

An integer programming approach to elective surgery scheduling

Analysis and comparison based on a real case

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Abstract The scope of this work covers a real case of elective surgery planning in a Lisbon hospital. The aim is to employ more efficiently the resources installed in the surgical suite of the hospital in question besides improving the functioning of its surgical service. Such a planning sets out to schedule elective surgeries from the waiting list on a weekly time horizon with the objective of maximizing the use of the surgical suite. For this purpose, the authors develop an integer linear programming model. The model is tested using real data obtained from the hospital's record. The non-optimal solutions are further improved by developing a custom-made, simple and efficient improvement heuristic. Application of this heuristic effectively improves almost all non-optimal solutions. The results are analyzed and compared with the actual performance of the surgical suite. This analysis reveals that the solutions obtained using this approach comply with the conditions imposed by the hospital and improve the use

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of the surgical suite. It also shows that in this case study the plans obtained from the proposed approach may be implemented in real life.

Keywords Health care · Operating rooms · Elective case scheduling · Integer programming

1 Introduction

The health sector has been progressively affected by restrictive budgets that, of necessity, not only call for an urgent need to promote a resource rationalization practice among hospitals but, above all, the demand for greater efficiency in the use of resources and the performance of each service.

The surgical suite is widely regarded as the hospital's central engine as it has a direct impact in many other hospital departments, such as surgical wards and recovery units. As such, it is deemed a priority to improve the efficiency of this component.

Improvement of the surgical suite's efficiency may lead to increased productivity, in terms of the number of surgeries undertaken, thus contributing to a reduction in surgery waiting lists. Costs involved in keeping a patient on the waiting list for surgery are high, both at the prevention and the maintenance level, even more so as considering the user's quality of life. In addition, according to Portugal's [General Direction of Health \(2004\)](#), reducing surgery waiting lists is one of the priorities of the National Health Service (SNS).

In the literature, operating room planning has been considered to be a three-stage process. [Magerlein and Martin \(1978\)](#), [Przasnyski \(1986\)](#), [Blake and Carter \(1997\)](#) and, recently, [Cardoen et al. \(2010\)](#) present literature reviews on operating room planning.

In a first stage, called *case mix planning*, operating room time is distributed among individual or groups of surgeons. At a strategic level of decision-taking, this stage defines the hospital's supply for surgery and is usually conducted on an annual basis, together with the definition of the annual hospital budget. At this stage, there are some linear or integer linear programming approaches to solve the planning problem ([Hughes and Soliman 1985](#); [Kuo et al. 2003](#); [Robbins and Tuntiwongpiboon 1989](#); [Testi et al. 2007](#)). Also a linear goal programming approach was presented by [Blake and Carter \(2002\)](#).

The second phase involves developing a surgery master schedule, a cyclic timetable that defines the number and type of operating rooms available, the hours that such rooms will be open, as well as determining the surgeons or surgical groups sharing priority in each operating room's time periods. This phase is referred to as *master surgery planning* and is related to a tactical level of hospital management. There is a greater range of approaches for this stage of operating room planning. Literature varies from integer or mixed integer linear programming models solved by general solvers ([Blake et al. 2002](#); [Santibáñez et al. 2007](#); [Testi et al. 2007](#); [Vissers et al. 2005](#)) or heuristic methods ([Blake and Donald 2002](#)), to a quadratic integer programming model solved by heuristic procedures, goal programming and

simulated-annealing in a stochastic approach (Belidn and Demeulemeester 2007; Belidn et al. 2009). Another such approach results from the study of van Oostrum et al. (2008).

The third phase lies at an operational level and is referred to as *elective case scheduling*. In this phase, each surgical case is scheduled for a specific operating room and day (sometimes referred to as *advance scheduling*). Then, either each surgery is scheduled for a specific period in the day or the surgeries scheduled for the same day are simply ordered (*allocation scheduling*).

Studies focusing only on *advance scheduling* result from the work of Marcon et al. (2003), Hans et al. (2008) and Fei et al. (2009). Marcon et al. (2003) provide an integer linear programming model based on the multiple knapsack problem to assign elective surgeries to operating rooms, thus minimizing the risk of non-performance. Hans et al. (2008) address the problem of assigning elective surgeries to operating rooms in such a way that not only surgical suite utilization is optimized but also total overtime is reduced to a minimum, thus avoiding surgery cancellations. The authors developed a robust surgery loading by assigning slack in addition to the planned surgeries on each operating room to cope with the stochastic durations of the planned surgeries. Fei et al. (2009) propose a column-generation-based heuristic in a deterministic approach to assign surgical cases to operating rooms and days. The integer programming model proposed by these authors complies with the availability of operating rooms and surgeons, and its objective seeks to minimize the cost of total unexploited opening hours and overtime.

For *allocation scheduling*, Hsu et al. (2003) present a tabu search approach for sequencing elective surgeries on a particular date such that the number of post-anaesthesia care unit nurses is minimized. Cardoen et al. (2009a,b) propose a multicriteria mixed integer linear model for sequencing elective surgeries. The authors use a weighted sum of the six objectives considered and solve the resulting problem by a branch-and-price approach. Lamiri and Xie (2006), Denton et al. (2007) and Lamiri et al. (2009) developed stochastic approaches for this part of the problem.

Guinet and Chaabane (2003) consider *elective case scheduling* in a unique problem. These authors propose an assignment model with resource and time-window constraints to assign patients to operating rooms, days and periods of time, in order to minimize patient intervention costs which stem from overtime cost and patient waiting time (defined as hospitalization cost). In this model, patients have a previous hospitalization date and thus planning surgery aims to minimize the patient's waiting time in the hospital prior to this hospitalization date. Therefore, the model allows for overtime to assign all the surgeries considered. The model proposed is solved using a primal-dual heuristic.

Ozkarahan (1995) and Jebali et al. (2006) tackle both *advance scheduling* and *allocation scheduling* in two separate problems. Ozkarahan (1995) presents an integer linear programming model similar to the job shop makespan problem to assign elective surgeries to operating rooms in order to minimize makespan, i.e. the length of time required to complete all the operations. A heuristic procedure is employed to sequence the surgeries assigned in the first step. Jebali et al. (2006) propose a mixed integer linear programming model for each of the two steps.

We should note that most of the approaches referred to for the third stage do not address the earlier stages of operating room planning. Only some authors present results from real case studies such as [Denton et al. \(2007\)](#), [Hans et al. \(2008\)](#) and [Cardoen et al. \(2009a,b\)](#).

