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Visions and Re-Visions of Charles Joseph Minard

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Charles Joseph Minard is most widely known for a single work—his poignant flow-map depiction of the fate of Napoleon's Grand Army in the disastrous Russian campaign of 1812. In fact, Minard was a true pioneer in thematic cartography and in statistical graphics; he developed many novel graphic forms to depict data, always with the goal to let the data "speak to the eyes." This article reviews Minard's contributions to statistical graphics, the time course of his work, and some background behind the famous March on Moscow graphic. This article also looks at some modern re-visions of this graph from an information visualization perspective and examines some lessons this graphic provides as a test case for the power and expressiveness of computer systems or languages for graphic information display and visualization.

Keywords: *data visualization, dynamic graphics, history, Mathematica, Napoleonic wars, statistical graphics, thematic cartography*

> RE-VISION *n. ri-'vizh-en* (ca. 1611) 1. To see again, possibly from a new perspective; *syn:* review, reconsideration, reexamination, retrospection. 2. An act of revising; *syn:* rewrite, alteration, transformation. (Merriam-Webster, 2002)

Readers of Tufte (1983) and Wainer (2000) have become acquainted with some early developments in the history of statistical graphics by Playfair, Florence Nightingale, and others. The "others" include Charles Joseph Minard, whose *Carte figurative des pertes successives en hommes de l'Armee Français dans la campagne de Russe 1812–1813* (hereafter, the "Napoleon's March on Moscow" graphic) is, some have claimed, "the best graphic ever produced" (Tufte, 1983).

This graph (see Figure 1, Figures 1–8 on pages 37–44) shows the catastrophic loss of life in Napoleon's Grand Army. The diminishing size of the army, initially 422,000 strong (including conscripts from his empire), is shown by the width of a steadily diminishing line, overlaid on the map of Russia, ending with 10,000 returning at the end of the campaign. A subscripted graph of declining temperature over the Russian winter shows the brutal conditions which accompanied the sol-

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diers on their terrible retreat. This graphic, as Marey (1878) put it, seemed to defy the pen of the historian by its brutal eloquence.

Later, Funkhouser (1937), in the first modern overview of graphical methods, devoted several pages to Minard's work and called him "the Playfair of France," to suggest the scope of his contributions. Tufte (1983) brought this image to wide attention, describing it as showing "multivariate complexity integrated so gently that viewers are hardly aware that they are looking into a world of six dimensions. . . . It may well be the best statistical graphic ever produced" (Tufte, 1983, p. 40). It is therefore ironic that this March on Moscow graphic is often seen as an isolated work. Apart from this, Minard's wider contributions to statistical graphics are little known, especially outside France.

My introduction to the larger context of Minard's work and the motivation for the present study arose from a copy of an 1883 volume of *l'Album de Statistique Graphique,* published annually by the Bureau de la Statistique Graphique of the Ministry of Public Works from 1879 to 1899. A large-format book (about 12 × 15 in.), each of its figures folds out to either three or four times that size, and contains exquisite detail and beautiful color tones. Most important, the figures display an astonishing range and depth of visual information, using all known graphic forms (map-based pie and star charts, mosaic plots, line graphs, bar charts, and, of course, numerous flow maps).

Collectively, these works may be considered the epitome of the "golden age of statistical graphics" (Palsky, 1996; Friendly & Denis, 2001). Some copies of these images may be seen on my Gallery of Data Visualization (GDV) (http://www.math.yorku.ca/SCS/Gallery/). That lovely volume was the springboard for this re-vision of Minard, presented in two parts corresponding to the Merriam-Webster definitions of "re-vision."

Re-Visions of Minard's Other Graphics

Minard died in 1870, and the well-known March on Moscow graphic (November 20, 1869) was, along with a similar graphic of Hannibal's army in Italy, among his last published works, yet all of the volumes of *l'Album* (carried out under the direction of Émile Cheysson) show the unmistakable signature of the graphic inventions of Charles Minard, whose role and inspiration are acknowledged in the preface. Re-examining some of Minard's works is the first sense of my title phrase "Re-visions of Minard."

Minard was a true pioneer in what geographers call "thematic cartography" and also in statistical graphics. The authoritative article by Robinson (1967) listed 51 *cartes figuratives* among his known works, but only four redrawings, pale imitations of the originals, are shown in that article. Several color reproductions appear in Robinson's *Early Thematic Mapping in the History of Cartography* (Robinson, 1982), and 15 appear in Palsky (1996), who lists 58 graphic works by Minard. A comprehensive, indexed bibliography of all of Minard's graphic works appears on the GDV website.

Thematic cartography always involves a tension and trade-offs between the confines of the map and the representation of data. Minard almost invariably chose accuracy of the representation of data over the "tyranny of precise geographical position" whenever conflict arose. "Accordingly, he revised coastlines, paid little attention to projections, and forced the scales of the geographic features on his maps to fit the data being portrayed, rather than *vice versa,* as is usually done today" (Robinson, 1967, p. 95). It is for this reason that he carefully labeled his maps *cartes figuratives et approximatives.* As one may see in the fine detail of the March on Moscow graphic, he also took considerable care to represent the quantitative information in both numerical and visual forms.

