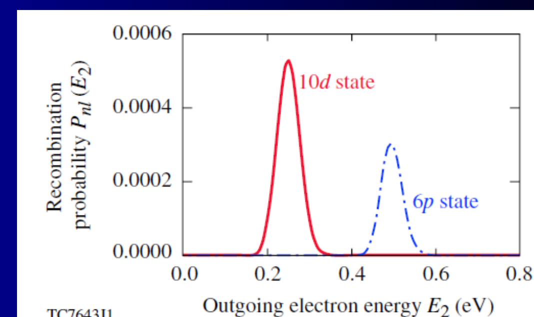


FIG. 1 (color online). Snapshots of electron probability distribution on the plane spanned by the radial coordinates r_1 and r_2 for different times: (a) $t = 0.0$ fs, (b) $t = 60$ fs, (c) $t = 100$ fs, (d) $t = 150$ fs, (e) $t = 194$ fs, and (f) (in log scale) $t = 260$ fs.



Thus, for the case of $K_E = 0.1$ eV considered in Figs. 1 and 2, the total system energy is about $E_{tot} \sim 0.12$ eV instead of $2K_E$. Hence, when one electron recombines to the 10d state ($|E_{10d}| \approx 0.136$ eV) of the H atom, the outgoing electron takes an initial total energy of 0.12 eV plus $|E_{10d}|$, thereby $P_{10d}(E_2)$ peaks at $E_2 \sim 0.256$ eV, as shown by the (red) solid line of Fig. 2. Similar energy conservation is also well satisfied for the recombination to the 6p state, as is illustrated by the (blue) dash-dotted line in Fig. 2. Our quantum calculations unambiguously reveal the essential feature of a TBR process.

$H^+ + e^- + e^- \rightarrow H + e^-$

K_E=0.1 eV

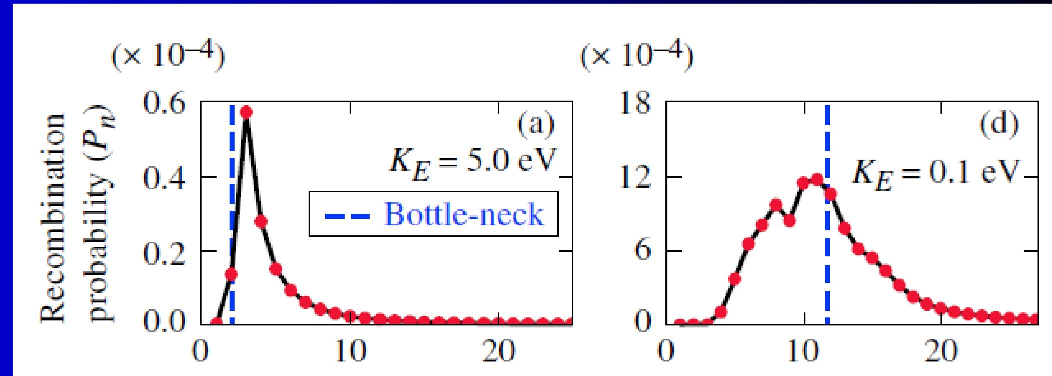
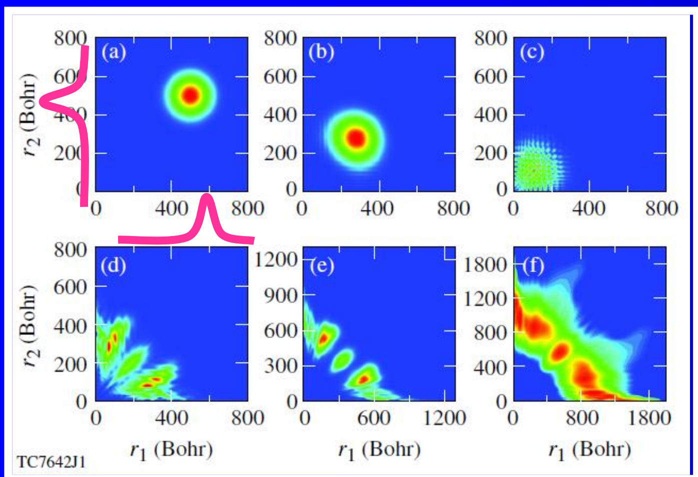
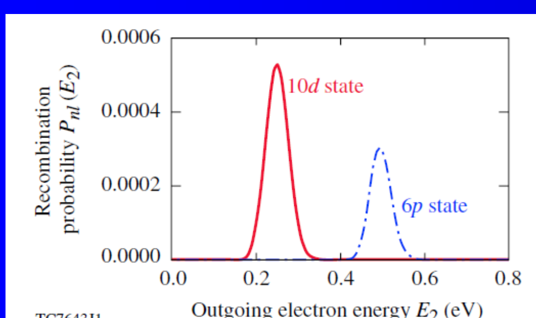


FIG. 3 (color online). The recombination probability P_n as a function of the energy level n, for different electron kinetic energies K_E marked in each panel.





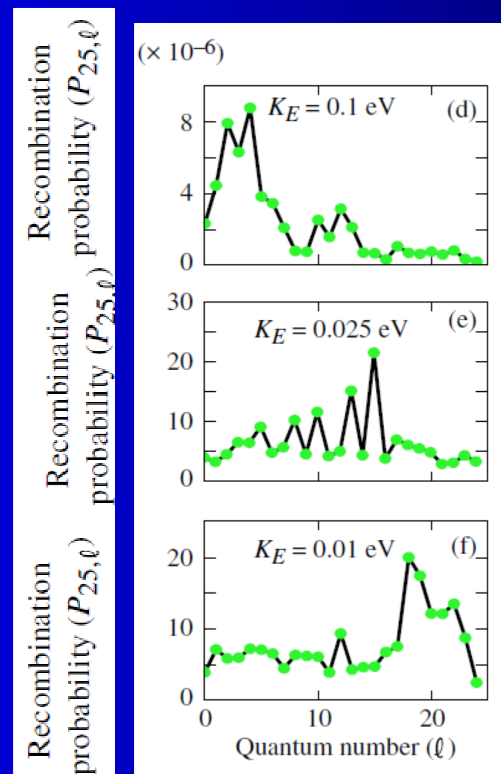


FIG. 4 (color online). The recombination probability $P_{n=25,l}$ as a function of the angular-momentum quantum number l, for different electron kinetic energies K_E marked in each panel.

$Ar^+ + e^- + e^-$

$H^+ + e^- + e^-$

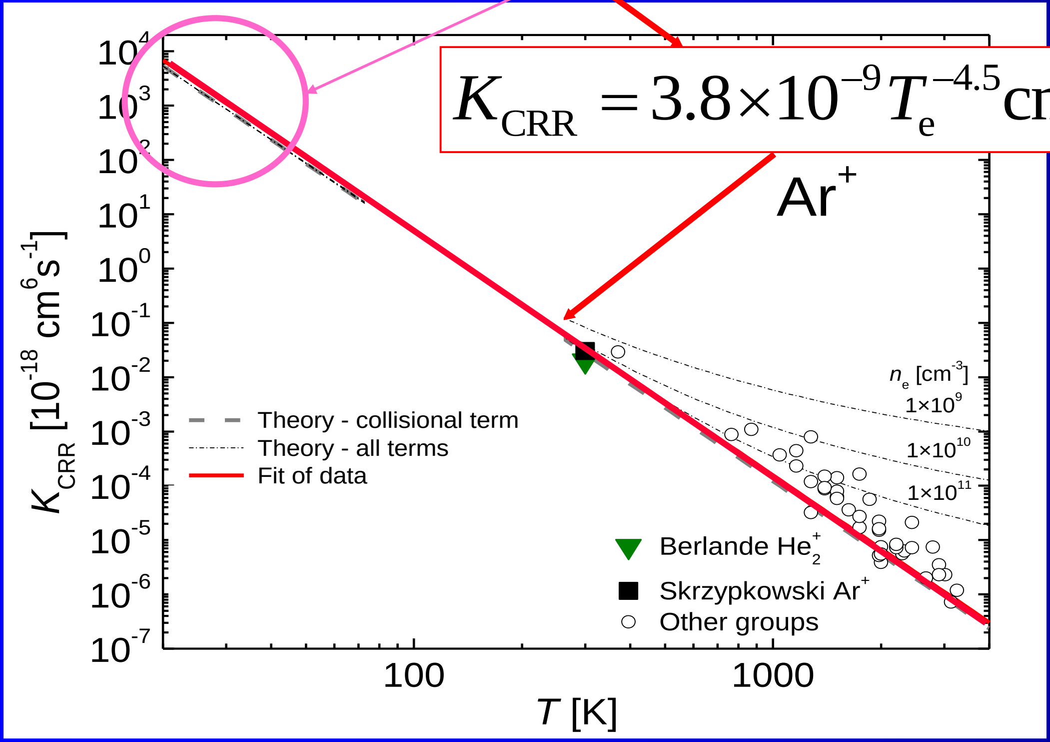
Anti hydrogen formation

$$\frac{dn_e}{dt} = -K_{CRR}\ [Ar^+] n_e^2 - \frac{n_e}{\tau_D} = -K_{CRR}\ \ n_e^3 - \frac{n_e}{\tau_D}$$

$$\alpha_{\mathrm{CRR}} = 3.8\times10^{-9} T_e^{-4.5} n_e + 1.55\times10^{-10} T_e^{-0.63} + 6\times10^{-9} T_e^{-2.18} n_e^{0.37} \mathrm{cm^3 s^{-1}}$$