Bearing in mind the definition of the operating room planning as a three stage process, this work falls within *elective case scheduling* and simultaneously considers two components which make up this planning phase: advance and allocation scheduling. We are engaged in a case study of a hospital in Lisbon, the purpose of which is to schedule elective surgeries for a time period and a room within a weekly planning horizon. The aim is to increase the efficiency of the hospital's surgical suite.

Bearing in mind the approach to the problem, that of jointly considering advance and allocation scheduling, plus the use of the same type of variables in the developed model, the closest work is that of [Guinet and Chaabane \(2003\)](#). Notwithstanding, the specificities of this problem are different from those observed in the case study mentioned above. [Cardoen et al. \(2009a,b\)](#) also resort to the same type of variables used in the developed model, albeit working only on allocation scheduling, and considering different specifications of the problem.

This paper proceeds in Sect. 2 with a description of the elective case scheduling problem (ECSP) and some hospital specifications. Section 3 proposes an integer linear programming model, followed by the solution approach in Sect. 4. Section 5 presents the 4 years' historical data collected at the hospital under study, along with the results of the computational experiments performed using the hospital data (Sect. 6). Finally, an analysis of the solutions is made in Sect. 7 and the conclusions are reported in Sect. 8.

2 Problem description

This work focuses on a general, central and university hospital in Lisbon, incorporated within the Portuguese National Health Service. It has no maternity or outpatient emergency service and performs about 5,000 surgeries per year. The hospital has five surgical specialties. Its surgical suite has six operating theatres, one of which is reserved for ambulatory surgeries. All rooms in the surgical suite are equipped with the same basic equipment. Specialized equipment is mobile, although it must be moved as little as possible in view of its sensitive, fragile nature. The practice of this hospital is to assign rooms to surgical specialties. We were requested by the hospital to consider the impossibility of exchanging the specialty of a room throughout the day. Although the exchange of surgical specialties in a room during the day is technically possible, this would require an operating room downtime of about an hour.

Between two surgeries performed in the same room, cleaning and disinfecting protocols, performed by auxiliary staff and taking about 30 min, must take place. Each operating room has a fixed, permanent nursing team assigned throughout the surgical suite's regular time. Each patient is assigned to a surgeon at waiting list booking time and, therefore, when planning, patient and surgeons are already assigned. Currently, the surgical suite's regular work schedule is between 8.30 am and 8 pm, from Monday to Friday.

Surgeries can be urgent or elective. Urgent surgeries do not fall within the scope of this paper. Elective surgeries are not limited to a specific time and can be planned. Elective surgery can be either conventional or ambulatory. According to Administrative Regulation nb 45/2008 of 15th January [Ministry of Health \(2008\)](#), ambulatory surgery is elective surgery with hospital admission and discharge in a period less than 24 h. Ambulatory surgery is often also called outpatient surgery and conventional surgery referred to as inpatient surgery.

Elective surgery has an associated priority level which defines its date due. Four priority levels are possible [Ministry of Health \(2008\)](#): deferred urgency (surgery must be completed in 72 h); high priority (surgery must be completed within a fortnight); priority (surgery must be completed within 2 months); and normal (surgery must be completed within 1 year).

A patient comes to the surgical suite from a hospital unit and moves to a recovery unit. Induction, surgical procedure and waking up are performed in the surgical suite. In the case study, these three procedures take place in the same location, the operating room. Hence, in this work, induction and waking up are included in surgery and treated as a single act. Induction and waking up require an anesthetist and a nurse trained in anesthetic functions (an anesthetist nurse), whereas a surgical procedure requires one or more surgeons, two nurses (a scrub nurse and a circulating nurse), auxiliary staff and equipment.

In the above context and in keeping with the hospital's requests maximum use of the surgical suite emerges as its key objective. This goal enables the hospital to increase returns on the resources installed and acquired, thus helping to increase the efficiency of this service.

Surgery planning is performed on a weekly base and is finalized on Friday for the following week. Planning is initially undertaken individually for each surgical specialty. The planning maps for each specialty ward are then sent to the surgical suite where they are conferred and verified for feasibility of the conjunction of plans from the different specialty wards by the head nurse of the surgical suite. Changes to the planning can occur during the week and must necessarily be proposed by 12 am of the previous day.

Note that, in the model developed, the problem was treated as a whole and all surgical specialties were considered as a whole for the surgery planning in the surgical suite. Practical experience in this hospital shows that the resources do not limit the activity of the surgical suite, namely beds, nurses, auxiliary staff and materials.

3 An ILP model

Every Friday, a set C of surgeries from the hospital's waiting list is selected, by increasing order of priority, for scheduling in the next planning week. The following subsets of C are defined: by specialty $j \in J$ (being J the set of surgical specialties)— C_j^{SP} ; and by priority level— C_1^{PR} and C_2^{PR} are the set of surgeries in C classified as deferred urgency and high priority, respectively. The set $C \setminus C_1^{PR}$ consists of surgeries considered for scheduling which are not classified as deferred urgency priority level. To perform these surgeries one has a set of surgeons H . Surgeries must be scheduled to a

room in the set of rooms R , and on a day in the set of days available for scheduling, D . Time has a discrete representation as a set of time periods available for scheduling, T . Let γ be the number of time periods corresponding to the 30 min needed to cleaning and disinfecting the operating room after each surgery.

Surgery $c \in C$ has surgeon h_c assigned ($h_c \in H$). Each surgery has an estimated duration that leads to the number of time periods required to execute surgery c , represented by p_c . Consequently, subset $T_c \subseteq T$ is defined, such that surgery c can start at the beginning of any time period in T_c , in order to be completed within the surgical suite regular time. Overtime is not permitted in planning and is made barred by restricting the variables domain to the respective set T_c .

Daily and weekly operating time limits for each surgeon $h \in H$ are represented, respectively, by T_{hd}^{MAXD} and T_h^{MAXW} . For each surgeon h , T_{hd}^{MAXD} may differ over the different days. The parameter i_{ctd} reflects the impossibility for surgery $c \in C$ to start at the beginning of period $t \in T_c$ and day $d \in D$ due to surgeon or patient unavailability. This parameter has a value of 0 when the surgeon or the patient is not available to start the respective surgery at the beginning of period t on day d ; otherwise, has value of 1. For all deferred urgency surgeries ($c \in C_1^{PR}$), $i_{ctd} = 0, \forall d > 1$.