Minard was first a civil engineer, then an instructor at the École Nationale des Ponts et Chaussées (ENPC), the premier training school for engineers responsible for building the roads, canals, and railroads of France. Later, he was an Inspector General of the Council des Ponts et Chaussées. As a result, a great deal of Minard's graphic work is concerned with the visual representation of the movement of goods and people. He made dozens of "flow maps," depicting passenger traffic on European railways (see Figure 2), the transport of meat and produce to feed the growing population of Paris (Figure 4), the international distribution of French wines, cotton, coal, and so forth. As in Figure 2, Minard typically included the numerical information on the map as well, either annotated or in tabular form, and included lengthy legends describing exactly what was being portrayed, the nature of the visual representation, and conclusions which could be drawn from these.

Minard did not invent the idea of using flow lines on a map. That honor belongs to Henry Drury Harness (1837), showing passenger traffic and the flow of goods in Ireland (Robinson, 1955, 1982). Whether Minard knew of Harness's work or developed the flow map independently is unknown. However, it is fair to say that Minard developed this graphic technique to an art form, and the March on Moscow graphic, his last such effort, represents his finest achievement, for reasons we shall see later.

These maps and charts were all made with the express purpose of informing decision makers responsible for planning in an era of rapid growth and development—what we might now call the "dawn of the age of globalization." In these works, Minard had moved from work as a *civil* engineer (designing canals and railways) to work as a *visual* engineer (designing informative visual data displays). For example, in 1865 the location for a new central post office was to be chosen in Paris. Minard's ingenious map (Figure 3) showed the population distribution in Paris as population-sized squares for each arrondisement, and located the post office at their visual center of gravity.

Minard's influence and contribution to visually based planning was such that, from about 1850–1860, *all* Ministers of Public Works in France had their portraits painted with one of Minard's creations in the background (Chevallier, 1871, p. 17). In 1861, some of Minard's works were presented to Napoleon III (a singular honor for an engineer of middle-class background), who received them with enthusiasm.

Robinson (1967) provided further insights into Minard's role in the development of thematic cartography in the latter half of the 19th century and his personal history.

Minard's Graphic Inventions

Minard's maps and charts used and extended a number of other graphic forms to display data: pie charts, the Coxcomb (or "rose diagram") attributed to Florence Nightingale, a shading scheme with two colors of varying lightness to show a bipolar dimension, and so forth, but always with the goal "to make the proportions of numerical relations apparent to the eye immediately". (Minard, 1862, p. 2 quoted in translation in Palsky, 1991, p. 114). Like Playfair's charts, Minard's graphic inventions were designed "to speak to the eyes" (Minard, 1862, p. 4 quoted in translation in Palsky, 1991, p. 114).

For example, Playfair (1801) invented the pie chart, but Minard was the first to use a divided circle in cartography (Wallis & Robinson, 1987), and he used it in a novel way—to show *both* amounts and relative proportions on a map. Figure 4 shows the quantities of butcher's meats supplied to the Paris market by each department. The total quantity of all meat from each department is shown by a circle whose area is directly proportional to this total by weight. Each circle is subdivided to show the relative proportion of beef (black), veal (red), and mutton (green). In the color original it is immediately apparent that regions close to Paris supplied large proportions of veal or mutton, while those far from Paris supplied mostly beef. The background shading also distinguishes those departments with no contribution (beige) from those with any nonzero contribution (yellow). The map was produced by lithograph and hand colored.

This idea of scaling an icon to a size that represents a regional total, while the internal subdivisions represent the relative proportions within the geographical region, is another significant contribution of Minard. Also noteworthy are a series of *tableaux graphiques,* using variable-width divided bars, shaded in different colors to display the numbers in a two-way table as proportions in relation to their row or column totals, an early precursor of the mosaic display (Friendly, 1994, 2001). Minard himself never used this graphic form on a map, but figures from *l'Album* in 1883 and 1884 depict the transportation of goods and passengers in France, where the data for *each* city is portrayed as a mosaic, with an overall size proportional to the total for that city, just as in the pie-map for meat. These figures are also remarkable in their use of spatial recursion—the transport out of each city is connected to its destination, where a smaller mosaic records the distribution from there.

The Ebb and Flow of Minard's Graphic Output

Some additional observations on Minard's career came from a visit to the Archives of the ENPC to view Minard's original and subsequent attempts to document his contributions. As Robinson (1967) pointed out, there were two, largely distinct, phases to Minard's remarkable career. The first, from 1810 (age 29), was as an engineer and later instructor and administrator for the ENPC, until his (mandatory) retirement at age 70 in 1851.

His second career as cartographer and graphical innovator—a visual engineer—dated from the 1844 publication of his *tableaux graphiques* and lasted until his death in 1870. The significant events and accomplishments in Minard's two careers are listed as .pdf timelines on the GDV website, which draws on the obituary by Chevallier (1871), and the accounts of Robinson (1967) and Palsky (1996). Events in his career as a thematic cartographer are shown here in Table 1.

As a practical engineer, Minard had no ciriculum vitae, but the catalog of his works held by the ENPC is impressive. Among 105 items, it lists eight short books (including *La Statistique,* 1869), 10 sets of course or lecture notes, 30 published brochures, and 17 articles in the *Annales des Ponts et Chaussées,* in addition to nearly 60 maps, tables, and other graphic work. Three graphic construction designs for bridges and arches executed while he was a student (circa 1812) show the early hand of a master draftsman; a later example of this work may be seen in Tufte (1983, p. 39). Minard (1862, p. 4) dates his first *statistical* graphic to 1826, but says he had just followed a form of graphic tables used earlier by Layton Cooke in England.