$$\alpha_{CRR} = K_{CRR}\ n_e$$

$$K_{\mathrm{CRR}} = 3.8\times10^{-9} T_e^{-4.5} \mathrm{cm^6 s^{-1}}$$

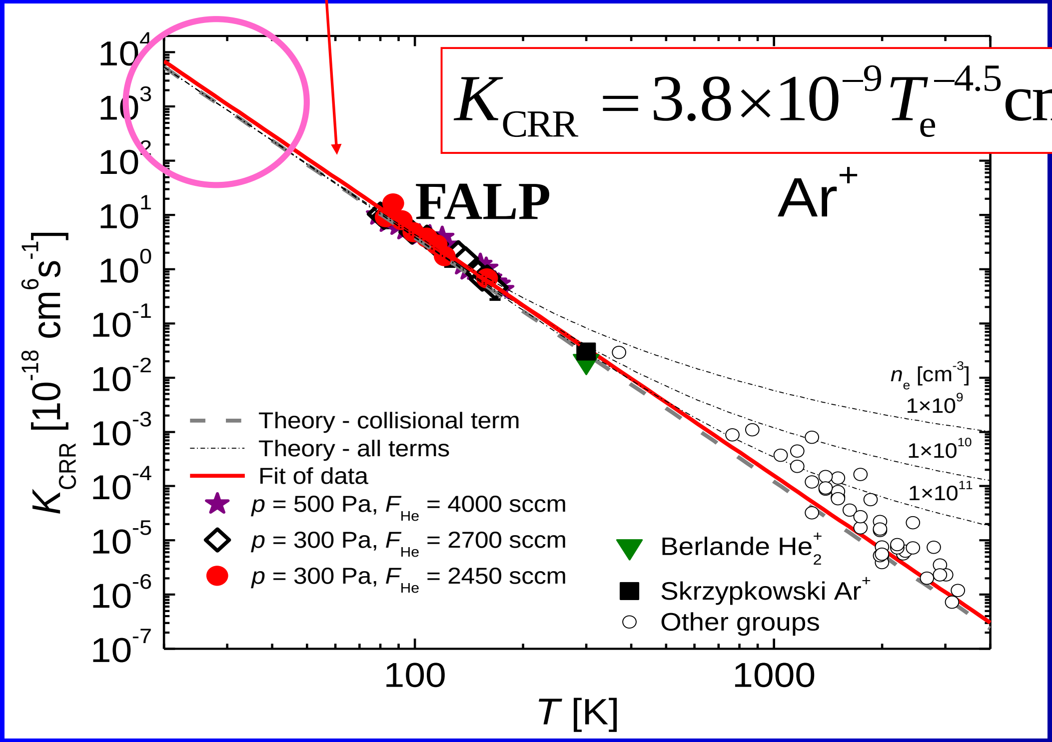


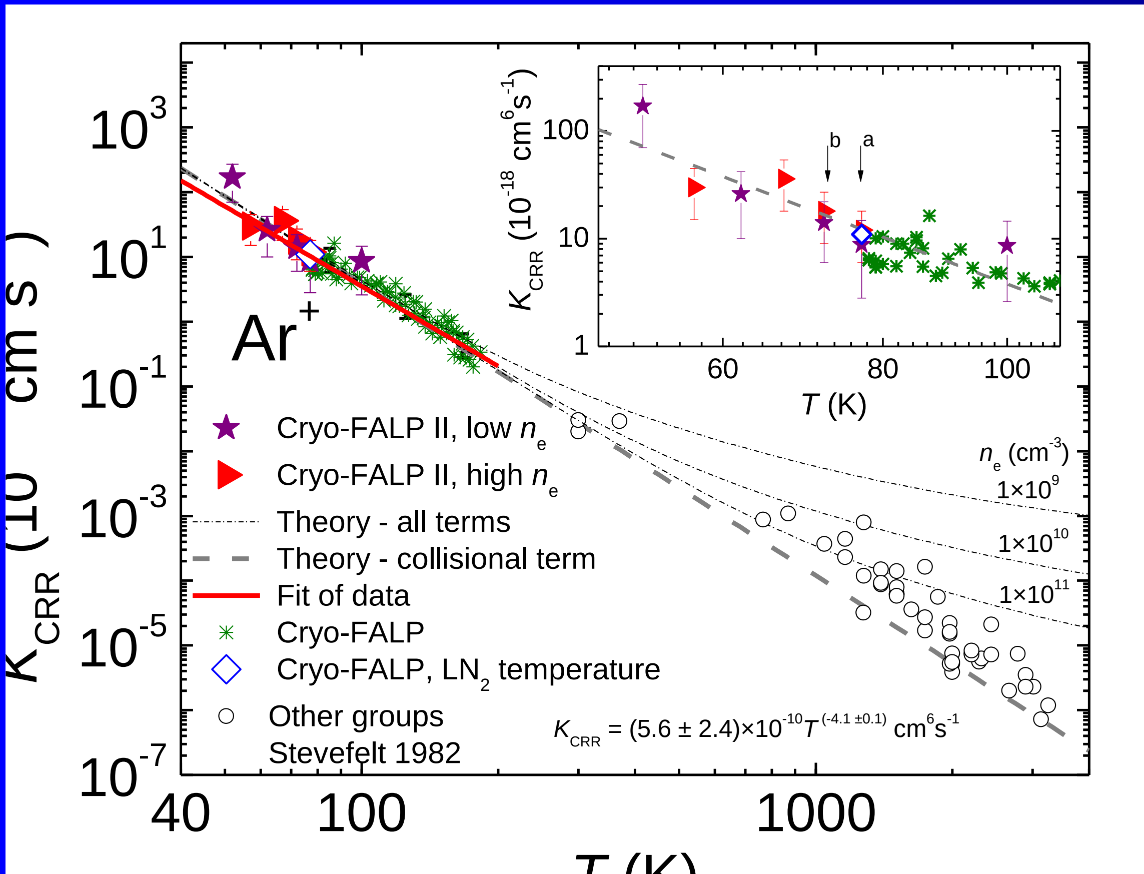
$Ar^+ + e^- + e^-$

$$\frac{dn_e}{dt} = -K_{CRR}\,[Ar^+]\,n_e^2 - \frac{n_e}{\tau_D} = -K_{CRR}\,n_e^3 - \frac{n_e}{\tau_D}$$

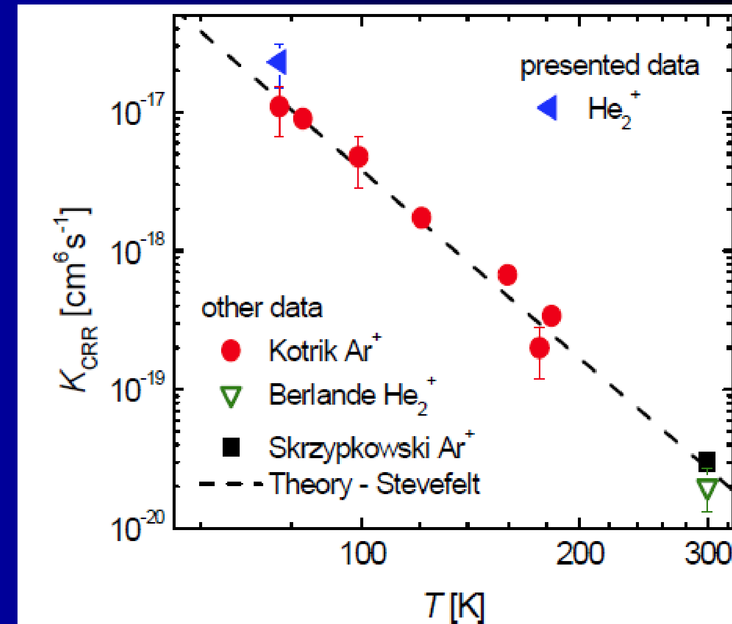
$$\alpha_{\mathrm{CRR}} = 3.8\times10^{-9}T_e^{-4.5}n_e + 1.55\times10^{-10}T_e^{-0.63} + 6\times10^{-9}T_e^{-2.18}n_e^{0.37}\,\mathrm{cm^3s^{-1}}$$

$$K_{\mathrm{CRR}} = 3.8\times10^{-9}T_e^{-4.5}\,\mathrm{cm^6s^{-1}}$$





Ar+
Cryo-FALP II, low ne
Cryo-FALP II, high ne
Theory - all terms
Theory - collisional term
Fit of data
Cryo-FALP
Cryo-FALP, LN2 temperature
Other groups
Stevefelt 1982
ne (cm-3)
1×10^9
1×10^10
1×10^11
KCRR = (5.6 ± 2.4)×10^-10 T^(-4.1 ±0.1) cm6s-1
KCRR (10-18 cm6s-1)
T (K)
b
a
KCRR
T (K)



presented data
He2+
other data
Kotrik Ar+
Berlande He2+
Skrzypkowski Ar+
Theory - Stevefelt
KCRR [cm6s-1]
T [K]

Zákony zachování – řešení B.

- A

Zákony zachování – řešení B.

- A

Eltest

The ELTEST Experiment

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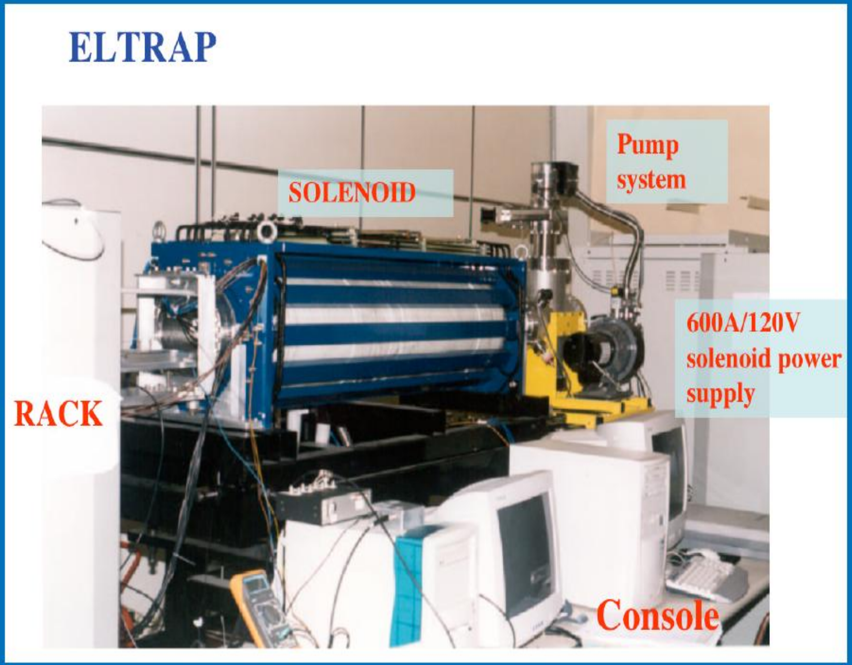
INFN Sezione di Milano and Dipartimento di Fisica Università di Milano
INFN Laboratori Nazionali di Legnaro

19 September 2005

The ELETRASP Experiment



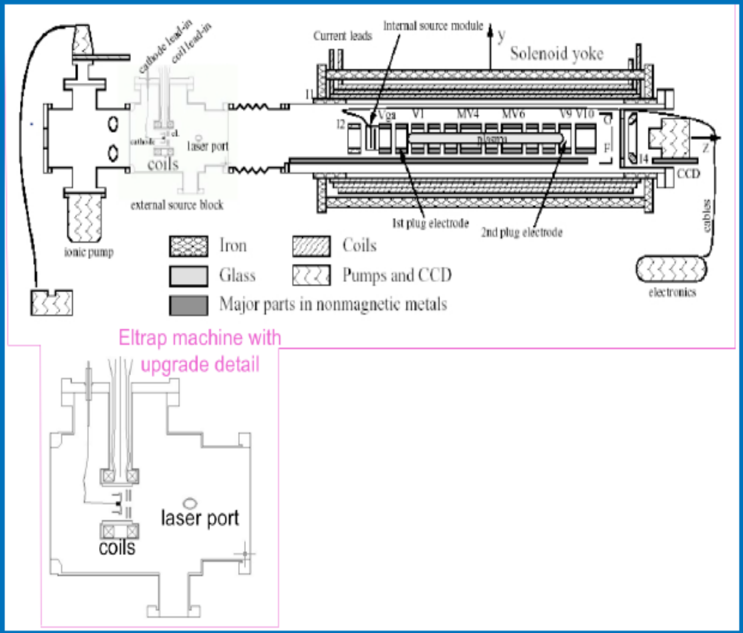


Figure 2: Eltrap schematics with the pulsed source

Eltest

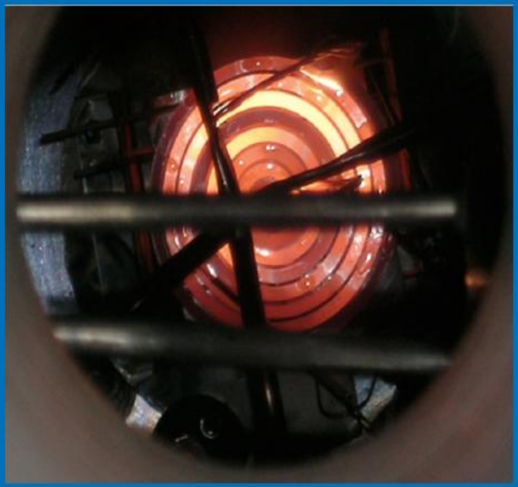


Figure 8: Planar source

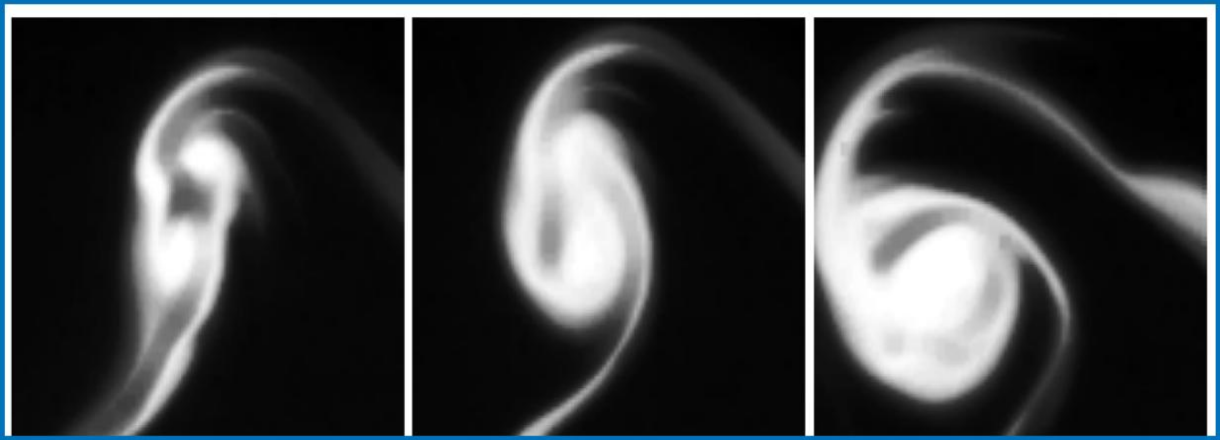


Figure 3: Evolution of a merger process observed in Eltrap (The enlargements of the three pictures are different)

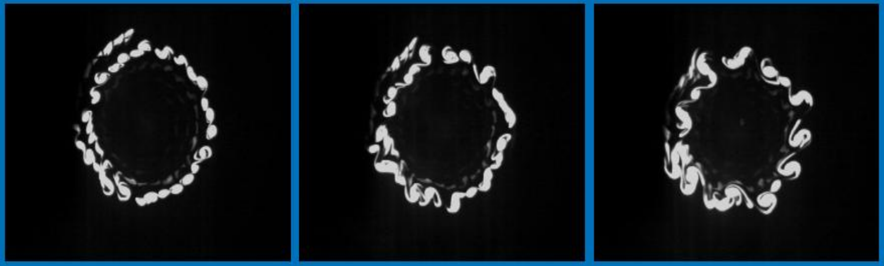


Figure 4: Evolution of an electron beam in ELTRAP in the space charge regime. The images have been taken using different values of the confining magnetic field (decreasing from left to the right picture).

Eltest

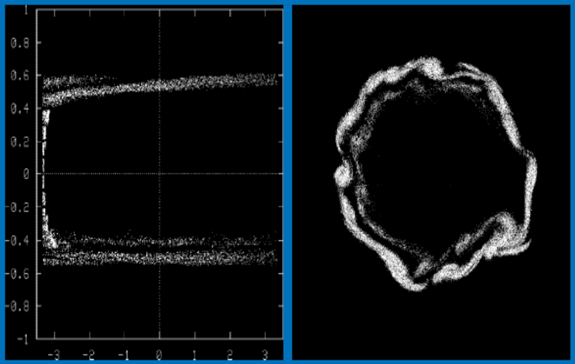


Figure 5: Computation of the beam evolution in ELTRAP using the PIC code MEP. Left: macroparticle distribution in a thin longitudinal layer containing the axis, computed at a time close to a stationary state. Right: macroparticle distribution in a thin layer on the phosphor screen, at the same time.