Since surgeries are non-preemptive jobs, starting time variables were considered in formulating the problem. Thus, the decision variables used in the model are:

$$x_{crt d} = \begin{cases} 1, & \text{if surgery } c \text{ starts at the beginning of period } t \text{ on day } d \text{ in room } r \\ 0, & \text{otherwise } (c \in C, r \in R, t \in T_c, d \in D) \end{cases}$$

Additional variables were also considered to register on a daily basis the surgical specialty assigned to each room:

$$y_{jrd} = \begin{cases} 1, & \text{if a surgery of specialty } j \text{ starts in room } r \text{ on day } d \\ 0, & \text{otherwise } (j \in J, r \in R, d \in D) \end{cases}$$

Variables y_{jrd} can be avoided in the formulation of the problem. However, introduction of these variables in the model, though slightly increasing the total number of variables, significantly reduces the number of constraints that prevent one from using any operating room for more than one surgical specialty on the same day. Moreover, preliminary experiments showed better results using these additional variables.

The integer linear programming model used to formulate the elective case scheduling problem within the context of this case study is:

$$\max \sum_{c \in C} \sum_{r \in R} \sum_{t \in T_c} \sum_{d \in D} p_c x_{crt d} \tag{1}$$

$$\text{subject to: } \sum_{r \in R} \sum_{t \in T_c} x_{crt 1} = 1, \forall c \in C_1^{PR} \tag{2}$$

$$\sum_{r \in R} \sum_{t \in T_c} \sum_{d \in D} x_{crt d} = 1, \forall c \in C_2^{PR} \tag{3}$$

$$\sum_{r \in R} \sum_{t \in T_c} \sum_{d \in D} x_{crt d} \leq 1, \forall c \in C \setminus (C_1^{PR} \cup C_2^{PR}) \tag{4}$$

$$\sum_{c \in C} \sum_{\substack{t'=t-p_c+1-\gamma \\ t' \in T_c}}^t x_{crt'd} \leq 1, \forall r \in R, t \in T, d \in D \tag{5}$$

$$\sum_{r \in R} x_{crt'd} \leq i_{ctd}, \forall c \in C, t \in T_c, d \in D \tag{6}$$

$$\sum_{j \in J} y_{jrd} \leq 1, \forall r \in R, d \in D \tag{7}$$

$$\sum_{c \in C} \sum_{j \in J} x_{crt'd} \leq y_{jrd} |T|, \forall j \in J, r \in R, d \in D \tag{8}$$

$$\sum_{\substack{c \in C: \\ h_c=h}} \sum_{\substack{t'=t-p_c+1 \\ t' \in T_c}}^t \sum_{r \in R} x_{crt'd} \leq 1, \forall h \in H, d \in D, t \in T \tag{9}$$

$$\sum_{\substack{c \in C: \\ h_c=h}} \sum_{r \in R} \sum_{t \in T_c} p_c x_{crt'd} \leq T_{hd}^{MAXD}, \forall d \in D, h \in H \tag{10}$$

$$\sum_{\substack{c \in C: \\ h_c=h}} \sum_{d \in D} \sum_{r \in R} \sum_{t \in T_c} p_c x_{crt'd} \leq T_h^{MAXW}, \forall h \in H \tag{11}$$

$$x_{crt'd} \in \{0, 1\}, \forall c \in C, r \in R, t \in T_c, d \in D \tag{12}$$

$$y_{jrd} \in \{0, 1\}, \forall j \in J, r \in R, d \in D \tag{13}$$

In the above model, objective function (1) maximizes surgical suite occupation. Constraint set (2) forces deferred urgency level priority surgeries to be scheduled on Monday to meet the 72h deadline for their completion. Constraint set (3) imposes high priority surgeries to be scheduled during the planning week. It should be noted that it is not certain that surgeries classified as deferred urgency and highly priority may all be scheduled within the respective periods. However, practical experience, as well as the number of surgeries of this type usually found in the waiting list and its distribution by the surgical specialties, shows that such impositions do not generally render the problem unfeasible. The model could easily be changed in order to include this explicitly, using slack variables in constraints (2) and (3) with the corresponding penalty terms in the objective function. The option of mandatory scheduling high priority surgeries during the week arises as a means of easing the following week’s planning. Constraints (4) state that the remaining surgeries, classified as priority or normal, may be scheduled or not during the planning week.

Constraints (5) guarantee that surgeries do not overlap in the same room. These constraints also impose γ empty periods for room cleaning at the end of each surgery (based on the definition of the lower sum limit). Constraints (6) provide the possibility to consider surgeons’ or patients’ unavailability periods. Constraint set (7) prevents assignment of more than one surgery specialty to each room and day. Therefore, it is

not permitted to exchange surgery specialty in the room during the day. Constraints (8) are the linking constraints for variables x and y . Constraints (9) ensure that surgeons do not overlap between rooms in the same time period and day. In the real situation of the hospital involved, surgeons may exchange operating rooms. On the one hand, this exchange is feasible as the rooms are physically side by side. On the other hand, permission to exchange operating rooms by surgeons allows them to work in another operating room during hygiene periods in the previous room (about 30 min idle). This is also the reason why the cleaning time is not incorporated in the surgeries' duration. Constraint sets (10) and (11) impose a daily and weekly operating time limit on each surgeon. The time limit on Monday (day 1 on the planning's horizon) can be enlarged in order to guarantee the scheduling of all deferred urgency surgeries of the respective surgeon. Finally, constraints (12) and (13) express the variables' domain.

4 Solution approach

Model (1)–(13) is highly complex and attains a large dimension in real instances. In fact, the model has $o(|C| \times |R| \times |T| \times |D|)$ variables and constraints, naturally assuming that the number of surgeries to be scheduled is at least equal to the number of operating rooms available for scheduling ($|C| \geq |R|$).

Hence, the elective surgeries' scheduling problem was decomposed into two hierarchical phases. With operating rooms clearly separated and distinct for the planning, the abovementioned division was made according to the nature of the surgeries: conventional and ambulatory. As conventional surgeries represent a greater number of surgeries (about 85% of the hospital's waiting list) and require the planning of five operating rooms, thus generating a high dimension problem, these type of surgeries are planned in the first phase. In the light of the plan obtained in the first phase, ambulatory surgeries are planned in the second phase. As one must assess only one operating room and since the number of ambulatory surgeries in the waiting list is much smaller, in a real instance, the second phase constitutes a problem of a rather reduced dimension, compared to the first planning phase. Division of the elective case scheduling problem into two hierarchical phases as described above allows the global problem dimension to be reduced.