TABLE 1
Minards Timeline: Later Years.

Later years (1843–1870): Thematic Cartographer, Visual Engineer for France

Concern with passenger travel: *parcours partial*	**1843**		
Tableaux graphique: travel vs. distance, area ~ income	**1844**		
1st flow map: travellers from Dijon to Mulhouse	**Mar. 1845**		
Appointed Inspector General of ENPC (age 65)	**1846**		
Retires from ENPC (age 70)	**Mar. 27, 1851**	**1851**	Map of transport of combustible minerals, presented to Ministers of Finance, Public Works
3rd Intl. Statist. Cong. (Vienna)	**1857**	**1857**	Forms of graphics includes flow maps
1st use of proportional pie charts on map (butcher's meats in Paris)	**1858**		
Minard's maps presented to Napoleon III	**1861**		
Des tableaux graphique et des cartes figurative: 4 plates	**1862**		
First use of comparative flow maps	**1863**		
Center-of-gravity map (Paris)	**1865**	**1865**	Flow map of movement of passengers on principal railways of Europe
Population density of Spain, shading ~ pop. density	**1866**		
Movement of ancient languages graphic	**1867**		
March on Moscow and Hannibal graphic	**1869**	**1869**	*La Statistique*: the applications of statistics
Leaves Paris to escape feared Prussian siege	**Sep., 1870**	**Oct. 24, 1870**	Dies in Bordeaux (age 89)

In this review, it seemed appropriate to attempt a *graphic* view of Minard's later career from the catalog and bibliographic materials. A smoothed time plot of his graphic output (see Figure 5) proved quite revealing, in both numbers and content. First, we see that Minard published about two graphic works per year over the 27-year period, but the rate varied systematically with events in his life. The rate drops precipitously after his promotion to Inspector General of the ENPC in 1846. After his retirement, it rises steadily over the next 10 years to a peak of twice his average rate in the early 1860s at age 80.

Minard suffered increasingly from arthritis and rheumatism in his later years, requiring crutches, which may account for the decline over the last nine years—but only down to his average rate. In September 1870, as the Prussian army moved steadily toward Paris, the 90-year-old Minard became increasingly fearful and fled with his family to Bordeaux, regretfully leaving behind the books and papers he had worked on for the last 25 years. Six weeks later, he took ill with a terrible fever and died three days later, on October 24 (Robinson, 1967, p. 99). He had taken with him several works in progress (Chevallier, 1871, p. 21), but these evidently have been lost.

As much as this quantitative portrayal tells a part of Minard's story, Minard's choice of subject matter and graphic format tells another part (Figure 5). After several initial *tableaux graphiques* depicting the movement of passengers on railways (designed to inform the choice of rate structure for short versus long journeys), Minard developed the flow map in 1846, and used this graphic form almost exclusively until 1864 to portray his bread and butter topics—the transportation of goods by water or by land. He wrote, "I realized, by substituting merchandise for voyagers, my maps and graphic tables acquired numerous commercial applications" (Minard, 1869, p. 8).

In 1862, he published a flow map showing patterns of emigration from Europe throughout the world (colored by origin of departure), and in 1863, a pair of beautiful flow maps comparing the importation of cotton to Europe in 1858 and 1862, showing dramatically the effects of the American Civil War on trade (Palsky, 1996, Fig. 46–47). His spurt of five graphs in 1865 includes three comparative maps, one relating the strategic disposition of the forces of Charlemagne against the Huns in 791 to those of Napoleon against the Austrians in 1805. These also include the center-of-gravity map for the Paris post office. In addition, a map showing the population density of provinces of Spain by cross-hatching, uses a visual scheme (1/spacing ~ population) in which the numerical value can be read *directly* (rather than using class intervals), one of the first such uses. In 1867, he produced an entirely different form of map, showing the movement of ancient languages throughout Europe. The "Napoleon's March on Moscow" graphic of 1869 was followed only by two *tableaux graphiques,* one showing the decline in the study of Latin from 1818 to 1864 and the other showing an increase in the mean age of students at each level of promotion at the École Polytechnique.

Thus, we see that the fluctuations in Minard's quantitative output were accompanied by a qualitative shift in emphasis. They also included wide experimentation with new graphic forms, an intellectual concern for the uses of historical data and social statistics, and the continuing desire to portray these graphically.

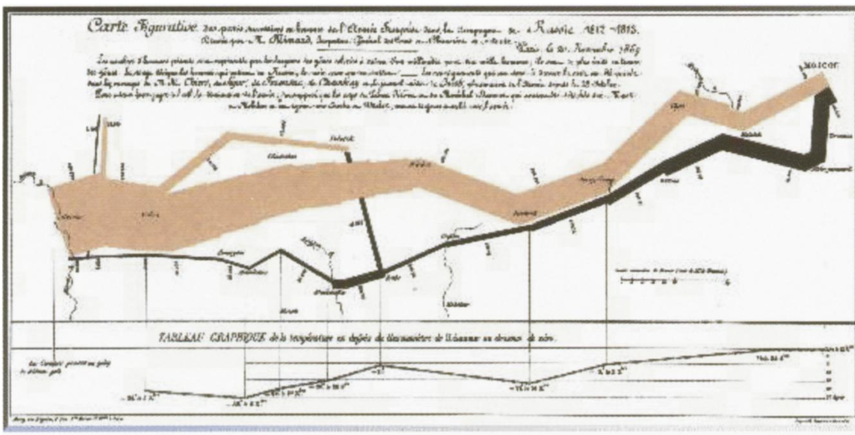

FIGURE 1 *Minard's* carte figurative *of Napoleon's 1812 campaign. Source: Tufte (1983).*