Eltrap

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Cylindrical Penning trap for the study of electron plasmas

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The ELTRAP device installed at the Department of Physics of the University of Milan is a Malmberg–Penning trap, with a magnetic field up to 0.2 T, equipped with charge coupled device optical diagnostics. It is intended to be a small scale facility for electron plasma and beam dynamics experiments, and in particular for the study of collective effects, equilibrium states, and the formation of coherent structures in these systems. The device features a relatively long solenoid, corrected by 4 shims and 16 dipole coils, in order to obtain a large uniform magnetic field region. The modular electrode design allows several variations of the experimental configuration. The first experiments which assess the operation of the facility are described. Plasma confinement times up to several minutes have been obtained and an electron temperature of 4–8 eV has been measured.
 [DOI: 10.1063/1.1602931]

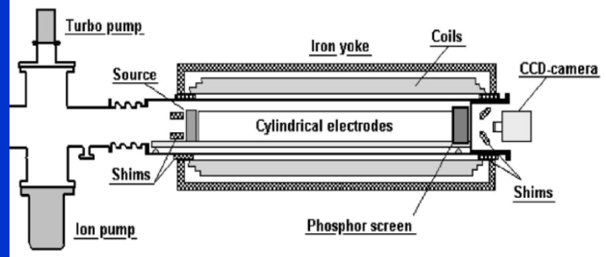


FIG. 1. Schematic of the ELTRAP machine, showing the solenoid with the iron yoke and the shims, the vacuum chamber with the reentrant flange, the pumping system and the main internal structures (not to scale). The cylindrical electrodes are shown in detail in Fig. 2.

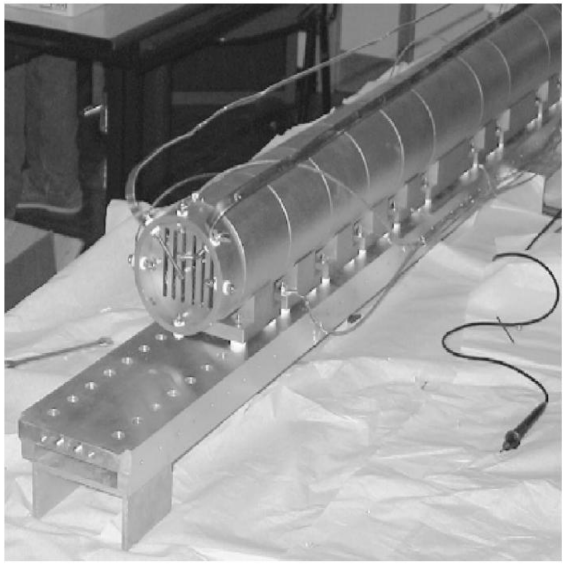


FIG. 3. Picture of the internal OFHC electrodes aligned and mounted on an aluminum bar. In the front, one can recognize the source OFHC cup containing the spiral filament and the grids.

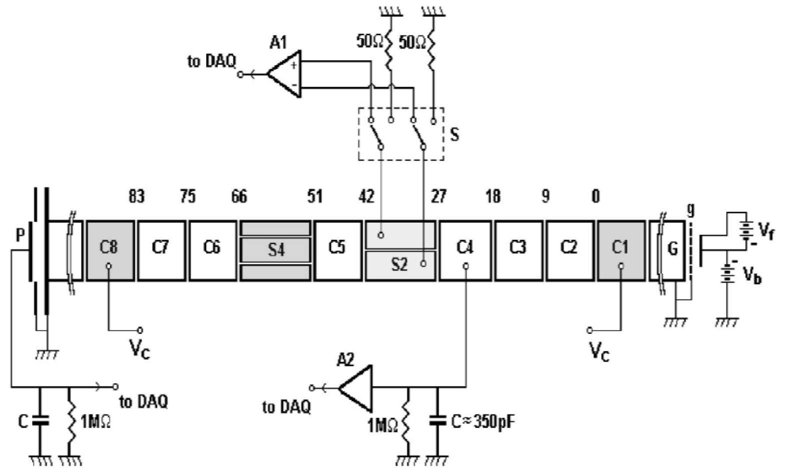


FIG. 2. Schematic of the internal cylindrical conductors, with the circuits used to measure the induced charge signals induced (e.g., on rings S2 and C4). Also shown are the supply circuit of the source (on the right) and the charge collector (on the left).

Eltest

The ELTEST Experiment

R. Pozzoli, M. Cavenago, F. De Luca, M. De Poli,
V. Petrillo, M. Romè, L. Serafini, G. Bettega, and A. Illiberi
INFN Sezione di Milano and Dipartimento di Fisica Università di Milano
INFN Laboratori Nazionali di Legnaro

19 September 2005

Difúze

- A

Vývoj plazmatu řízený difúzi

Difúze – Od Kracíka k Chenovi

- Difúze
- Elektrický proud

$$\frac{\partial}{\partial t}(n\overline{mv}) + \nabla_r(\tfrac{1}{3}n\overline{mv^2}) - nm[\Gamma + (\omega_c \times \bar{v})] = -mn\overline{vv_1}$$

$$\frac{\partial}{\partial t}(n\overline{mv}) + \nabla_r(\tfrac{1}{3}n\overline{mv^2}) - nZeE - J \times B = -n\overline{mvv_1}$$

$$p_{xx} = p_{yy} = p_{zz} = \tfrac{1}{3}(p_{xx} + p_{yy} + p_{zz}) = p = nkT$$

$$\mathbf{p} = nkT\mathbf{I}\,.$$

Předpokládejme stacionární stav $\frac{\partial}{\partial t} = 0$

$B = 0, Z = 1$

$$0 + \nabla_r(\frac{2}{3}n\frac{3}{2}kT) \pm ne\vec{E} + 0 = nm\overline{\vec{v}}\,\nu_1$$

$T = const.$

pozor na závislost na v

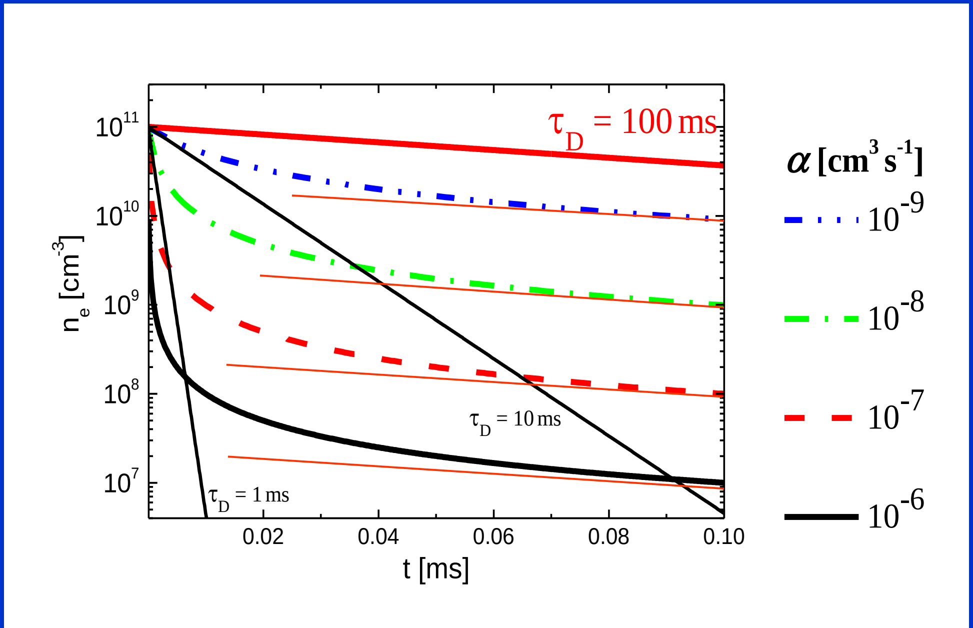
$$kT\nabla_r n \pm ne\vec{E} = nm\overline{\vec{v}}\,\nu_1$$

$$\nu_1 = 2\pi N v \int_0^\pi (1 - \cos\chi)\,\sigma(\chi, v)\sin\chi\,d\chi$$

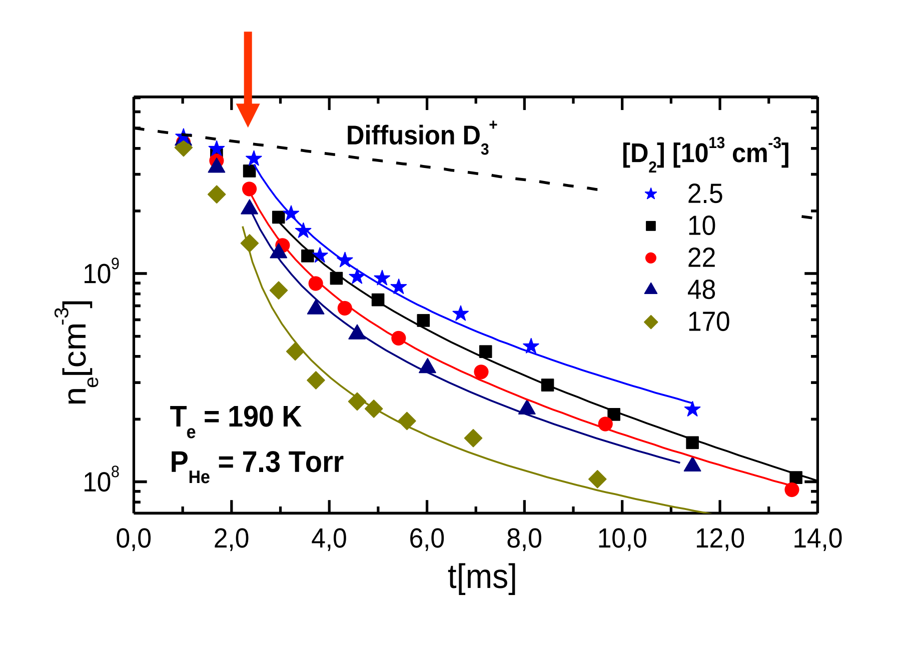
$$\overline{\vec{v}} = \frac{1}{nm\nu_1}(\pm ne\vec{E} - kT\nabla_r n) = \pm\frac{e}{m\nu_1}\vec{E} - \frac{kT}{m\nu_1}\frac{\nabla_r n}{n}$$

AISA vypočtené difúzní ztráty

- Difúze a rekombinace



Experimentální data



- A

Děkuji Vám za pozornost

Monte-Carlo calculation

B=0.1T | r=6mm | p=10^{-6} Torr

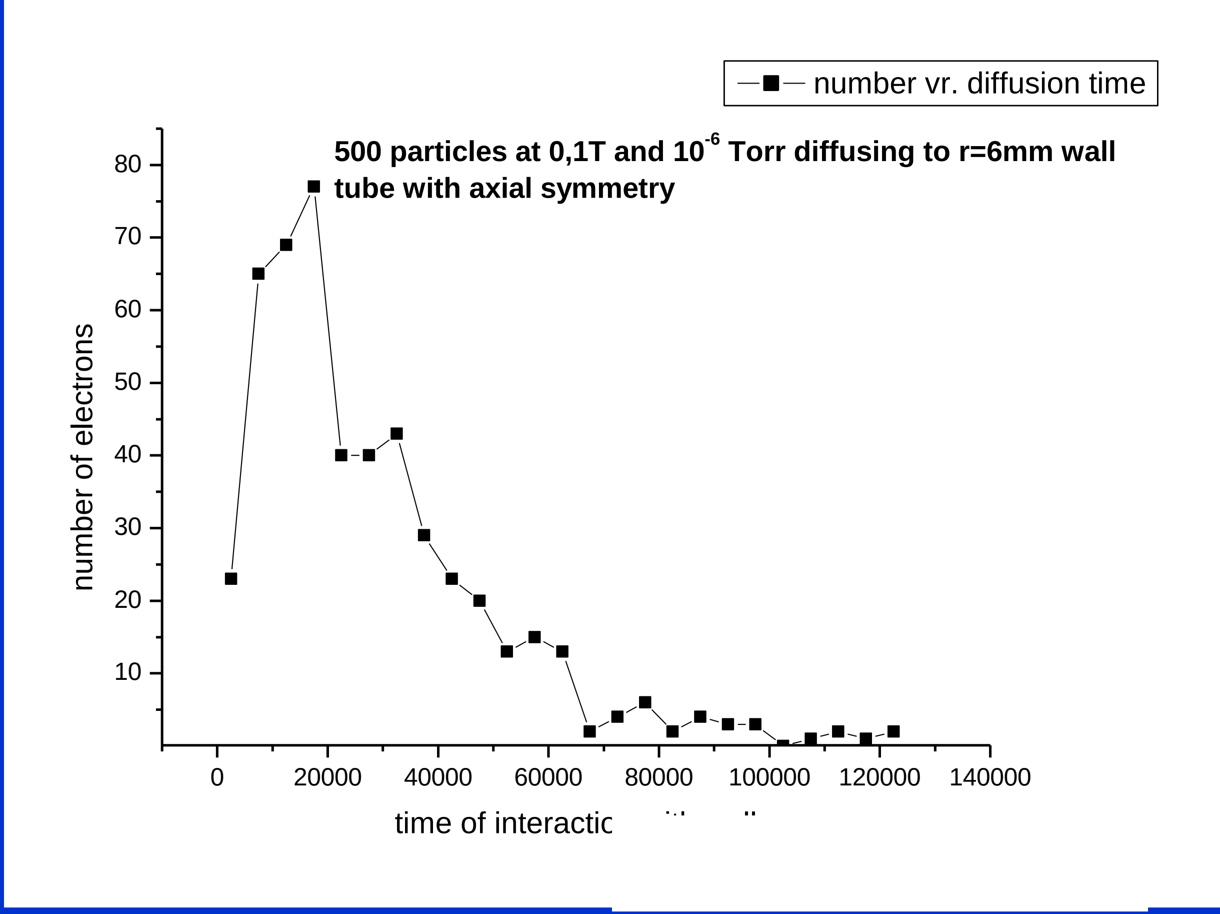
500 e^- in He | t~2×10^4 s

B

Magnetic "radial trap"