In each phase, an integer linear programming (ILP) solver was used with limited time. In the event of its stopping without optimality, the best feasible integer solution obtained was improved using a simple improvement heuristic. The improvement heuristic developed can be summarized in the following four steps:

1. Re-schedule surgeries as early as possible in the day, while retaining the same order.
2. Try to schedule unscheduled surgeries in the time available at the end of each day, respecting each room's surgery specialty and ensuring that each surgery is completed within surgical suite regular time.
3. Try to exchange two or three consecutive scheduled surgeries, with priority or normal level of priority, for one unscheduled surgery whose duration is no greater.

4. If the last surgery scheduled at the end of the day is classified as priority or normal, try to exchange this last surgery for one unscheduled surgery occupying the remainder of the regular time in the day.

Every heuristic step must be performed taking into account feasibility, defined by constraints (2)–(13). In particular, the heuristic satisfies surgery priorities as described in the model.

The first step rearranges surgeries scheduled to enable one to schedule more surgeries in step 2, to allow the surgical suite to be used more efficiently and thus improve the value of the objective function. Step 3 refers directly to the objective function, since the exchange of two or three consecutive surgeries by another one with a non-superior duration avoids the empty periods for cleaning the operating room between any pair of surgeries and, therefore, the value of the objective function increases. The last step aims to complete the daily use of each operating room in the surgical suite, by replacing the last scheduled surgery by another, thus occupying the room until the end of regular time.

Note that while step 1 only serves to support step 2, and does not contribute directly to any change in the solution value, each of the following three steps directly permits the objective function value to increase.

5 Data

The hospital under study provided a historical record containing information on all surgeries performed in the hospital surgical suite from 1 January 2004 to 28 December 2007. In this period, 21,050 surgeries were performed.

Tables 1 and 2, respectively, describe the duration of the conventional and ambulatory surgeries performed, aggregated by surgical specialty. These tables show that the median value is lower than the mean value for all surgical specialties. The difference is less substantial in the case of ambulatory surgeries. The minimum values point to data insertion errors.

In addition to this historical record, another one was provided, and refers to the waiting list for surgery in seven different moments in which decisions about the weeks' planning occurred (Friday) as well as the respective hospital week planning and hospital record of each of those weeks. The weeks provided are those starting on 12 and 26 February 2007, 5, 12, 19 and 26 March 2007 and 2 April 2007.

Table 1 Descriptive of conventional surgeries duration in the historical record (in minutes)

Surgical specialty	Mean	Median	St.dev.	Minimum	Maximum	Number
Otorhinolaryngology	92	77	61	0	410	2,371
Digestive and general surgery	75	61	53	0	562	6,529
Thorax surgery	110	96	62	5	476	2,148
Urology	72	54	58	2	473	4,165
Angiology and vascular surgery	72	54	54	1	413	1,652

Table 2 Descriptive of ambulatory surgeries duration in the historical record (in minutes)

Surgical speciality	Mean	Median	St.dev.	Minimum	Maximum	Number
Otorhinolaryngology	28	27	13	2	107	1,484
Digestive and general surgery	41	40	22	1	155	1,359
Urology	31	27	18	0	147	412
Angiology and vascular surgery	40	38	18	5	132	488

Table 3 Descriptive of waiting lists for conventional and ambulatory surgeries in 4 weeks

	9 February		23 February		2 March		9 March	
	Conv.	Amb.	Conv.	Amb.	Conv.	Amb.	Conv.	Amb.
Number of surgeries	2,043	264	1,984	274	1,944	265	1,899	287
Surgical speciality (%)								
Otorhinolaryngology	9.9	15.5	11.7	19.0	11.2	20.8	10.3	21.3
Digestive and general surgery	52.9	36.8	53.1	37.6	55.6	35.8	55.5	37.6
Thorax surgery	5.0	–	4.6	–	4.0	–	4.3	–
Urology	18.2	3.4	16.9	2.9	15.2	40.0	16.0	37.3
Angiology and vascular surgery	14.0	44.3	13.7	40.5	14.0	3.4	13.9	3.8
Priority level (%)								
Deferred urgency	0.98	0.7	1.2	0	1.0	0	0.8	0
High priority	0.05	0	0	0	0	0	0	0
Priority	3.67	2.3	2.8	2.9	2.7	3.4	2.9	2.8
Normal	95.3	97.0	96	97.1	96.1	96.6	96.3	97.2

Table 3 presents a brief description of the waiting list for surgery in the respective decision-taking moments (Friday) for the first 4 weeks.

6 Computational experiments

A computational experiment was developed to test the solution approach described in Sect. 4 with real data from the hospital. Tests focused on the four weeks described above (the remaining weeks were also tested and led to similar results).

Surgeon and patient unavailability, represented by constraints (6), were not included in the computational tests for conventional surgery planning (first phase scheduling) since it was not possible to obtain the relevant data. However, these sets of constraints were included in computational tests for ambulatory surgery planning (second phase scheduling) to ensure feasibility of the whole week's planning, linking conventional and ambulatory surgeries planning. In addition, total time spent by each surgeon in the conventional surgeries' schedule was reduced in the total daily and weekly operating time limit to be used in constraints (10) and (11) for ambulatory surgeries planning.

Despite the different instances concerned, the model used to schedule ambulatory surgeries is the same as the one employed to schedule conventional surgeries.

The expected duration for each surgery was based on mean and median values obtained in the historical data for the same surgical procedure. When there is no surgery in the historical data performed with the same surgical procedure, the expected surgery duration is computed as the mean or median values for the corresponding surgery specialty. Time periods of 15 min were employed, thus creating 46 daily time periods in regular working time, whilst overtime is not provided for planning. The choice of the time period's duration results from the balance between precision in the duration of surgeries and the respective impact on the model's dimension. For instance, by reducing the dimension of time periods to 10 min one arrives at 69 daily time periods in regular working hours, thus significantly increasing the number of variables and constraints to the model. On the other hand, as there is considerable uncertainty inherent in the plan and its performance, diminishing the dimension of the time periods at the cost of increasing the model's dimension is unwarranted.

The daily and weekly operating time limit for each surgeon was based on a percentage of his/her daily and weekly working hours. For all surgeons the operating time limit of 75% of an 8 h working day and 60% of a 42 h working week was considered.