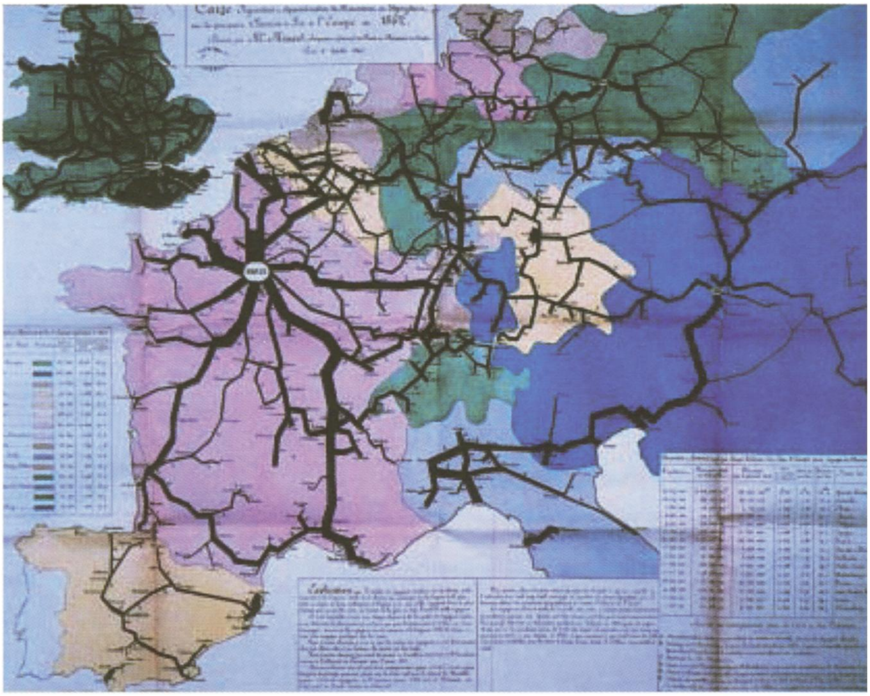

FIGURE 2 *Minard's 1865 flow map, showing the movement of travelers on the principal railroads of Europe.*
Note. (Col. et cliché ENPC, 5862/C351; used by permission.)

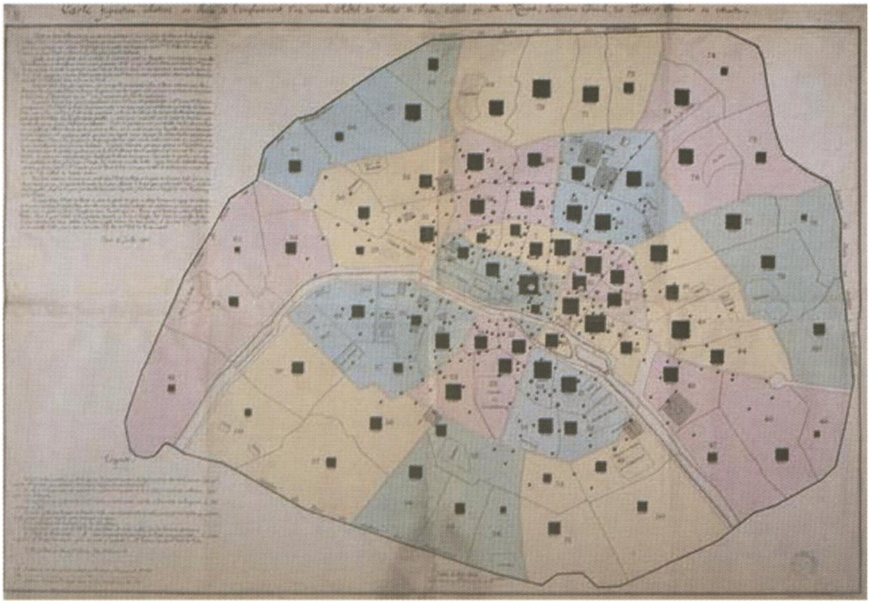

FIGURE 3 *Minard's center-of-gravity map of Paris.*
Note. The visual center of gravity of population density was used to choose location for a new post office. (Col. et cliché ENPC, 10970/C589; used by permission.)