$$n = n_0 e^{-\frac{D}{\lambda^2}t} \cos\frac{x}{\lambda}$$

$$n = n_0 e^{-\frac{D}{\lambda^2}t}$$



Monte-Carlo calculation arrival time

B=0.1T

r=6mm

$p=10^{-6}$ Torr

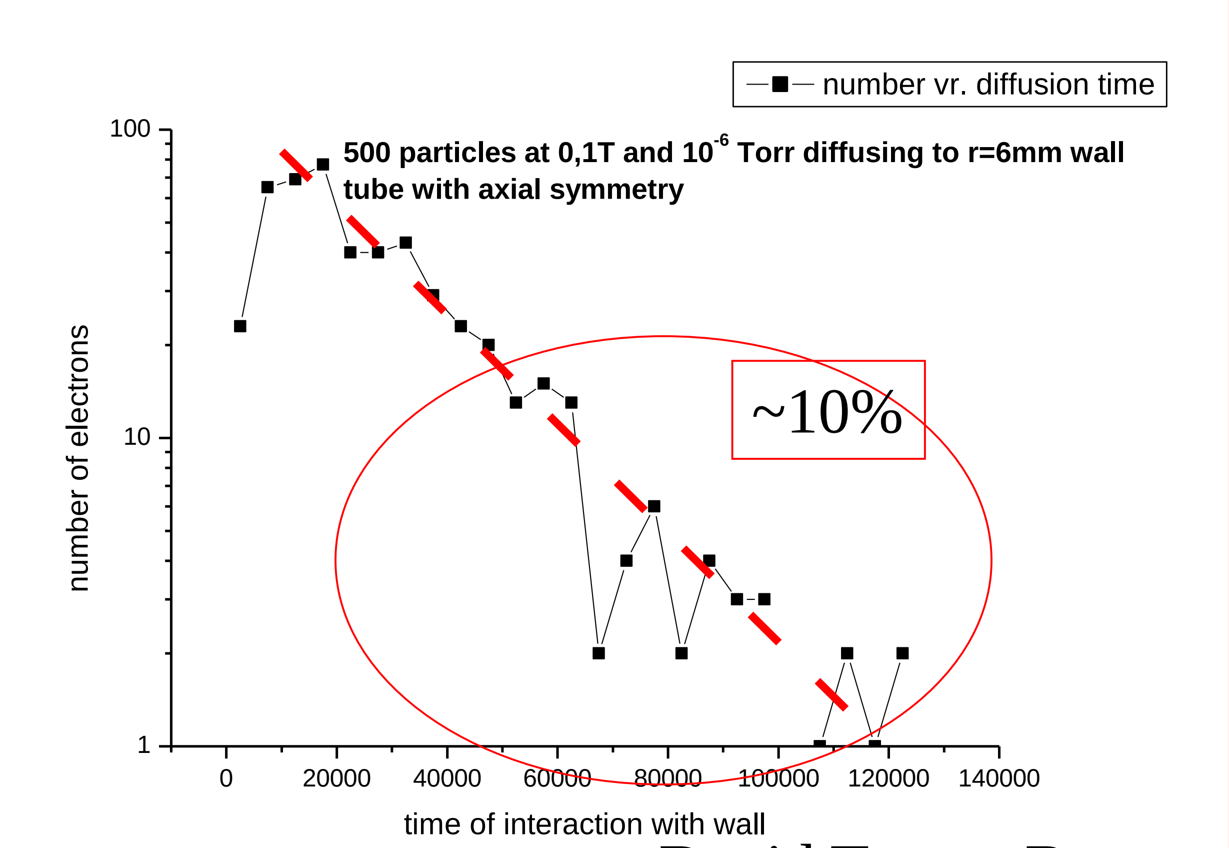
500 e^- in He

$t \sim 2 \times 10^4$ s

B

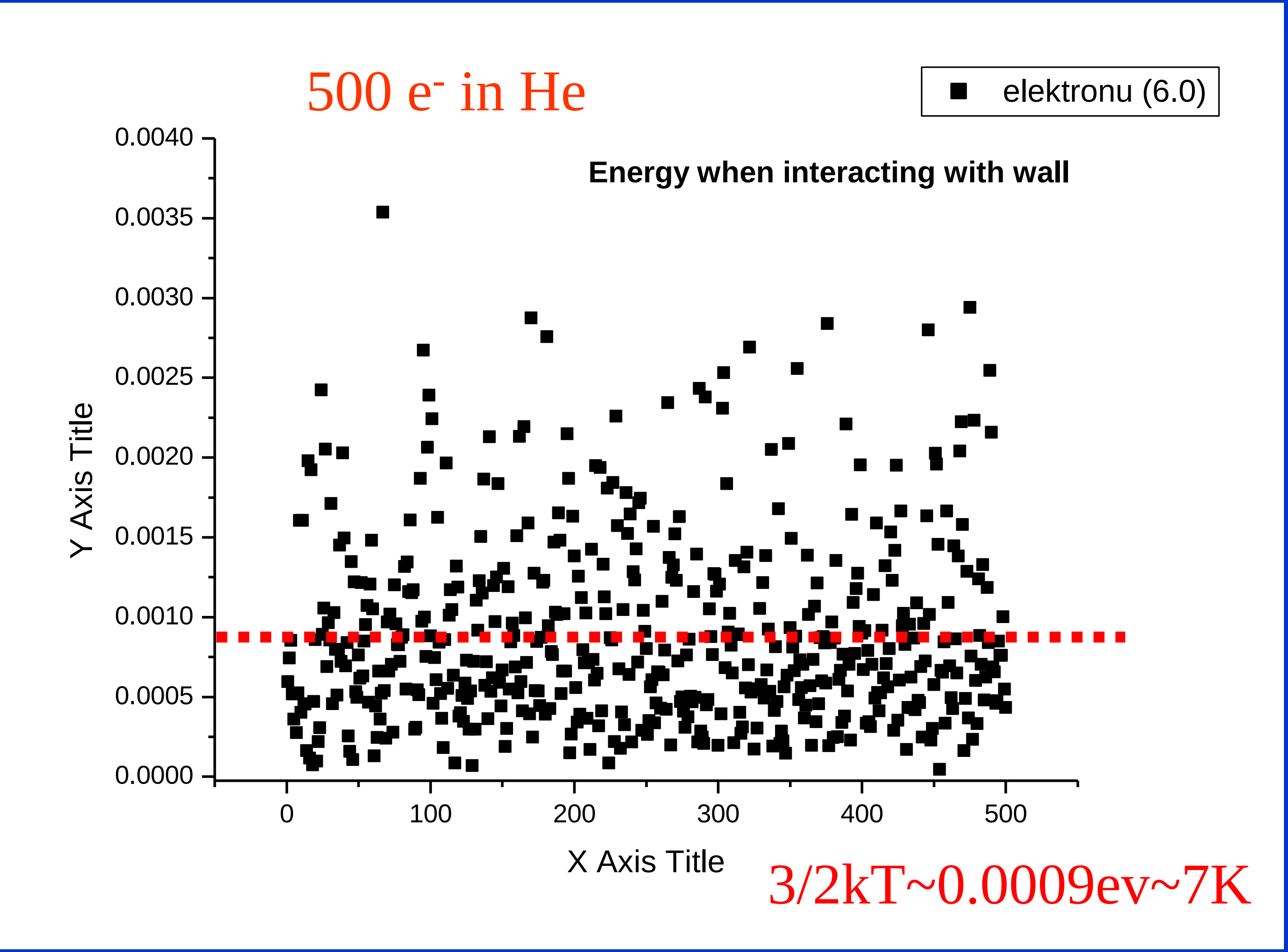
Magnetic "radial trap"

$$n = n_0 e^{-\frac{D}{\lambda^2}t}$$



David Trunec Brno

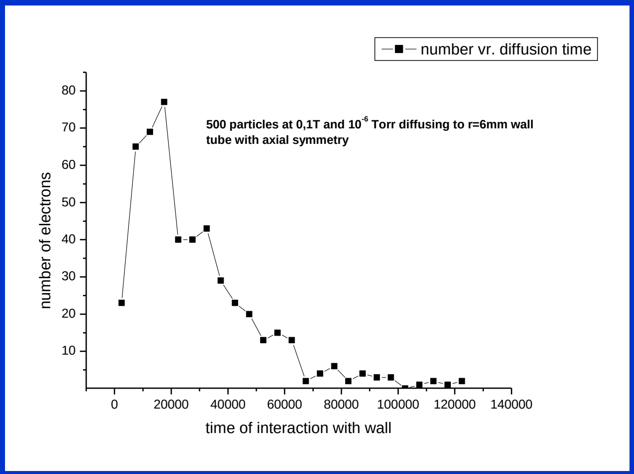
Monte-Carlo calculation energy distribution at impact



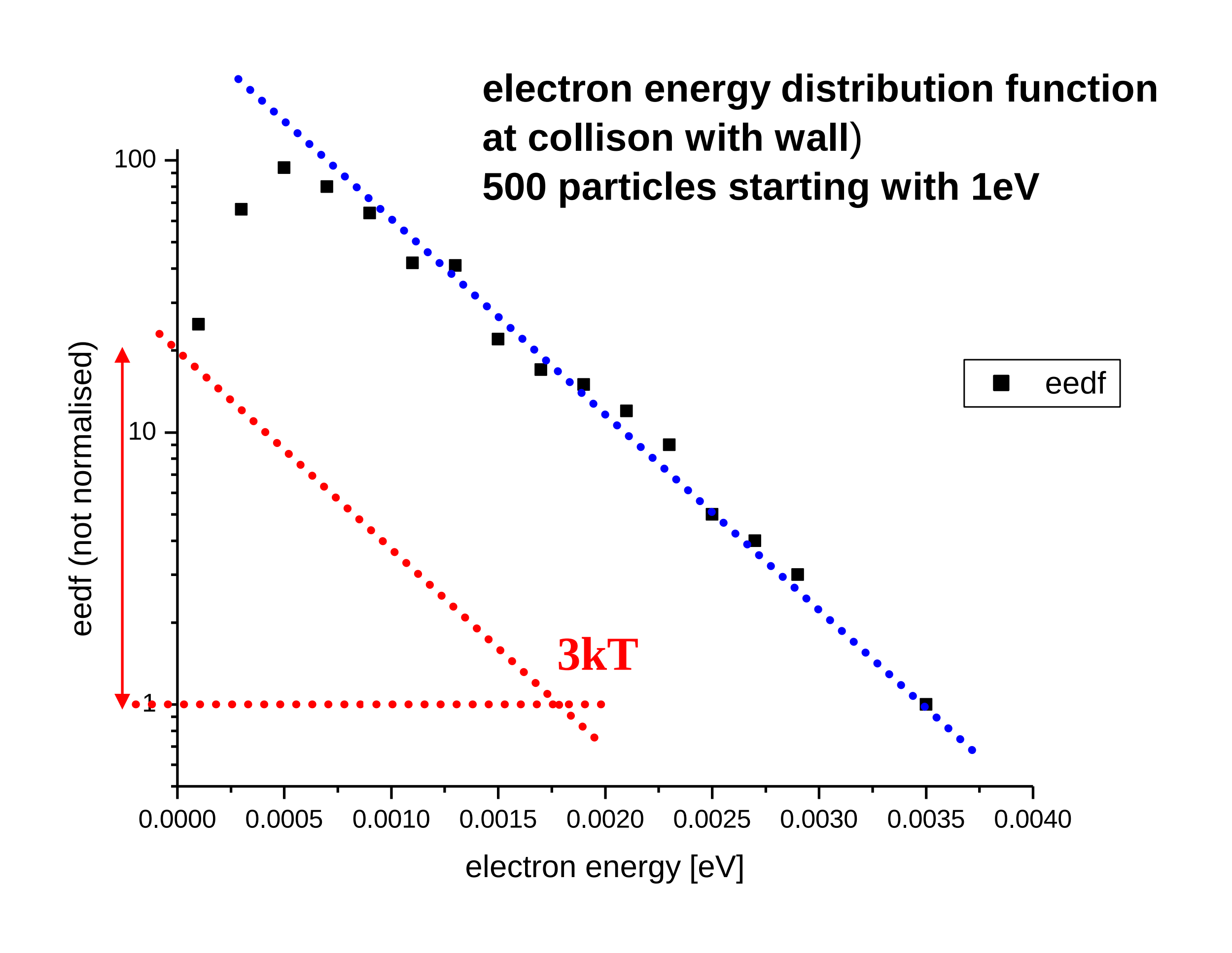
Energy distribution at impact

$$F(\varepsilon) = 2n\sqrt{\frac{1}{\pi}}\left(\frac{1}{kT}\right)^{3/2} m\frac{1}{m}\exp\left(-\frac{mV^2}{2kT}\right)d\varepsilon$$

$$\ln(F(\varepsilon)) \sim cons. - \frac{mV^2}{2kT} \sim cons. - \frac{\varepsilon}{kT}$$