Instances are identified by a reference to the number of surgeries considered for planning ($|C|$) and, in subscript, a reference to the type of expected duration used (respectively, 1 and 2 for mean and median values). Symbols \mathcal{A} and \mathcal{C} respectively denote ambulatory and conventional planning. Since the waiting list for conventional surgery is much greater than the number of surgeries of this type that can be scheduled weekly, a subset of surgeries was considered as input for the model proposed for ECSP, constituting set C . The selection was made on the basis of the surgeries included in hospital planning, in order to enable a comparative basis, followed by a selection of surgeries by order of priority until the required number of surgeries ($|C|$) was obtained. Bearing in mind this guideline, sub-sets of the waiting list have been selected for conventional surgery with 250, 300, 500 and 1,000 surgeries. In this way, instances of a higher dimension include instances of minor dimension for the same planning week.

The ILP models were solved using CPLEX 11.0 with CONCERT 2.5 [ILOG \(2003, 2004\)](#). The improvement heuristic was coded in C++ language. Tests were performed in a Core2 Duo, 2.53 GHz computer with 4 GB of RAM. Time limit to run ILP model with Cplex was set to 30,000 s (about 8 h of running time).

The results are shown in Tables 4, 5 and 6. In Tables 4 and 5, column 1 refers to the test instance and columns 2 and 3 to its dimension given by the number of variables and constraints, respectively. *LP Time* is the time taken to solve the linear relaxation. Columns 5 and 6 refer to the gap obtained within Cplex time limit and to the time needed to reach this gap (when the latter does not match Cplex's time limit, it means that as of the time identified in the aforesaid table's column up until the final of the 30,000 s, the gap value was not improved). Table 4, *H Time* displays the time employed by the improvement heuristic and *Final Gap* is the gap associated with the heuristic solution. Table 6 presents the final results obtained for elective surgeries' planning, aggregating the results of the planning phases for conventional and ambulatory surgeries. The first column refers to the instance. The second and third columns indicate

Table 4 Results obtained for conventional surgeries planning

Instance	Variables	Consts.	LP time (s)	IP gap (%)	Time to gap (s)	H time (s)	Final gap (%)
Planning week: 12–16 February 2007							
Pw1C_250 ₁	242,200	11,934	1,571.76	9.97	13,895.5	0	6.55
Pw1C_250 ₂	243,525	11,934	435.71	5.27	12,865.3	0	5.17
Pw1C_300 ₁	294,225	11,984	616.66	10.02	6,526.5	0	3.78
Pw1C_300 ₂	295,800	11,984	595.17	7.65	9,131.5	0	5.76
Pw1C_500 ₁	503,625	12,184	444.52	3.74	18,743.2	0	3.46
Pw1C_500 ₂	506,725	12,184	355.71	12.98	9,677.8	0	5.83
Pw1C_1000 ₁	1,035,500	13,864	746.54	<i>o.m.</i>	–	–	–
Pw1C_1000 ₂	1,041,525	13,864	672.01	<i>o.m.</i>	–	–	–
Planning week: 26–2 March 2007							
Pw2C_250 ₁	240,370	11,698	459.21	8.03	5,592.4	0	6.13
Pw2C_250 ₂	241,720	11,698	1,704.54	–	30,000	–	–
Pw2C_300 ₁	293,570	11,984	619.68	–	30,000	–	–
Pw2C_300 ₂	295,195	11,984	613.91	7.01	8,017.3	0	5.19
Pw2C_500 ₁	500,845	12,420	363.84	<i>o.m.</i>	–	–	–
Pw2C_500 ₂	503,620	12,420	678.96	12.19	11,933.2	0	6.36
Pw2C_1000 ₁	1,030,895	13,864	825.46	<i>o.m.</i>	–	–	–
Pw2C_1000 ₂	1,037,095	13,864	821.22	<i>o.m.</i>	–	–	–
Planning week: 5–9 March 2007							
Pw3C_250 ₁	242,515	10,518	476.15	3.32	7,291.5	0	2.87
Pw3C_250 ₂	243,680	10,518	387.33	3.24	16,866.6	0	1.09
Pw3C_300 ₁	294,315	11,040	556.94	2.93	26,608.9	0	2.81
Pw3C_300 ₂	295,680	11,040	545.96	1.29	27,924.7	0	1.29
Pw3C_500 ₁	503,790	11,712	324.34	1.45	22,430.8	0	1.45
Pw3C_500 ₂	506,255	11,712	425.34	3.18	5,212.2	0	2.97
Pw3C_1000 ₁	1,032,690	13,628	815.35	<i>o.m.</i>	–	–	–
Pw3C_1000 ₂	1,038,605	13,628	626.67	<i>o.m.</i>	–	–	–
Planning week: 12–16 March 2007							
Pw4C_250 ₁	245,975	12,406	459.14	1.06	23,867.4	0	1.06
Pw4C_250 ₂	247,895	12,406	469.55	1.08	15,352.0	0	1.08
Pw4C_300 ₁	298,525	12,692	776.23	5.15	28,327.6	0	5.15
Pw4C_300 ₂	300,570	12,692	513.31	2.49	6,783.7	0	1.83
Pw4C_500 ₁	508,650	13,128	1,152.74	1.34	25,254.2	0	1.13
Pw4C_500 ₂	511,770	13,128	420.55	4.18	19,034.7	0	3.62
Pw4C_1000 ₁	1,036,100	14,572	859.77	<i>o.m.</i>	–	–	–
Pw4C_1000 ₂	1,042,670	14,572	689.10	<i>o.m.</i>	–	–	–