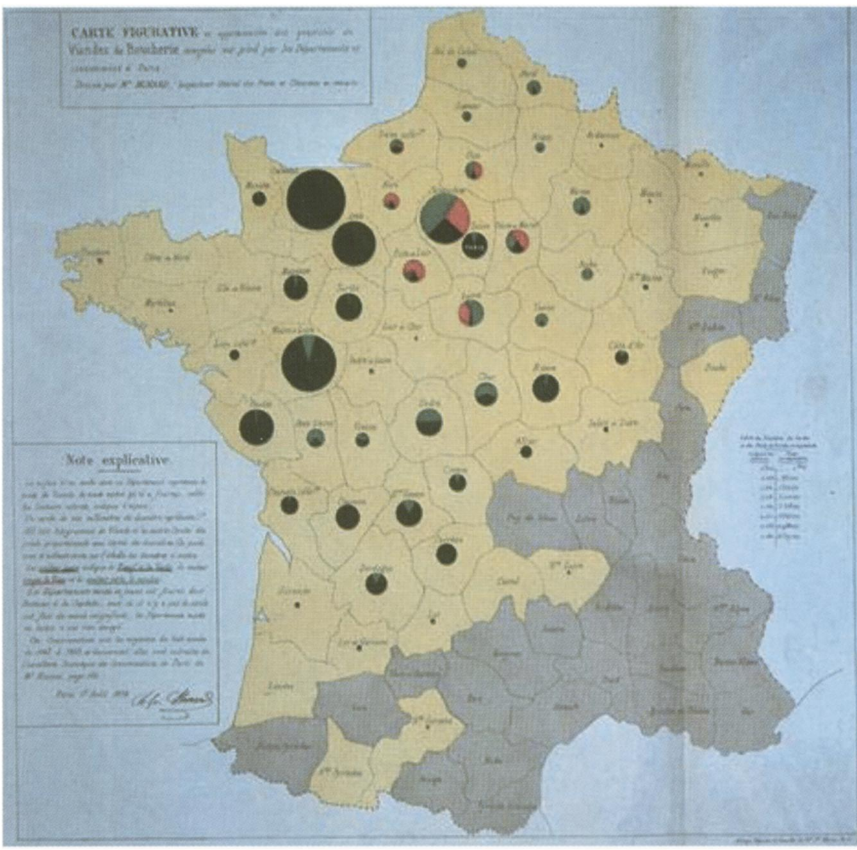

FIGURE 4 *Minard's 1858 divided-circle map, showing the amounts and proportions of butcher's meats supplied to the Paris market.*
Note. This is the first known use of pie charts in cartography. (Col. et cliché ENPC, 10969/C590; used by permission.)

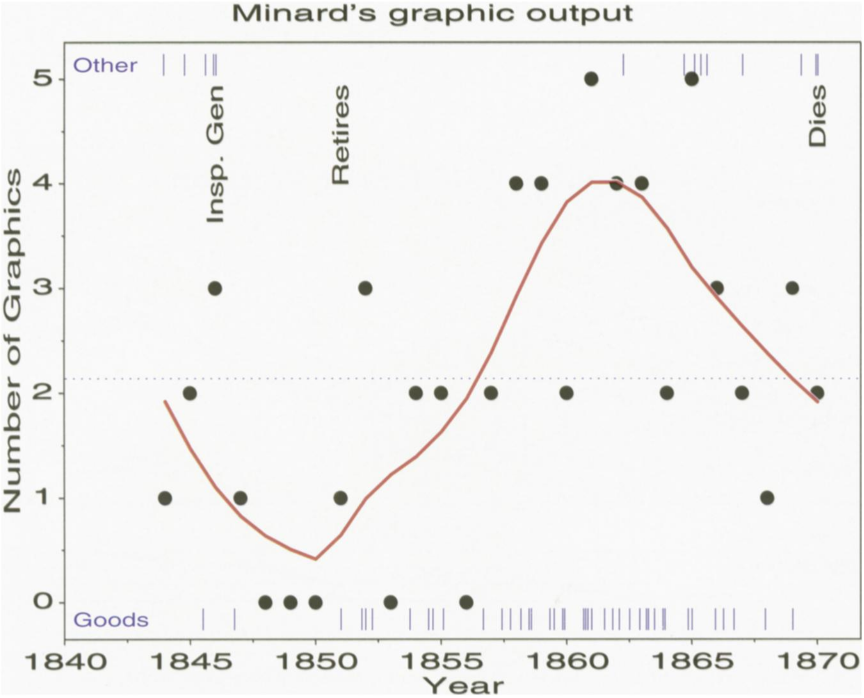

FIGURE 5 *The ebb and flow of Minard's graphic output. The dotted horizontal line shows the mean number of graphics produced each year from 1844–1870. The smoothed curve is a nonparametric quadratic lowess smoother. Vertical lines at the top and bottom show individual graphics, classified by content: Goods vs. Other.*