3/2kT~0.0009ev~7K



Arrival time

B=0.1T

r=6mm

$p=10^{-6}$ Torr

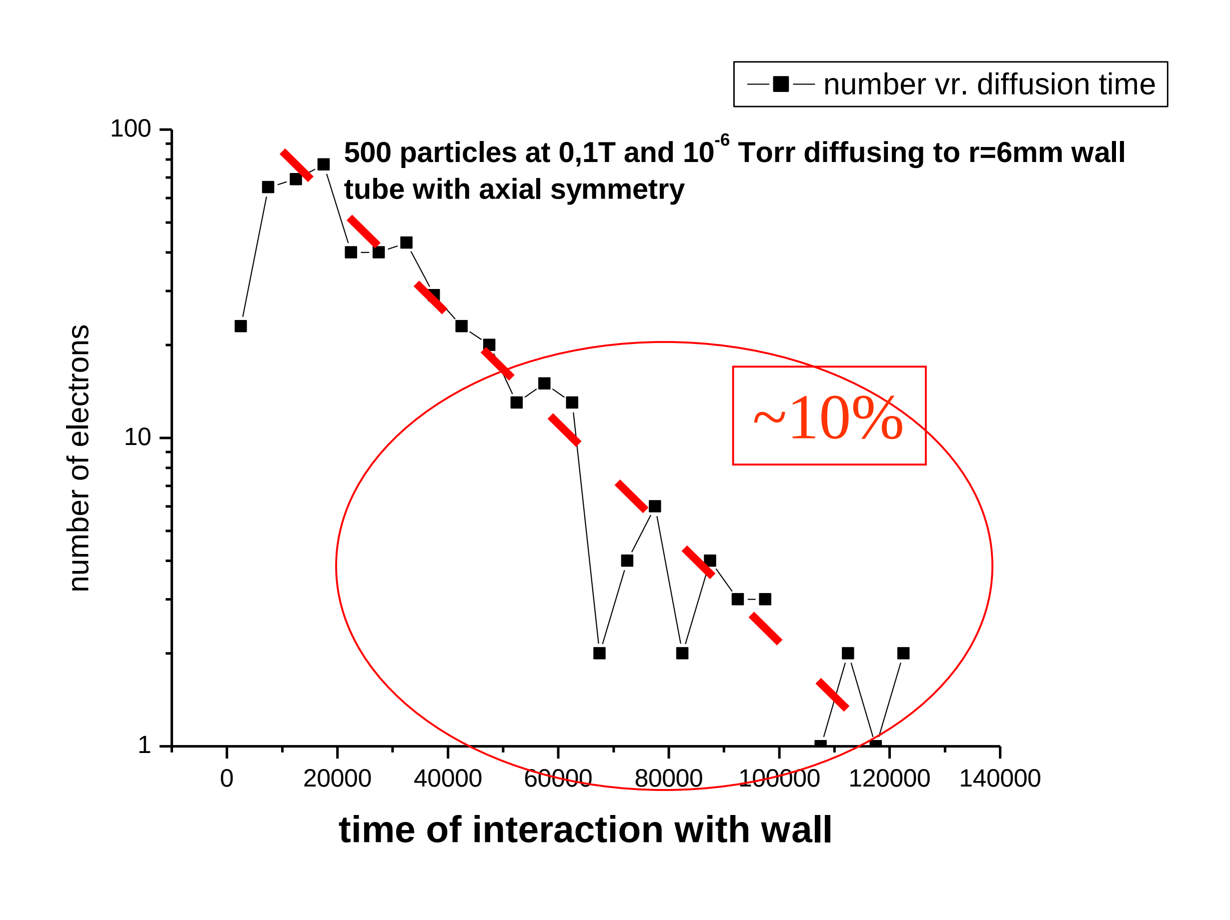
$t \sim 2 \times 10^4$ s

B

Magnetic "radial trap"

$$n = n_0 e^{-\frac{D}{\lambda^2}t}$$

$$D_\perp = D \frac{1}{1+(\varpi_c / v_1)^2}$$



Conclusions

B.R.Beck, J. Fajans, J.H.Malmberg, Phys. Plasmas 3(1996) 1250

Electron cooling by cyclotron radiation

A. Cyclotron radiation cooling theory

A single classical electron, orbiting in a magnetic field, loses energy via cyclotron radiation at a rate given by the Larmor formula:

$$\frac{dE_\perp}{dt} = \frac{2e^2}{3c^3} a_\perp^2 = \frac{4e^2\Omega^2}{3mc^3} E_\perp , \qquad (1)$$

where $a_\perp = \Omega v_\perp$ is the perpendicular acceleration, Ω is the cyclotron frequency, and $E_\perp$ is the perpendicular energy $mv_\perp^2/2$. Averaging Eq. (1) over a Maxwellian distribution yields

$$\frac{dT_\perp}{dt} = -\frac{3T_\perp}{2\tau_r}, \qquad (2)$$

where the radiation time τ_r is defined to be

$$\tau_r \equiv \frac{9mc^3}{8e^2\Omega^2} \simeq \frac{4\times 10^8}{B^2} \text{ s.} \qquad (3)$$

When $\nu \gg \tau_r^{-1}$, as is the case for our plasmas, and the plasma is quiescent, then $T_\perp(t) = T_\parallel(t) = T(t)$ to a good approximation, and one obtains

$$\frac{dT}{dt} = \frac{-T}{\tau_r}. \qquad (4)$$

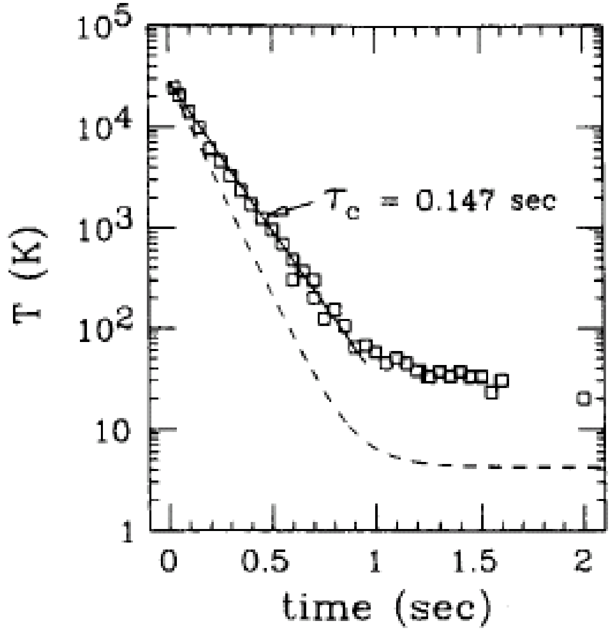


FIG. 3. Measured plasma temperature vs. time for a magnetic field of 61.3 kG. The dashed curve is a plot of Eq. (5).

Recombination of ultracold plasma

$$H^+ + e^- + M \rightarrow H + M$$

$$H^+ + e^- + e^- \rightarrow H + e^-$$

ATRAP	ATHENA
Three-body recombination (TBR)	radiative (stimulated) recombination
$\overline{e} + \overline{e} + \overline{p} \rightarrow \overline{e}$, $\overline{H}(nl)$	$\overline{e} + \overline{p} + h\nu_r \rightarrow h\nu + h\nu_r$, $\overline{H}(nl)$
$\overline{H}(1s)$	$\overline{H}(1s)$
Gabrielse et al. Phys. Lett. A (1988)	Neumann et al. Z. Phys. A (1983)

TBR is efficient at low T and higher positron densities

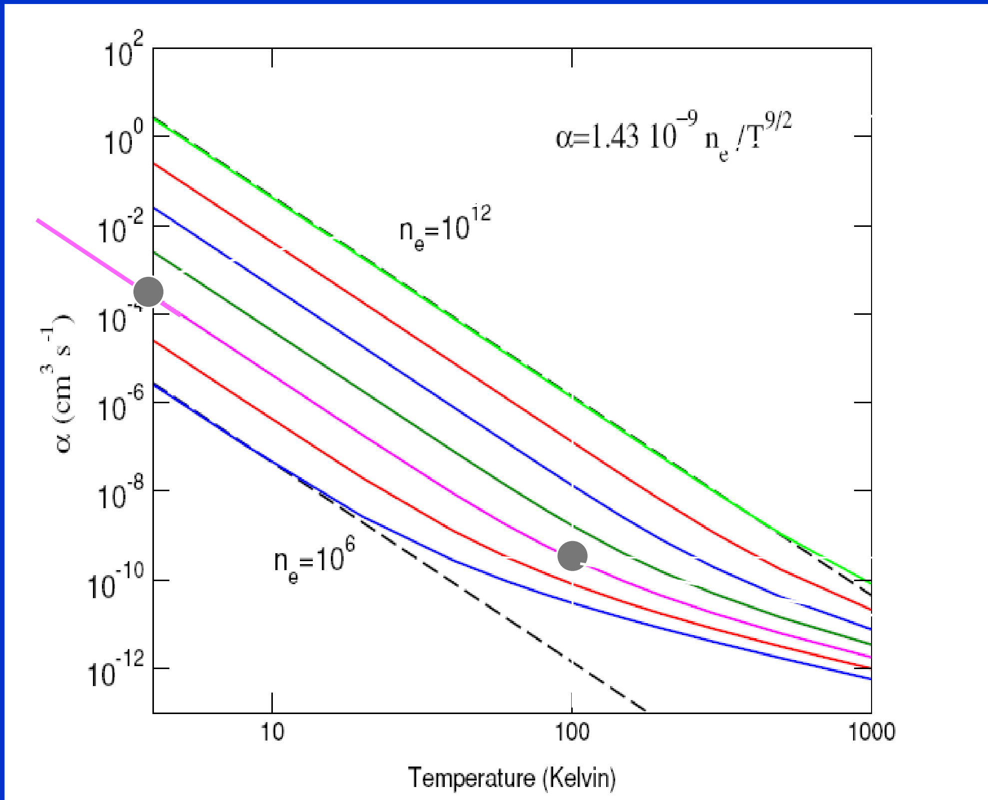
$$R[s^{-1}] = n_e\,\alpha_{TBR} = C\frac{n_e^2}{T^{9/2}}$$

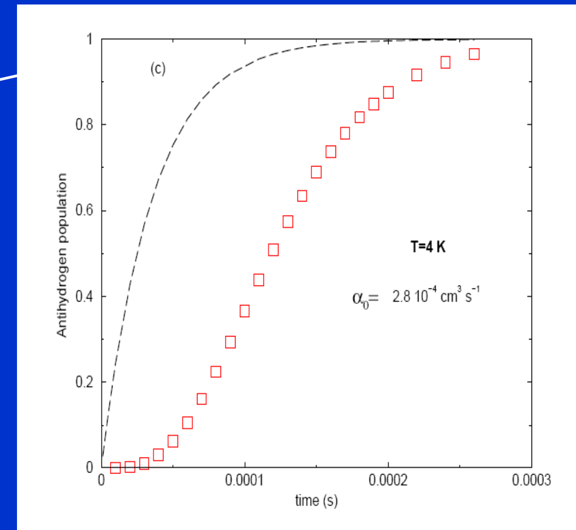
$$\alpha = 3.8\times10^{-9}\,T^{-9/2} n_e + 1.55\times10^{-10}\,T^{-0.63} + 6.0\times10^{-9}\,T^{-2.18}\,n_e^{0.37}$$