o.m. Out of memory

Table 5 Results obtained for ambulatory surgeries planning

Instance	Variables	Consts.	LP time (s)	IP gap (%)	Time to gap (s)
Planning week: 12–16 February 2007					
Pw1C_250 ₁ + Pw1.A_264 ₁	57,768	69,595	2.45	0	11.81
Pw1C_250 ₂ + Pw1.A_264 ₂	57,803	69,630	2.07	0	7.92
Pw1C_300 ₁ + Pw1.A_264 ₁	57,768	69,595	1.92	0	7.72
Pw1C_300 ₂ + Pw1.A_264 ₂	57,803	69,630	1.75	0	19.78
Pw1C_500 ₁ + Pw1.A_264 ₁	57,768	69,595	2.15	0	10.62
Pw1C_500 ₂ + Pw1.A_264 ₂	57,803	69,630	2.42	0	8.42
Planning week: 26–2 March 2007					
Pw2C_250 ₁ + Pw2.A_274 ₁	60,455	72,056	2.20	0	56.38
Pw2C_300 ₂ + Pw2.A_274 ₂	60,475	72,076	2.37	0	90.34
Pw2C_500 ₂ + Pw2.A_274 ₂	60,475	72,312	2.15	0	12.40
Planning week: 5–9 March 2007					
Pw3C_250 ₁ + Pw3.A_265 ₁	58,490	69,610	2.20	0	170.29
Pw3C_250 ₂ + Pw3.A_265 ₂	58,535	69,655	2.09	0	263.14
Pw3C_300 ₁ + Pw3.A_265 ₁	58,490	69,846	2.18	0	27.24
Pw3C_300 ₂ + Pw3.A_265 ₂	58,535	69,891	2.54	0	1,122.04
Pw3C_500 ₁ + Pw3.A_265 ₁	58,490	70,082	2.25	0.62	8.30
Pw3C_500 ₂ + Pw3.A_265 ₂	58,535	70,127	2.36	0	125.66
Planning week: 12–16 March 2007					
Pw4C_250 ₁ + Pw4.A_287 ₁	63,360	75,682	2.81	0	181.09
Pw4C_250 ₂ + Pw4.A_287 ₂	63,400	75,722	2.43	0.62	1,529.12
Pw4C_300 ₁ + Pw4.A_287 ₁	63,360	75,918	2.57	0	213.49
Pw4C_300 ₂ + Pw4.A_287 ₂	63,400	75,958	2.25	0	262.67
Pw4C_500 ₁ + Pw4.A_287 ₁	63,360	76,154	2.36	0	197.51
Pw4C_500 ₂ + Pw4.A_287 ₂	63,400	76,194	2.29	0.62	11.31

respectively the total time and total gap of the global solution. *Time periods booked* includes the number of time periods booked for conventional and ambulatory surgeries and the respective sum. The last two columns display the corresponding surgical suite occupation rate in regular time, respectively, without and with the cleaning time.

As seen in columns 2 and 3 of Tables 4 and 5, the model reaches high dimensions in real instances. The instances that used the median value for the expected surgery duration possess more variables than the instances that used the mean value. This observation is justified by the fact that the median value is, in general, lower than the mean value, thus increasing the number of time periods in T_c . Ambulatory surgery instances (column 2 of Table 5) have far less variables than conventional surgery instances (column 2 of Table 4) since they have fewer surgeries and include only one operating room. However, the number of constraints in ambulatory surgery instances (column 3 of Table 5) increases significantly due to the inclusion of constraints (6)

Table 6 Results obtained for elective surgery planning

Instance	Total time (s)	Total gap (%)	Time periods booked			OR occup. rate (%)	
			Conv.	Amb.	Total	Without cl. time	With cl. time
Planning week: 12–16 February 2007							
Pw1C_250 ₁ + Pw1.A_264 ₁	15,481.5	5.54	885	162	1,046	75.87	96.23
Pw1C_250 ₂ + Pw1.A_264 ₂	13,311.0	4.39	889	160	1,049	76.01	96.01
Pw1C_300 ₁ + Pw1.A_264 ₁	7,152.8	3.22	927	161	1,088	78.84	99.35
Pw1C_300 ₂ + Pw1.A_264 ₂	9,748.2	4.89	903	160	1,063	77.03	97.75
Pw1C_500 ₁ + Pw1.A_264 ₁	19,200.5	2.95	955	164	1,119	81.09	98.91
Pw1C_500 ₂ + Pw1.A_264 ₂	10,044.4	4.96	927	162	1,089	78.91	98.33
Planning week: 26–2 March 2007							
Pw2C_250 ₁ + Pw2.A_274 ₁	6,110.2	5.21	897	158	1,055	76.45	96.30
Pw2C_300 ₂ + Pw2.A_274 ₂	8,723.9	4.42	906	158	1,064	77.10	96.81
Pw2C_500 ₂ + Pw2.A_274 ₂	12,626.7	5.43	927	160	1,087	78.77	97.32
Planning week: 5–9 March 2007							
Pw3C_250 ₁ + Pw3.A_265 ₁	7,940.2	2.44	907	160	1,067	77.32	97.90
Pw3C_250 ₂ + Pw3.A_265 ₂	17,519.2	0.93	914	158	1,072	77.68	99.42
Pw3C_300 ₁ + Pw3.A_265 ₁	27,195.3	2.40	924	161	1,085	78.62	98.19
Pw3C_300 ₂ + Pw3.A_265 ₂	29,595.2	1.10	929	159	1,088	78.84	99.13
Pw3C_500 ₁ + Pw3.A_265 ₁	22,765.7	1.33	967	160	1,127	81.67	99.49
Pw3C_500 ₂ + Pw3.A_265 ₂	5,765.5	2.54	944	159	1,103	79.93	98.91
Planning week: 12–16 March 2007							
Pw4C_250 ₁ + Pw4.A_287 ₁	24,510.4	0.91	940	162	1,102	79.86	99.42
Pw4C_250 ₂ + Pw4.A_287 ₂	17,353.1	1.01	927	160	1,087	78.77	99.35
Pw4C_300 ₁ + Pw4.A_287 ₁	29,319.9	4.38	912	162	1,074	77.83	96.38
Pw4C_300 ₂ + Pw4.A_287 ₂	7,562.0	1.56	928	160	1,088	78.84	98.55
Pw4C_500 ₁ + Pw4.A_287 ₁	26,606.8	0.97	974	162	1,136	82.32	99.71
Pw4C_500 ₂ + Pw4.A_287 ₂	19,468.9	3.18	939	160	1,099	79.64	98.49

in ambulatory surgeries planning, which are not considered in conventional surgeries planning.

In conventional surgeries planning (Table 4), the instances that considered 1,000 surgeries were unsolved. In the second planning week, three more instances were not solved, two of which failed to obtain an integer solution at the end of the 30,000 s and the other stopped on running out of memory. This is due to the fact that the percentage of surgeries classified as deferred urgency is higher in the second week tested (see Table 3). Nevertheless, a feasible solution was obtained on all tested weeks, giving rise to a valid operating plan for each of the planned weeks. On the other hand, in all tested instances for conventional surgeries planning, the improvement heuristic used zero seconds and effectively improved the *IP Gap* obtained in almost all of the instances tested (only five exceptions in the third and fourth planning week).