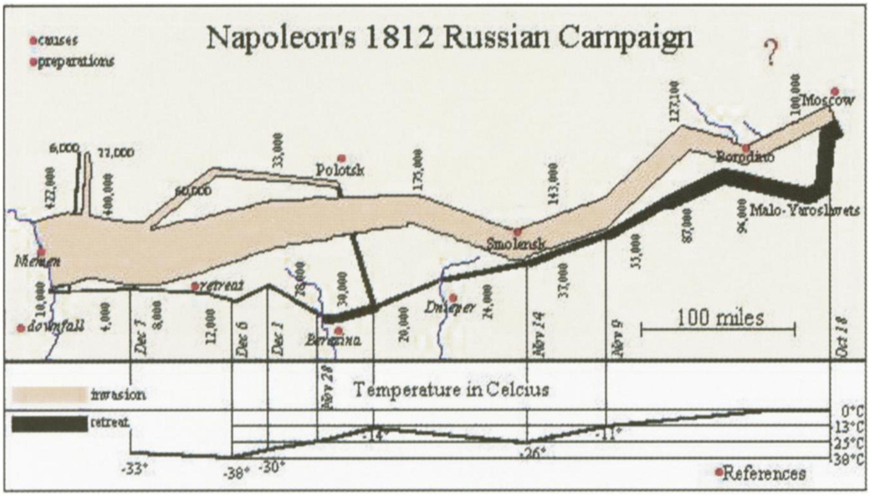

FIGURE 6 *Re-vision of Minard's graphic as a clickable web image map.*

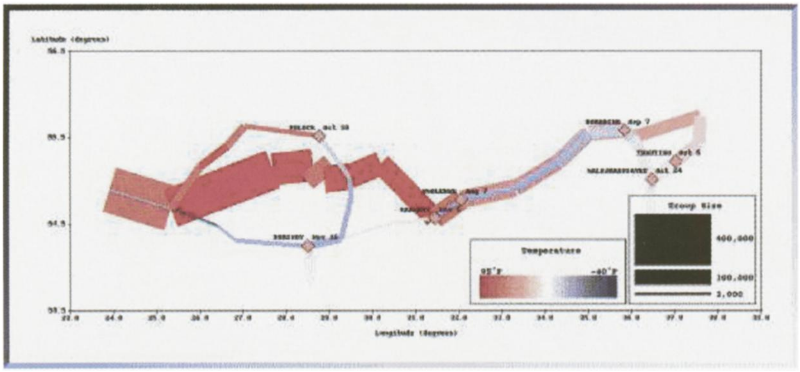

FIGURE 7 *SAGE graphic, re-visioned to link temperature information directly to the path and strength of Napoleon's army.*

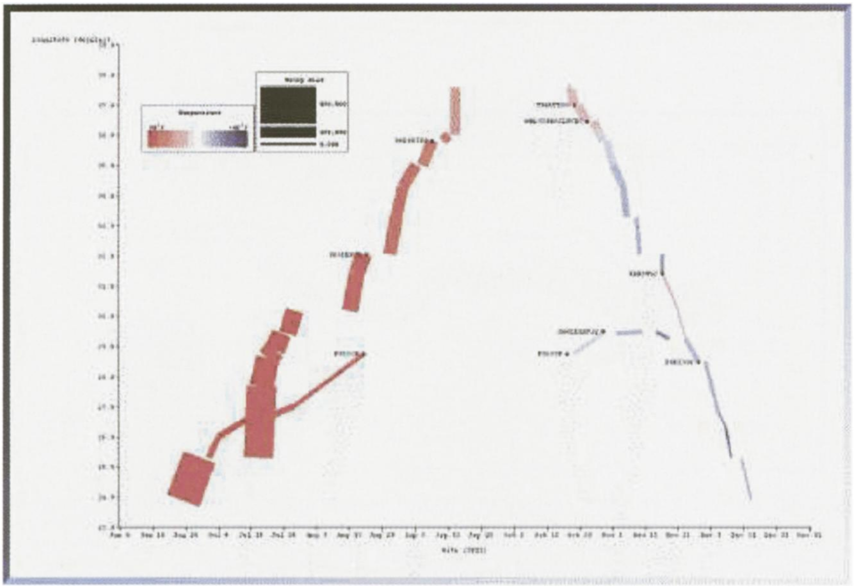

FIGURE 8 *SAGE graphic, re-visioned to show the relations between longitude, time and temperature explicitly.*

Napoleon's March Re-visited

A final historical understanding came as I viewed Minard's original of the "the best graphic ever produced." The Napoleon's March on Moscow graphic and the flow map of Hannibal's army were in fact printed together, and it is certain that Minard meant for them to be compared, as in his earlier comparison maps. Hannibal's campaign begins in southern Spain with 96,000 troops, crosses southern France, and ends in Italy with 26,000 troops. The loss of life was not nearly as dramatic as that suffered by Napoleon's troops, but the map does draw visual attention to the relatively large loss as Hannibal crossed the Alps. Together, the maps of these two campaigns provide a visual lesson to historians and generals, which might have been subtitled, "Some things to avoid in planning a military campaign."

In fact, there is a more personal and more emotive meaning to this graphic—as an antiwar statement by an engineer who had witnessed the horrors of war in his youth and who, in his final year, was forced to flee his home as the Prussian army approached Paris in September 1870. Chevallier says, "Finally, . . . as if he could sense the terrible disaster that was about to disrupt the country, he illustrated the loss of lives that had been caused by . . . Hannibal and Napoleon. . . . The graphical representation is gripping; . . . it inspires bitter reflections on the human cost of the thirst for military glory" (Chevallier, 1871, p. 18). It may well be for this reason that Minard's most famous graphic defied the pen of the historian.

Nevertheless, Marey (1878, p. 72) printed only an initial draft of the Napoleon portion in his book on graphical methods (one of the first general books on this subject); although he refers to the map of Hannibal's campaign, he does not make the thematic link between them. Funkhouser (1937) apparently relied on Marey, and, with Marey's effusive praise, it appears that he treated the Napoleon's March graphic as a separate, isolated work. Robinson (1967) described both together and, having studied Chevallier's necrology, appreciated their significance. Tufte (1983, p. 176), who reintroduced Minard to modern statistical graphics, printed the combined Napoleon-Hannibal graphic, but he used it to illustrate graphical aesthetics, rather than interpreting its meaning, either regarding Minard or the history of graphics.

Modern Re-Visions

My second sense of "Re-visions of Minard" stems from the thought that it would be a wonderful (if somewhat irreverent) challenge to take "the best graphic ever produced" and to try to improve it—or even, to reproduce it. As it has turned out, there are already a number of entries in this contest, and they prove instructive—both for appreciation of the past, and as sign-posts for the future. These may be seen in color in the GDV web site mentioned earlier; a few are worth reproducing and describing here.