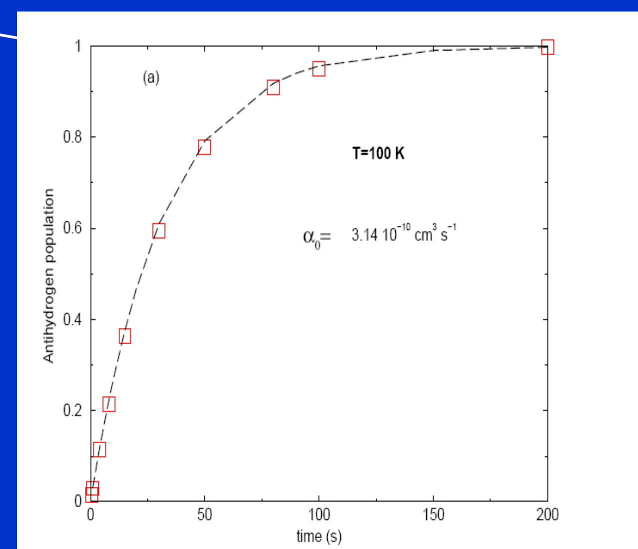
$$\alpha \rightarrow \alpha_{TBR} \quad T \rightarrow 0$$

(Note: we replace $\overline{e} \rightarrow e, \overline{p} \rightarrow p$)

Recombination of ultracold plasma



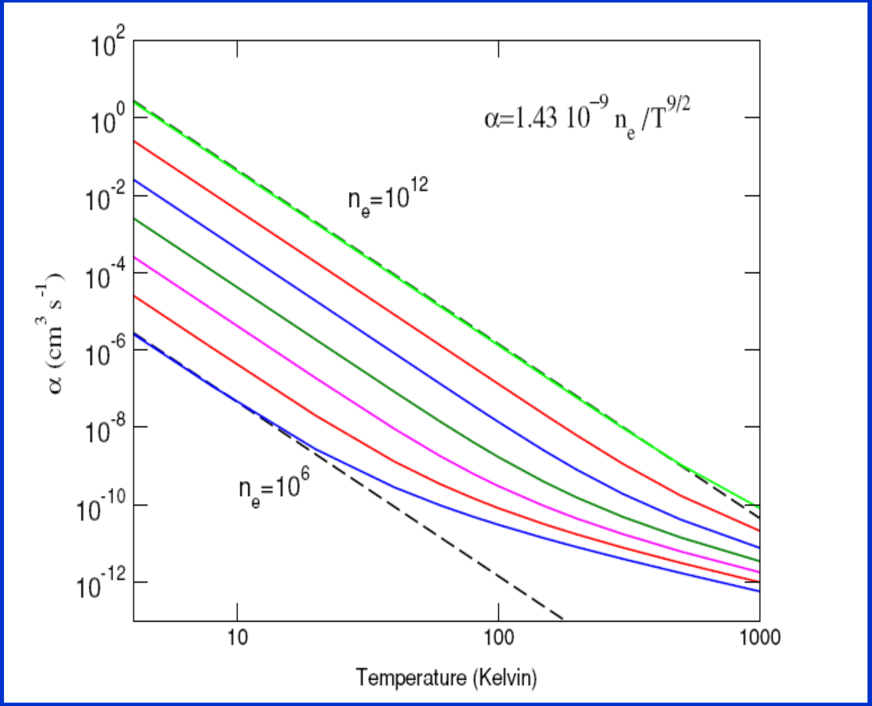




$$\alpha = 3.8 \times 10^{-9} T^{-9/2} n_e + 1.55 \times 10^{-10} T^{-0.63} + 6.0 \times 10^{-9} T^{-2.18} n_e^{0.37}$$

$$\alpha \to \alpha_{TBR} \quad T \to 0$$

Recombination of ultracold plasma



$$\alpha = 3.8\times 10^{-9}\,T^{-9/2} n_e + 1.55\times 10^{-10}\,T^{-0.63} + 6.0\times 10^{-9}\,T^{-2.18}\, n_e^{0.37}$$

$$\alpha \to \alpha_{TBR} \quad T \to 0$$

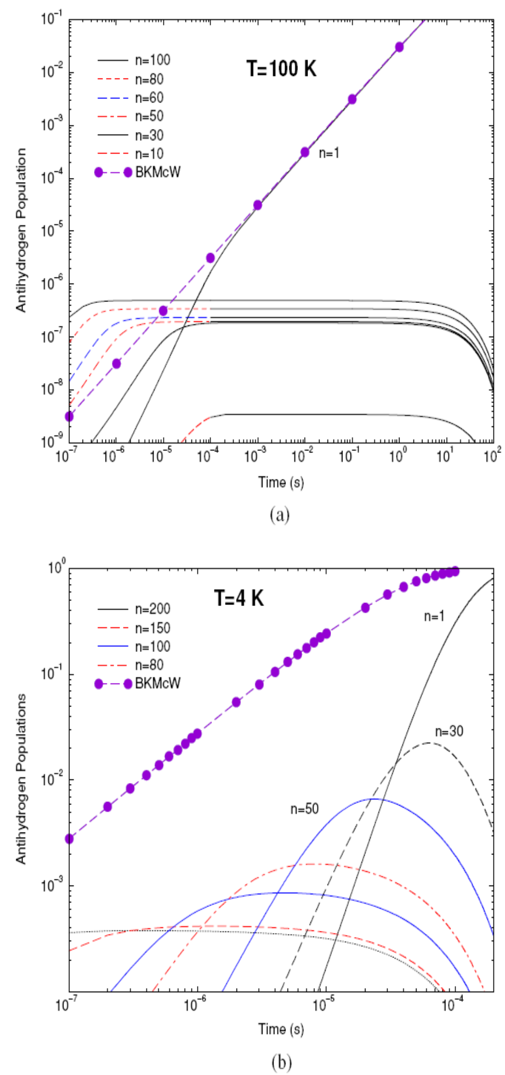
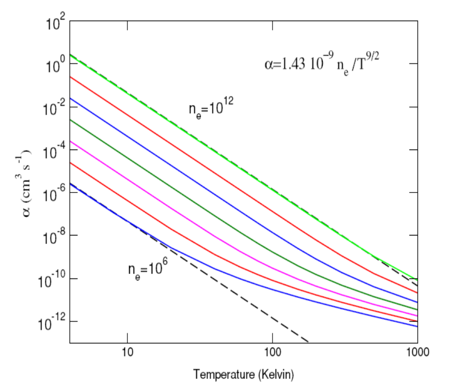


Figure 3. (a) Fractional population of antihydrogen in state with principal quantum numbers ranging form $n = 100 - 1$ as a function of time. The line labelled BKMcW is the time dependence of the ground state population predicted by the quasi-steady-state theory. (b) Same data as shown in (a) but at plasma temperature $T = 4\,\mathrm{K}$.

Recombination of ultracold plasma



$$\alpha = 3.8 \times 10^{-9} T^{-9/2} n_e + 1.55 \times 10^{-10} T^{-0.63} + 6.0 \times 10^{-9} T^{-2.18} n_e^{0.37}$$

$$\alpha \to \alpha_{TBR} \quad T \to 0$$

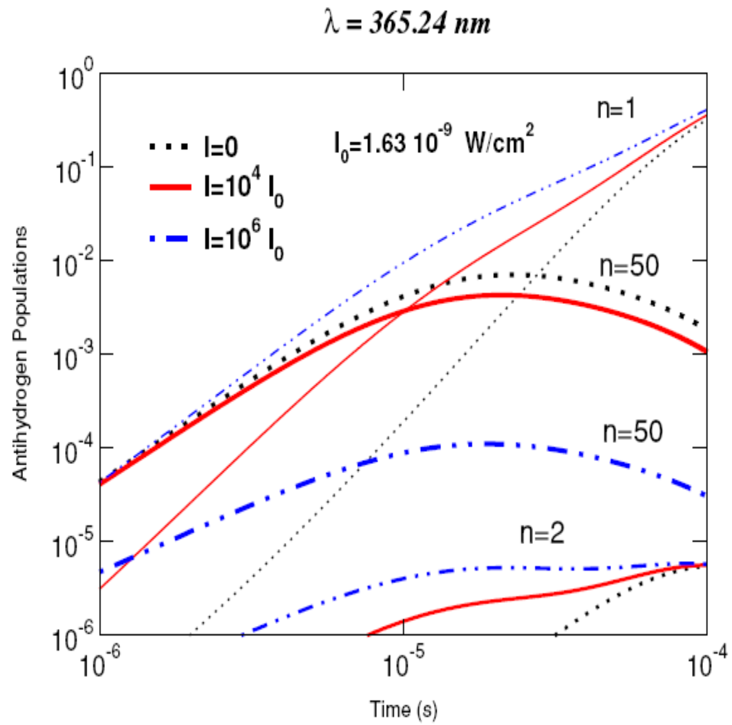


Figure 5. Response of the $n = 2$ and $n = 50$ populations in a resonant laser field corresponding to different laser intensities.

ATHENA

radiative (stimulated) recombination

$$\overline{e} + \overline{p} + h\nu_r \to \overline{H}(nl) + h\nu + h\nu_r$$

$$\overline{H}(1s)$$

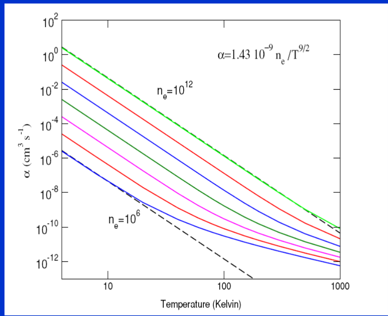
Neumann et al. Z. Phys. A (1983)