Bearing in mind the results, it is not possible to establish a conclusion on the impact of the length of the waiting list of conventional surgeries that is taken into account in each instance ($|C|$) on the results obtained in this planning phase. One is reminded that higher dimension instances contain minor dimension instances for the same planning week. However, consideration of more surgeries on the waiting list in set C provides greater flexibility to planning and, as such, enables one to obtain better results in terms of gap value. Such results are not visible in the computational test made for conventional surgeries scheduling for the following reason. Given the dimension and complexity of the model, Cplex encounters some difficulty in instances with higher dimension. On the other hand, bearing in mind the dimension of the hospital's waiting lists and the fact that less than 10% of these surgeries will be scheduled, it is not deemed necessary to consider all conventional surgeries on the waiting list for planning (set C).

The results obtained in the planning phase of ambulatory surgeries may be seen in Table 5. Since ambulatory surgeries planning is of smaller dimension, the model obtained an optimal solution in less than five minutes in almost all instances tested. In only three test instances the model was unable to prove, within the time limit, the optimality of the best integer solution attained and, in these cases, the improvement heuristic could not improve the gap value.

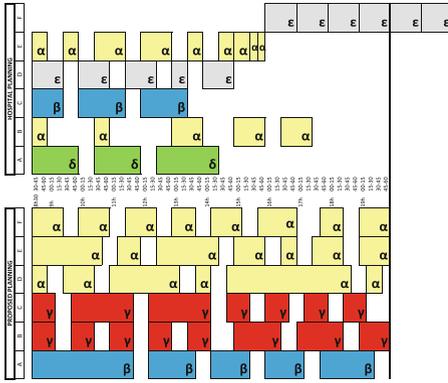
In Table 6 one may analyze the final results for the ECSP study, by joining the results of the planning phases for conventional and ambulatory surgeries. We can conclude that this approach has allowed us to find a valid operating plan for all tested weeks, producing a potential surgical suite occupation rate in regular time superior to 75% (not including the cleaning time). This value rises to 96% if cleaning time is included. This table illustrates that the method proposed in this paper is good even with instances where less conventional surgeries are considered for scheduling in the first planning phase.

7 Comparative analysis

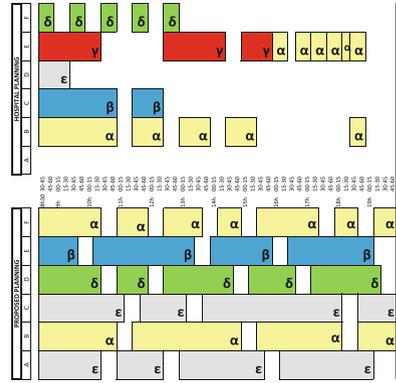
To analyze week plans resulting from the approach proposed in this paper for our case study, the authors explored the final solution from $Pw1C_{300_1} + Pw1A_{264_1}$ instance. It refers to the planning week from 12 to 16 February 2007. This planning week was arbitrarily chosen. The remainder displayed a similar behavior.

Figure 1 shows the proposed schedule, as well as the respective hospital planning for the corresponding week. For each day, Fig. 1 indicates the hospital plan (above) and the proposed plan (below).

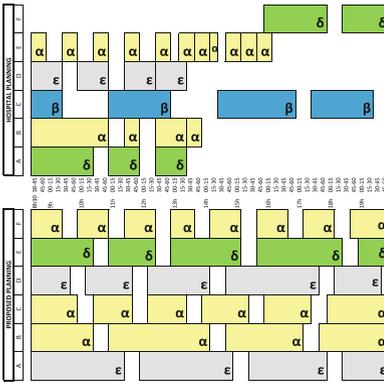
One can see that the proposed plan verifies all conditions imposed by the hospital, namely those regarding surgical suite regular working hours, bounded by the black line at the end of each day, and periods for room cleaning after each surgery (30 min corresponding to two periods in white after each surgery). However, Fig. 1 also shows that the hospital is planning without considering the time to clean the room (see e.g. Monday rooms E and F) and is using surgical suite overtime (see e.g. Monday room F). Time for preparation of the operating room to enable the surgical specialty to be exchanged, amounting to about an hour, is also not being considered in the hospital plan (see e.g. Tuesday and Thursday room E). As requested by the hospital director,



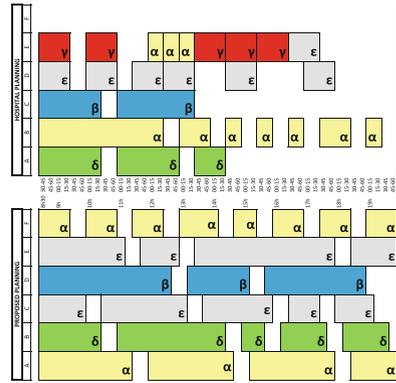
(a) Monday - 12 February 2007



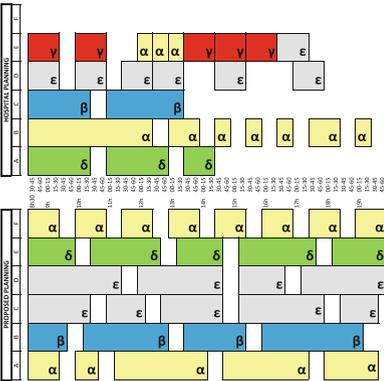
(b) Tuesday - 13 February 2007



(c) Wednesday - 14 February 2007



(d) Thursday - 15 February 2007



(e) Friday - 16 February 2007



Fig. 1 Hospital plan and proposed plan

Table 7 Week balance: production indicators

	Time periods booked/used			Cause			Surg.
	Regular time	Over-time	Total	Different duration (%)	Surgery canceled (%)	Surgery added (%)	
Hospital plan	518	10	528				131
Hospital record	590	16	606	28.9	33.2	37.9	127
IP plan	1,039	0	1,039				162
IP simulation	920	52	972	71.6	28.4	–	158
Proposed plan	1,088	0	1,088				161
Prop. plan simulation	991	42	1,033	34.5	65.5	–	154

this exchange is not allowed in the plan we proposed. Figure 1 also shows many idle periods in the hospital plan indicating an inefficient use of the surgical suite.