Have You Ever Seen Voice-Mail?

In fact, I do see voice-mail, now that my telephone has call-display, but this question, from *The Hacker's Test* (Lee, Hayes, & Thomas, 1989), is meant to suggest the wealth of new possibilities for statistical graphics now being created by new technology. The World Wide Web, Java, CD ROMs, and DVDs provide a glimpse of future systems which will allow the easy integration of text, graphics, sound, and other media.

As one illustration, an Information Design course at the University of Texas at Austin taught by Andrew Donoho, asked students to design a web presentation based on Minard's graphic. Sunny McClendon's page (http://www.ddg.com/LIS/InfoDesignF96/Sunny/Napoleon/welcome.html) is one example of the graphic linked (as a clickable image map, or graphic menu) to pages of text describing the major battles of the campaign. The graphic itself, Figure 6, is quite similar to Minard's original, but the text labels for dates, troop strength, temperature, and battle sites have been made more visible. The design work of other students may be seen at http://www.ddg.com/LIS/.

It is not too large a jump from these student projects to imagine an authoritative, historical account (e.g., Chandler, 1996) of Napoleon's 1812 campaign linked dynamically to Minard's graphic.

Re-Visioning the Relations Among Time, Space, and Temperature

Tufte (1983) cited Minard's graphic as a narrative graphic of time and space which illustrates "how multivariate complexity can be subtly integrated" (p. 40). It is instructive to ask how this multivariate complexity can be re-visioned.

Steven Roth and others (Roth, Chuah, Kerpedjiev, Kolojejchick, & Lucas, 1997) in the Visualization and Intelligent Interfaces Group at Carnegie Mellon University have developed, a System for Automated Graphics and Explanation (SAGE) (http://www.cs.cmu.edu/groups/sage/). Several illustrations of the use of SAGE redesign the relations among the graphic elements portraying space, time, and temperature, in relation to the diminishing strength of Napoleon's army. (Exactly how much of the redesign was done automatically and what criteria were used are not described.)

Figure 7 attempts to link the information about temperature directly to the path and strength of Napoleon's army, emphasizing their interrelations more directly. The (x, y) coordinates are still map-based (latitude and longitude), but temperature is shown by the color of the bands in continuous tones, from full red (hot) to full blue (cold); the width still portrays troop strength. The SAGE group says, "The use of color clearly shows the heat wave during the advance and the steady decline in temperature through the retreat. The exception, a spell of temperatures above freezing, is clearly visible when the retreating army is between the cities of Krasnyj and Bobr."

A second redesign (Figure 8) makes the temporal characteristics of the march explicit, by replacing the map coordinates with a graph of longitude (because the campaign was essentially East-West-East) versus date. Now, the peculiar side-march to Polotsk (designed to cut the Russians' communication with St. Petersburg on the outbound leg, and return there at the end) stands out more clearly than in Minard's original. The Bavarian troops, commanded by St. Cyr, captured Polotsk in August, and remained there until October, when they rejoined the main campaign. The horizontal gaps between rectangles also serve to distinguish the lengthy stays at certain locations, the siege of Moscow being the longest, from the periods of steady march on the disastrous retreat.

Dynamic Graphics: Interactivity and Linking

Part of the wonder of Minard's graphic is how seamlessly he linked the multiple dimensions of map locations, troop strength, advance and retreat, and temperature into a coherent and poignant display. A modern approach to high-dimensional, complex data is to use dynamic, interactive graphics, with multiple, linked views to provide: selection (brushing), subdivision (drill-down), aggregation (roll-up), and so forth.

These capabilities are illustrated as well by SAGE, as described by Roth et al. (1997). The two graphs in Figures 7 and 8 (and others) can in fact be linked, so that selection in one graph highlights or paints corresponding data in all graphs. Moreover, one can drag and drop various tools, including a slider, onto the SAGE frames. Embedding a slider linked to date allows the viewer to see the changes in troop strength, for example, dynamically over the course of Napoleon's campaign. Roth et al. (1997) pointed out that this technique (using another linked bar graph of troop strength over time) reveals (a) differences in casualties among the major battles at Smolensk, Borodino and Trautino, and (b) the fact that most of the losses in troops were unrelated to battles, and occurred early in time. Chandler (1966) and others pointed to desertion and disease as major contributors.

Lessons for Data Visualization

Minard's graphic has as least one more lesson as a test case for the power and expressiveness of computer systems or languages for graphic information display and visualization. Computer languages and systems, unlike natural languages, differ widely in "elegance factors" of power and expressiveness. A language for data graphics is elegant, in the sense used here, to the extent that it provides representations of data and processes which are both simple and transparent, so that the link between *specification* and *display*—between the program statements and the visible result—is readily apparent. What can we learn from the structure and process of programming to duplicate Minard's March on Moscow graphic? Two examples provide some answers to this question.