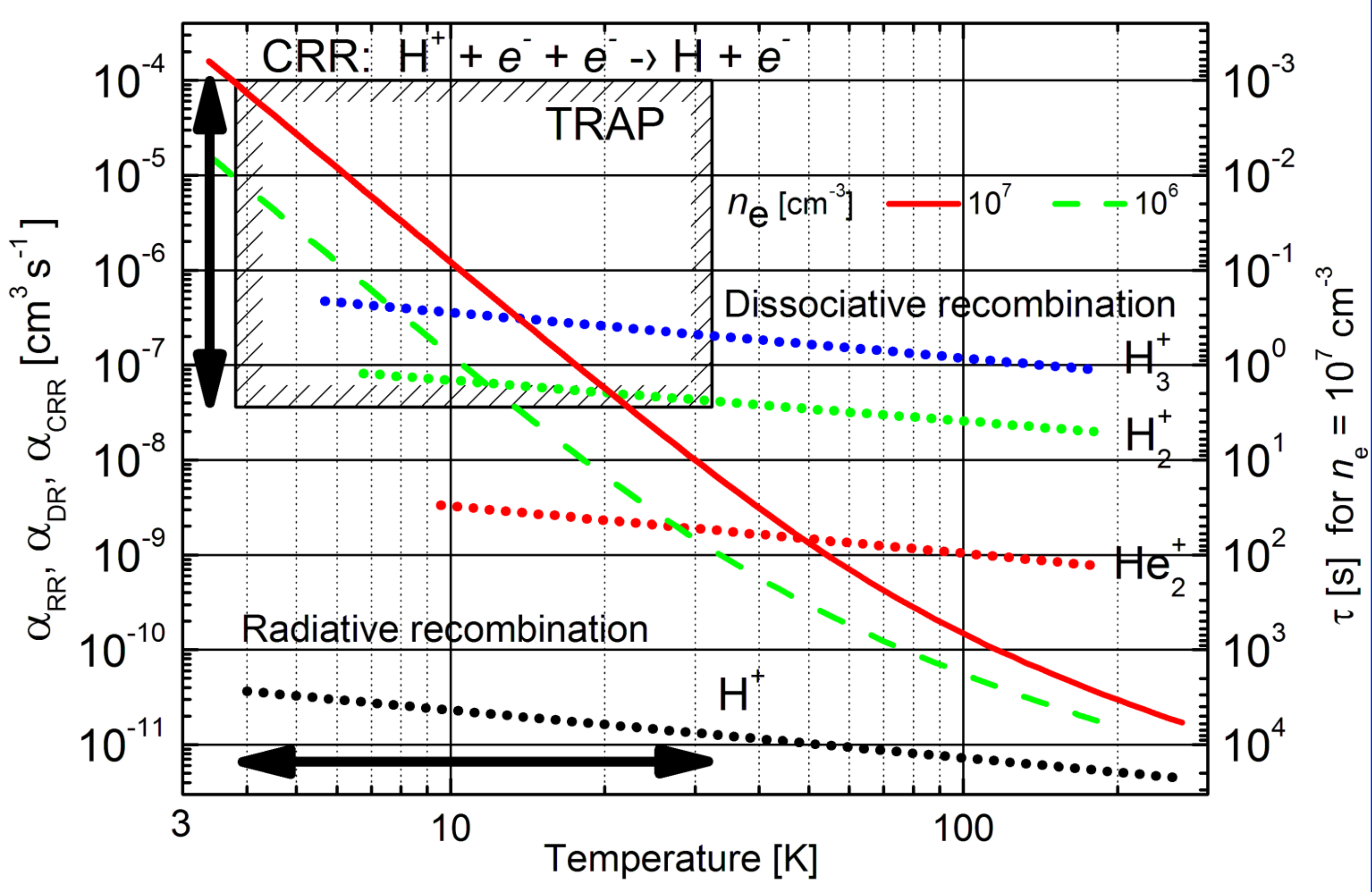
El. in magnetic field

- A

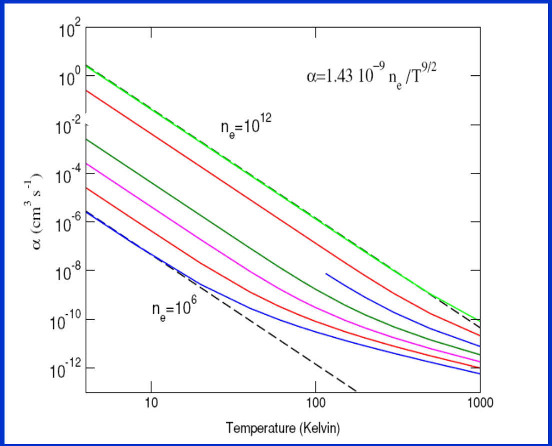
Go to antiproton lecture

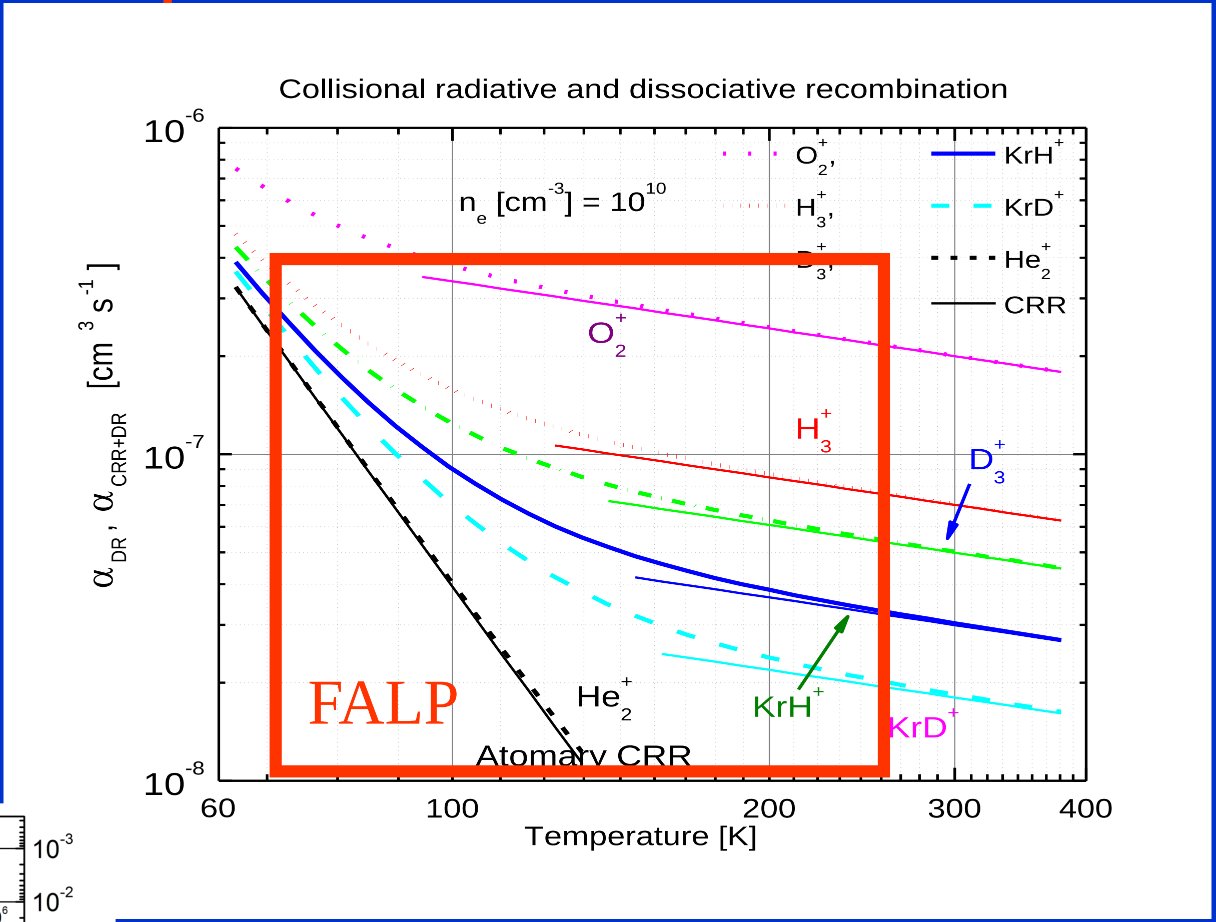
Recombination of ultracold plasma

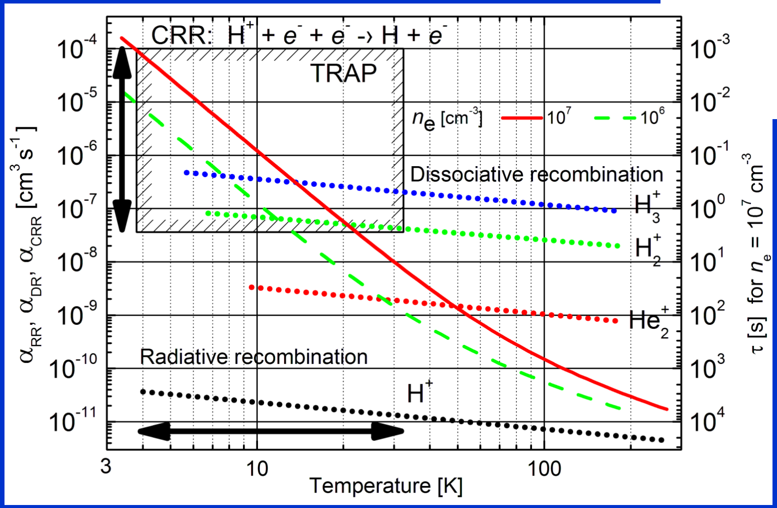




Recombination of ultracold plasma







Zákony zachování – řešení B.

- A

$$\tau_r \equiv \frac{9mc^3}{8e^2\Omega^2} \simeq \frac{4\times 10^8}{B^2}\ s$$

B given in Gaus

for B given in Tesla

$$\tau_r = \frac{4}{B^2}s \quad \Rightarrow \tau_r = \frac{4}{25} = 0.16s \quad at \quad 5Tesla$$

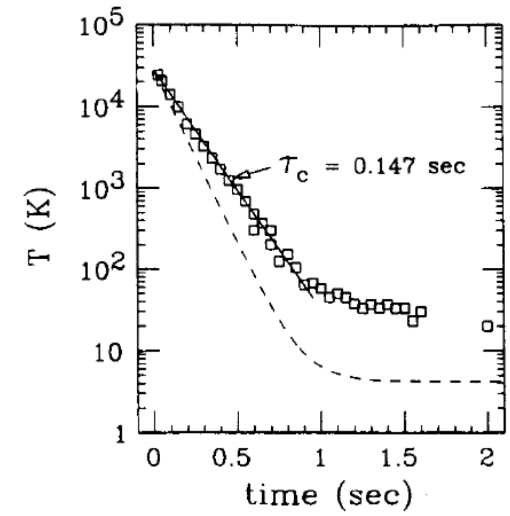
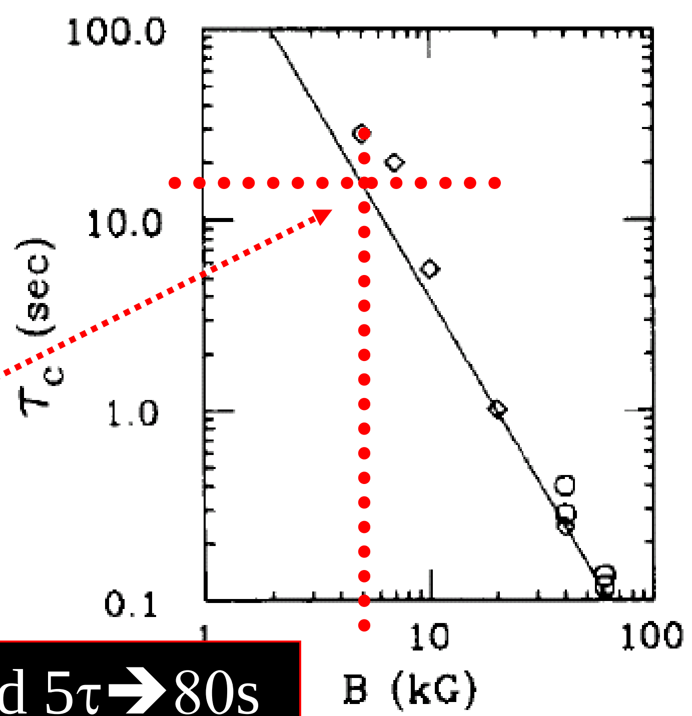


FIG. 3. Measured plasma temperature vs. time for a magnetic field of 61.3 kG. The dashed curve is a plot of Eq. (5).



$$\tau_r = \frac{4}{B^2}s \quad \Rightarrow \tau_r = 4s \quad at \quad 1Tesla$$

At realistic 0.5 T τ~16s to cool down we need 5τ→80s

Magnetic field - terminology

Diffusion

$$m\frac{d\mathbf{v}}{dt} = q\mathbf{v} \times \mathbf{B}\,.$$

$$\omega_c \equiv \frac{|q|\,B}{m}$$

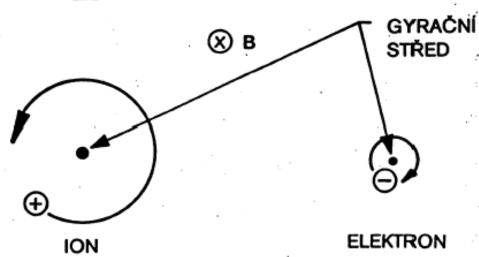
$$r_L \sim \frac{m\sqrt{kT/m}}{B}$$

Položíme-li $\hat{z}$ * do směru $\mathbf{B}$ $(\mathbf{B} = B\hat{z})$, dostáváme

$$m\dot{v}_x = qBv_y\,, \qquad m\dot{v}_y = -qBv_x\,, \qquad m\dot{v}_z = 0\,,$$

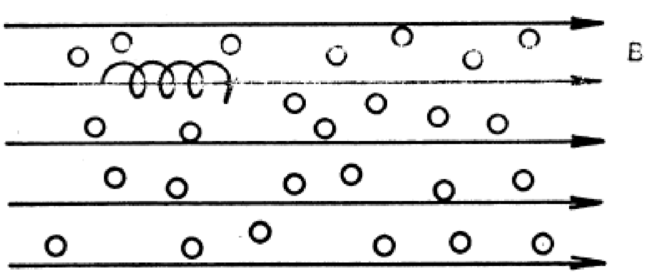
$$\ddot{v}_x = \frac{qB}{m}\dot{v}_y = -\left(\frac{qB}{m}\right)^2 v_x\,, \qquad \ddot{v}_y = -\frac{qB}{m}\dot{v}_x = -\left(\frac{qB}{m}\right)^2 v_y\,.$$

$$r_L \equiv \frac{v_\perp}{\omega_c} = \frac{mv_\perp}{|q|\,B} \qquad [2\text{-}6]$$

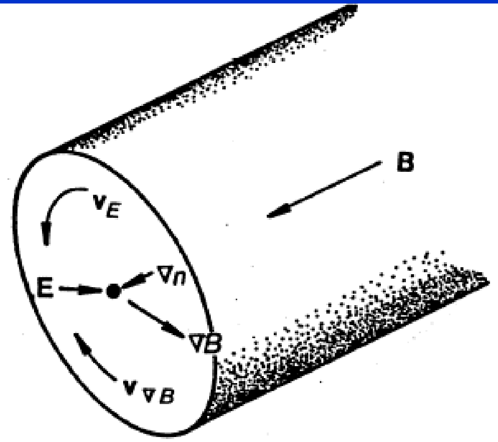


Larmorovy orbity v magnetickém poli.

Ve směru z $\qquad \Gamma_z = \pm\mu n E_z - D\frac{\partial n}{\partial z}\,.$



Nabitá částice bude v magnetickém poli rotovat okolo jedné siločáry, dokud se nesrazí.



Drifty částic v cylindricky symetrickém sloupci plazmatu nevedou ke ztrátám.