In order to evaluate the approach suggested in this paper, it is also interesting to understand how the proposed plan can behave in “reality”. With all the data collected, the proposed plan viability can be analyzed by taking into account the difference between the duration of real surgery and the anticipated duration used for planning and its impact on the plan’s implementation. So, the real duration of the scheduled surgeries was used for a proposed plan simulation. Surgeries start at the scheduled time unless previous surgeries took more time than expected. Starting surgeries in an overtime period was not allowed in this simulation. Should this occur, surgeries were canceled and would therefore not be held, with the exception of surgeries classified as urgency deferred priority level: as they have to be performed on the first planning day in order to respect the term, an overtime start was allowed. Another possibility requiring that a surgery be delayed occurs when the respective surgeon was performing another delayed surgery in another operating room at the time it was expected to begin.

Founded on the same instance under analysis, Tables 7 and 8 provide a balance of the week addressed on the basis of some indicators. As such, the analysis was performed on the week plan and the hospital information record (*Hospital plan* and *Hospital record*), besides the week plan and record simulation for both the solution obtained from the model developed using Cplex (without using improvement heuristic—*IP plan* and *IP simulation*), and the proposed solution (the solution obtained from the model with onwards application of the improvement heuristic—*Proposed plan* and *Prop. plan simulation*).

Table 7 displays the number of time periods booked and used (meaning, respectively, the number of occupied time periods on the plan and on the record/simulation) in regular time and overtime, as well as the weight of each of the contributors to the difference verified in the number of time periods between the plan and the record/simulation. These contributors constitute the differences between the real duration and the expected duration, duration of canceled surgeries and of added surgeries in the case of hospital information. As such, the three columns in *Cause* reveal a distribution of the

Table 8 Week balance: general surgical suite performance indicators

	Surgeons with no surgeries scheduled (%)	Operating time periods per surgeon			Regular time occup. rate (%)	
		Min	Mean	Max	Without cl. time	With cl. time
Hospital plan	34.4	2	12.9	48	37.54	51.38
Hospital record	36.1	1	15.2	54	42.75	52.97
IP plan	34.4	3	26.0	97	75.29	95.00
IP simulation	34.4	3	24.3	102	66.67	86.09
Proposed plan	34.4	3	27.2	98	78.84	99.35
Prop. plan simulation	36.1	3	26.5	97	71.81	91.09

percentage which justifies the difference acknowledged in the surgical record (whether simulated or real) regarding the total number of time periods booked and used.

It is easy to verify that the plan resulting from the approach proposed in this paper allows a better use of the hospital surgical suite, planning more than twice the regular time periods compared with hospital plan, thus contributing for the increase of planned efficiency of the service (see column 2 of Table 7).

In the hospital information, Table 7 indicates that the number of time periods used in hospital record exceeds the number of time periods planned (column 4 for the hospital information). This is accounted for not only by the possibility of adding surgeries during the week, but also by underestimation of surgery durations anticipated in hospital planning. Indeed, among the three cases compared, the hospital plan/record is the only one with a positive balance in the total difference between the real and expected duration of the surgeries performed, i.e. the anticipated duration is below the respective real duration in the weekly balance. Both in the approach excluding the use of the improvement heuristic, and in the proposed approach, the number of time periods in simulation was lower than in the plan. This was due not only to canceled surgeries, but also to the negative balance of the total difference between the real and expected duration of the surgeries performed. This suggests an overestimation of the surgery duration in this planning approach. Note that the instance analyzed used the mean value for the same type of surgical procedure as the duration anticipated for each surgery, and the median is lower than the mean. The use of the improvement heuristic enables one to increase the number of time periods booked and used, and complies with the problem objective function, although in this case the number of surgeries performed is slightly lower than in the simulated solution obtained without using the improvement heuristic.

The number of surgeries performed was always less than the number of surgeries planned (see column 8 of Table 7). The greatest difference verified in the case of the proposed approach causes the percentage of the number of time periods corresponding to canceled surgeries to be higher than in the approach where the improvement heuristic is not used (column 6 of Table 7). Note that in the case of the proposed plan, the main reason for the difference found in the number of booked and used time peri-

ods corresponds to the time of seven canceled surgeries, which constitute the biggest percentage in relation to the time periods associated with over prediction of surgeries duration in the proposed plan.

Table 8 provides generally used indicators for the surgical suite performance. The table shows the percentage of surgeons who do not have surgeries scheduled throughout the week (column 2) and, for the remainder, the minimum and maximum number of time periods planned/performed (columns 3 and 5, respectively) and the respective mean value (column 4). Columns 6 and 7 display the occupancy rate in regular time (without and with the cleaning time). It is possible to verify that the percentage of surgeons with no surgeries planned during the week is almost the same in the case of the three comparisons presented (see column 2). Where the surgeons with a surgical service planned during the week are concerned, the table indicates that the proposed approach significantly increases both the average and the maximum weekly operating time periods (see columns 4 and 5, respectively). From the values presented in column 4 (in the lines corresponding to hospital plan and proposed plan), it is possible to conclude that mean operating time per surgeon in the hospital solution is about 39 min on average per day and in the proposed approach it increases to 1 h 22 min on average per day.

8 Conclusions

This paper jointly considers *advance scheduling* and *allocation scheduling*, and thus assigns elective surgeries to an operating room, a day and a specific period of time, on a weekly planning horizon, in order to maximize the use of the surgical suite. In the literature, no other work was found involving the same problem specificities and approach and hence, the methodologies and results are not comparable.

The approach allowed us to obtain an operating plan for all the weeks under study. The operating plans are feasible and meet the requirements imposed by the hospital in question. Furthermore, this work includes more conditions, which, in practice have not been considered to date. As shown, the approach enables the hospital surgical suite to be more efficiently used. Moreover, the methodologies contribute to a reduction in the waiting list for surgery.

The analysis also shows that the proposed plan simulation makes use of more periods in overtime hours. However, this can easily be overcome by using accurate estimates for surgery duration. This is therefore not the aim of the approach. Moreover, the surgeries on the waiting list which are not scheduled for a certain period of time are transferred to other hospitals.

Thus, the approach proposed in this work provides an effective and innovative response to the current requirements in this specific area of health care, by proposing an alternative method in planning elective surgeries in a real case.

The scope of this case study calls for further research. A relevant question arises as to whether the increase in the hospital's production, which motivated our approach, will imply resource considerations which have, until now, not limited the planning of the surgery suite's current activity. Namely the impact on the capacity of recov-

ery rooms to receive the result of the surgical suite's increase in efficiency will be analyzed.

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