Mathematica (Wolfraam, 1991) was designed as a computer system for doing symbolic mathematics, and also provides powerful facilities for visualization. It is similar to *Lisp* in certain respects, particularly in its emphasis on list processing, recursion, modularity, function mapping, and the syntactic identity between user-defined and built-in procedures. Shaw and Tigg (1994) describe a function, `NapoleonicMarchOnMoscowAndBackAgainPlot[ ]`, as a tour-de-force of *Mathematica* graphics. Their figure and the complete code (Minard.m) may be found on the GDV website. They use nested lists containing map (x, y) coordinates plus additional information to represent the strength of Napoleon's army, temperature, rivers, labeled points, and so forth. For example, the temperature scale at the bottom of the graphic is represented as the list:

```
TempData = {
    {0.955, 0.306, 0},    {0.885, 0.304, 0},    {0.700, 0.259, -9},
    {0.612, 0.228, -21},  {0.433, 0.177, -11},  {0.372, 0.170, -20},
    {0.316, 0.201, -24},  {0.279, 0.181, -30},  {0.158, 0.195, -26}};
```

where the final element in each sub list is the temperature in degrees Reaumur as used by Minard. Using these lists, the program (in slightly simplified form) is just:

```
NapoleonicMarchOnMoscowAndBackAgainPlot[ ] :=
  Show[Graphics[
    {ProcessStrength[StrengthData],
    ProcessTemp[TempData],
    ProcessRivers[RiverData],
    ProcessBoxes[BoxData],
    ProcessTitle[TitleData],
    ProcessPoints[PointData],
    ProcessText[TextData]}
  ] ]
```

Some of the Process functions are complex, but most are simple uses of the idea of mapping a function over a list. For example, all the rivers are drawn simply by mapping the `Line [ ]` function over the list of coordinates representing the rivers:

```
ProcessRivers[riverdata_] :=
  Map[({RGBColor[0, 0, 1], Thickness [0.001], Line
[#]}&), riverdata]
```

Each `Process` function returns a list of graphics primitives to the `Graphics[ ]` function, which are rendered on the display device by `Show[ ]`. Such simplicity, I believe, tells us something about both the structure of Minard's thematic map, and about the capability of list processing and functional programming to recreate it.

A quite different form of specification is used by Wilkinson (1999), who describes a grammar for representing the data (variables, their attributes, transformations) and graph elements (coordinates, frames, scales, and guides) which compose a graphic, along with hints for how these may be implemented in an object-oriented graphic display system. Wilkinson represents the data as seven variables which derive from a ragged data table giving the map locations (latitude, longitude) and name of cities, locations, dates, and temperatures for the temperature scale, and the locations, direction (advance/retreat), and troop strength for the main display of successive losses over the campaign. A particular feature of his analysis is to add a three-category "group" variable corresponding to (a) the main army, (b) the left-flank group going to and from Vilnius, and (c) the side-march group to Polotsk. (In the *Mathematica* data, this structure is represented implicitly in the `StrengthData` using six sublists—one for each group, and for advance/retreat.) The particular features of the data structure may or may not matter, but it is clear that parsing Minard's graphic into these three groups is essential.

Wilkinson notes that the graphic consists of two sub-graphics, the march and temperature, linked by the common horizontal scale of longitude. His analysis of the march portion represents the graphic by the grammatical specifications,

```
FRAME: lonc*latc
GRAPH: point(label(city), size (0))
GRAPH: path(position(lonp*latp), size(survivors),
       color(direction), split(group))
GUIDE: legend(color(direction))
```

where `lonc` and `latc` give the city locations, `lonp` and `latp` give locations along the paths in each `direction`, and `survivors` gives troop strength. Wilkinson says, "The plot frame is determined by longitude and latitude; plot the names of the cities at their locations, and plot one path for each group, whose width reflects troop strength, and is colored according to the direction of the march." The economy of this description is again noteworthy. (Wilkinson's re-vision also highlighted several anomalies in Minard's graphic or the historical data; the most striking is the large gap between the locations labeled "Dec. 6" and "Dec. 7" on the temperature graph, suggesting the improbable movement of more than 50 miles in one day at the end of the retreat.)

There are certainly further programming details necessary to display the complete graphic from these specifications, just as there are with the *Mathematica* version, but these attempts have something useful and important to say about the connection between specification and display (and data) required in each. As we develop data visualization systems in the future, it is well to remember that a computer language can be a tool for learning and thinking, as well as for doing. The kind of comparative analysis I have sketched here may prove helpful in the analysis and design of computer systems for graphics and visualization, and Minard's March graphic may be a diamond standard against which all should be scratched.

Conclusions

Charles Joseph Minard was most definitely not a "one-hit wonder." His graphic inventions were numerous, his thematic maps meticulously designed to aid graphic communication, "to convey promptly to the eye the relation not given quickly by numbers" (Minard, 1862). This work influenced several generations of statisticians and cartographers and still has deep lessons from which we may learn, but regrettably it is little known today. It is hoped that this re-visioning of Minard will correct that to some degree and perhaps help rekindle a wider interest in the history of statistical graphics (Friendly & Denis, 2001). Some say, "Those who do not know their history are doomed to repeat it", but, in the case of Minard, my only reply is, "If only we could!"

Note

Reproductions of Minard's graphic works may be obtained from the Multimedia Library (Médiathèque) at the École Nationale des Ponts et Chaussées (ENPC): 6 et 8, avenue Blaise Pascal, Cité Descartes, Champs-sur-Marne, 77455 Marne-la Vallée CEDEX 2, France. Tel: +33 1.64.15.34.68. Fax: +33 1.64.15.34.79. lsaye@enpc.fr; www.enpc.fr/docparis/media.htm.

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