No losses

Calculations of parameters of movement

$$m\frac{d\mathbf{v}}{dt} = q\mathbf{v} \times \mathbf{B}\,.$$

Položíme-li $\hat{z}$ * do směru $\mathbf{B}$ ($\mathbf{B} = B\hat{z}$), dostáváme

$$m\dot{v}_x = qBv_y\,, \qquad m\dot{v}_y = -qBv_x\,, \qquad m\dot{v}_z = 0\,,$$

$$\ddot{v}_x = \frac{qB}{m}\dot{v}_y = -\left(\frac{qB}{m}\right)^2 v_x\,, \qquad \ddot{v}_y = -\frac{qB}{m}\dot{v}_x = -\left(\frac{qB}{m}\right)^2 v_y\,.$$

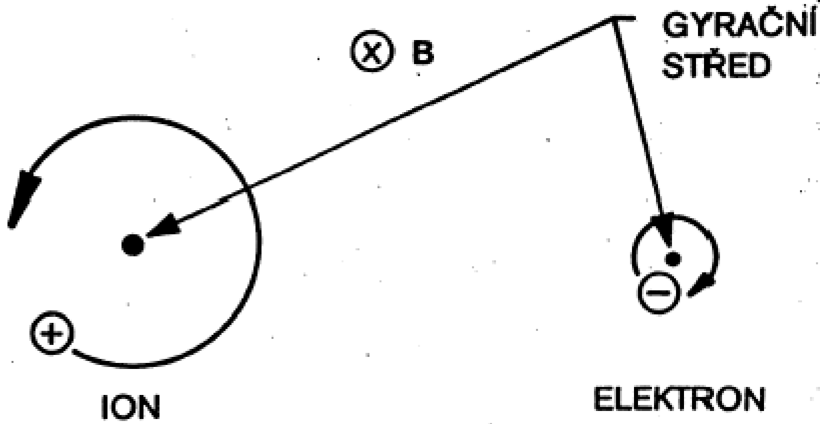
$$\omega_c \equiv \frac{|q|\,B}{m}$$

$$\varpi_c = \frac{eB}{m} = \frac{1.6x10^{-19}}{9.1x10^{-31}}B = 1.8x10^{11}B$$

$at \quad B = 1T \quad \varpi_c = 1.8x10^{11} \sim 180\ GHz$

$at \quad B = 0.1T \quad \varpi_c = 1.8x10^{10} \sim 18\ GHz$

$$r_L \equiv \frac{v_\perp}{\omega_c} = \frac{mv_\perp}{|q|\,B} \qquad [2\text{-}6]$$



Larmorovy orbity v magnetickém poli.

$$r_L \sim \frac{m\sqrt{kT/m}}{B}$$

$at \quad T = 300K \quad \& \quad B = 1T \quad r_l = 1\mu m$

$at \quad T = 3K \quad \& \quad B = 1T \quad r_l = 0.1\mu m$

$at \quad T = 300K \quad \& \quad B = 0.1T \quad r_l = 10\mu m$

Pro termální plazmu

$$r_L \sim \frac{m\sqrt{kT/m}}{B}$$

Diffusion in magnetic field

$$\vec{v} = \mu\vec{E} + \frac{e}{m\nu_1}\vec{B}x\vec{v} - D\frac{\nabla n}{n}$$

- Chen

$$\tau = \nu^{-1}$$

$$v_y(1+\omega_c^2\tau^2) = \pm\mu E_y - \frac{D}{n}\frac{\partial n}{\partial y} - \omega_c^2\tau^2\frac{E_x}{B} \pm \omega_c^2\tau^2\frac{KT}{eB}\frac{1}{n}\frac{\partial n}{\partial x}$$

$$v_x(1+\omega_c^2\tau^2) = \pm\mu E_x - \frac{D}{n}\frac{\partial n}{\partial x} - \omega_c^2\tau^2\frac{E_y}{B} \mp \omega_c^2\tau^2\frac{KT}{eB}\frac{1}{n}\frac{\partial n}{\partial y}$$

$$v_{Ex} = \frac{E_y}{B}, \qquad v_{Ey} = -\frac{E_x}{B},$$

$$v_{Dx} = \mp\frac{KT}{eB}\frac{1}{n}\frac{\partial n}{\partial y}, \qquad v_{Dy} = \pm\frac{KT}{eB}\frac{1}{n}\frac{\partial n}{\partial x}$$

$$v_\perp = \pm\mu_\perp E - D_\perp\frac{\nabla n}{n} + \frac{v_E + v_D}{1 + (\nu^2/\omega_c^2)}$$

$$D_\perp = \frac{D}{1+\omega_c^2\tau^2}$$

$$\mu_\perp = \frac{\mu}{1+\omega_c^2\tau^2}$$

Just introduction

$\tau = \nu^{-1}$

$$v_\perp = \pm\mu_\perp E - D_\perp \frac{\nabla n}{n} + \frac{v_E + v_D}{1 + (\nu^2/\omega_c^2)}$$

$$v_{Ex} = \frac{E_y}{B}, \qquad v_{Ey} = -\frac{E_x}{B},$$

$$v_{Dx} = \mp \frac{KT}{eB}\frac{1}{n}\frac{\partial n}{\partial y}, \qquad v_{Dy} = \pm \frac{KT}{eB}\frac{1}{n}\frac{\partial n}{\partial x}$$

$$\omega_c \tau = \omega_c/\nu = \mu B \cong \lambda_s/r_L$$

$$\mu_\perp = \mu \frac{1}{1+(\varpi_c/v_1)^2} \qquad D_\perp = D\frac{1}{1+(\varpi_c/v_1)^2}$$

Perpendicular to gradients

Parallel with gradients

$$\sim \frac{1}{1+.(v_1/\varpi_c)^2}$$

Diffusion without B

$$D = KT/m\nu \sim v_t^2 \tau \sim \lambda_s^2/\tau\ .$$

$(\varpi_c/v_1)^2 >> 1$

Decreasing diffusion in B

$$D_\perp = \frac{KT\nu}{m\omega_c^2} \sim v_t \frac{r_L^2}{v_t^2}\nu \sim \frac{r_L^2}{\tau}$$

„Diffusion step “

$(\varpi_c/v_1)^2 << 1$

Small influence on diffusion

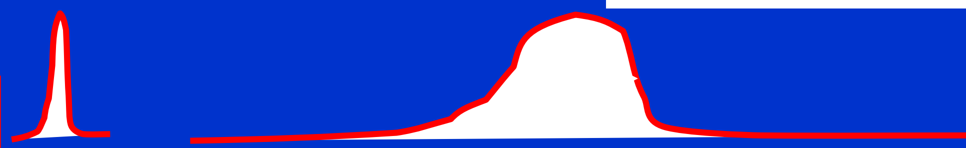
Analytical calculation of diffusion losses in magnetic field

$$D_\perp = \frac{D}{1+\left(\frac{\varpi}{\nu_1}\right)^2}$$

perfect agreement with Monte-Carlo

radial movement by diffusion

$$r \sim \sqrt{4Dt}$$



at 4K, 0.1Tesla and 10^{-6} Torr

$$\varpi = 1.7x10^{10}\, s^{-1} \sim 17GHz$$

$$v = 1.34x10^4\, m/s = 1.34km/s$$

Chyba

$$r_L = 7.66x10^{-7}\, m = 0.8\mu m$$

$$\nu_1 = 1.95x10^3\, s^{-1}$$

$$\left(\frac{\varpi}{\nu_1}\right)^2 = 8.1x10^{13}$$

$$D_e = 2.65x10^5\, m^2 s^{-1}$$

$$D_\perp = \frac{D_e}{1+\left(\frac{\varpi}{\nu_1}\right)^2}$$

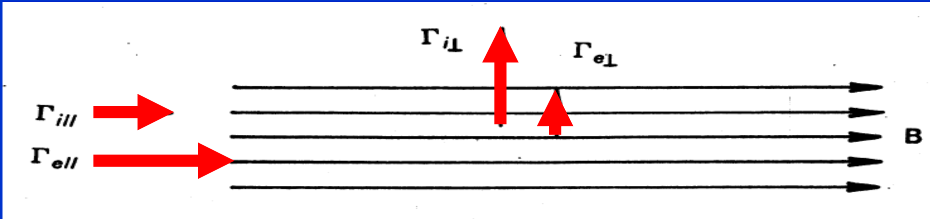
$$D_\perp = 2.65x10^5 \frac{1}{8x10^{13}}\, m^2 s^{-1} = 3.27x10^{-9}\, m^2 s^{-1}$$

at t=10^4s

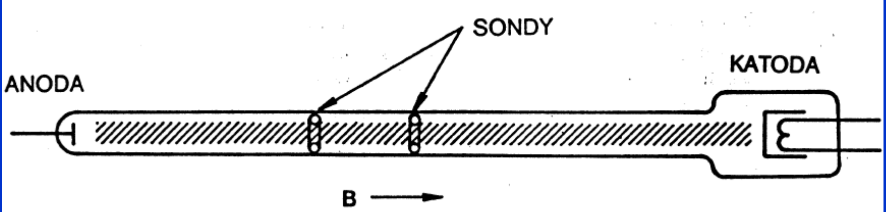
$$r \sim \sqrt{4Dt} = \sqrt{4.3.10^{-9}.10^4} = 1.1x10^{-2}\, m$$

Ambipolar diffusion

Experiment at high gas pressure



OBR. 5-13 Toky částic rovnoběžné s magnetickým polem a kolmé na něj.



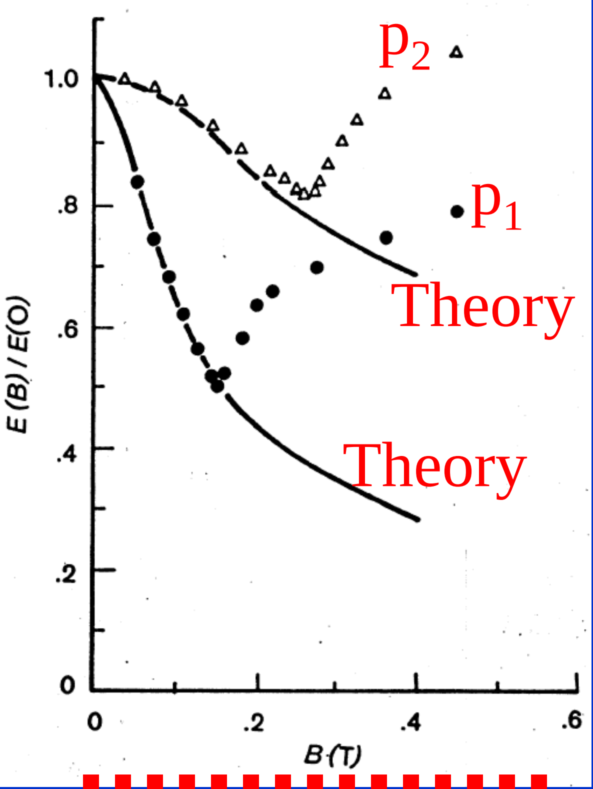
OBR. 5-14 Lehnertův–Hohův experiment ověřující vliv magnetického pole na difúzi v slabě ionizovaném plynu.

Otázka, zda magnetické pole zmenšuje příčnou difúzi v souhlase s rov. [5-51], se stala předmětem četných zkoumání. Prvý experiment uskutečněný v dostatečně dlouhé trubici, takže difúzi ke koncům bylo možno zanedbat, provedli Lehnert a Hoh ve Švédsku. Kladný sloupec héliového výboje měl průměr 1 cm a byl dlouhý 3,5 m (obr. 5-14). V takovém plazmatu elektrony plynule unikají radiální difúzí ke stěnám a jsou nahrazovány ionizací neutrálního plynu elektrony z konce rychlostního rozdělení. Tyto rychlé elektrony jsou zase nahrazovány urychlením v podélném elektrickém poli.

Ionty unikající difúzi jsou nahrazovaný ionizaci

➔ E_z bude úměrné difúzi.

Můžeme proto očekávat, že E_z bude zhruba úměrné velikosti příčné difúze. Dvěma sondami u stěn výbojové trubice měřili E_z při různém B. Na obr. 5-15 je vynesen poměr $E_z(B)$ ku $E_z(0)$ jako funkce B. Pro malé B experimentální body velmi dobře sledují předpovězenou křivku vypočtenou na základě rovnice [5-52]. Při určitém kritickém poli B_k okolo 0,1 T se však experimentální body vzdalují od teorie a vykazují ve skutečnosti *vzrůst* difúze



↔➔ Plasma??

Axial trap with electrostatic mirrors

$$D_\perp = \frac{D}{1+\left(\frac{\varpi}{\nu_1}\right)^2}$$

axial movement

at 4K, 0.1Tesla and 10^{-6} Torr

collision frequency

$\varpi = 1.7x10^{10} s^{-1} \sim 17GHz$

$\nu_1 = 1.95x10^3 s^{-1}$

$\left(\frac{\varpi}{\nu_1}\right)^2 = 8.1x10^{13}$

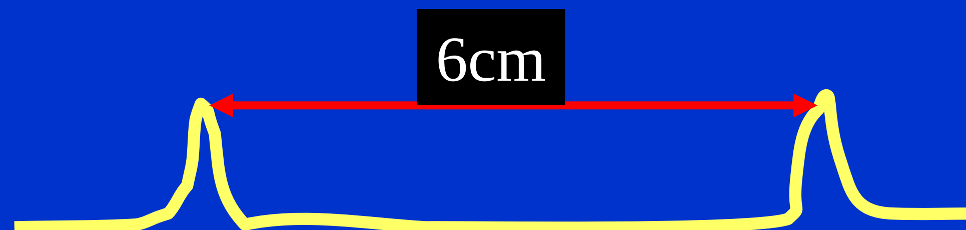
$r_L = 7.66x10^{-7} m = 0.8\mu m$

$D_e = 2.65x10^5 m^2 s^{-1}$

$v = 1.34x10^4 m/s = 1.34km/s$ Chyba ?

$D_\perp = 3.27x10^{-9} m^2 s^{-1}$

$\nu_1 = 1.95x10^3 s^{-1}$



Chyba ?

between collisions 0.5ms . 1300m/s= 65cm → 10x along the trap

Je tu uvedena rychlost iontov a nie rychlost elektronov