NMAI057 – Linear algebra 1

Tutorial 4

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Problem 1. Compute the following expressions:

(a) $2A$
(b) $A+B$
(c) $A-B$
(d) C^T
(e) Cv
(f) AB
(g) BC

for

$$A=\begin{pmatrix}1 & 2\\ 2 & -1\end{pmatrix},\ B=\begin{pmatrix}-1 & -1\\ 0 & 3\end{pmatrix},\ C=\begin{pmatrix}3 & 0 & 1\\ 2 & -2 & 0\end{pmatrix},\ v=\begin{pmatrix}1\\ 2\\ 3\end{pmatrix}.$$

Problem 2. Prove or disprove the following:

(a) For all matrices $A\in\mathbb{R}^{m\times n}$, $A+A=2A$.
(b) For all square matrices $A\in\mathbb{R}^{m\times m}$, $A=A^T$.

Problem 3. Compute $(-1)A+2BC$ for matrices

$$A=\begin{pmatrix}3 & 1\\ 4 & 1\end{pmatrix}, B=\begin{pmatrix}5 & 9\\ 2 & 7\end{pmatrix}, C=\begin{pmatrix}1 & -1\\ 2 & 2\end{pmatrix}.$$

Problem 4. Solve the systems of linear equations $(A \mid b)$ and $(B \mid c)$ given by

$A=\begin{pmatrix}2 & 3\\ 1 & 2\end{pmatrix}$ a $b=\begin{pmatrix}2\\ 1\end{pmatrix}$, and

$B=\begin{pmatrix}1 & 1 & 2\\ 1 & 1 & 1\\ 2 & 2 & 0\end{pmatrix}$ a $c=\begin{pmatrix}3\\ 1\\ -2\end{pmatrix}$.

Verify the correctness of your result x (resp. y) by computing the matrix product $Ax=b$ (resp. $By=c$).

Problem 5. Express the elementary row operations as matrix products, i.e., for each elementary row operation, find a matrix $E\in\mathbb{R}^{m\times m}$ such that EA is the result of applying the operation to matrix A for all matrices $A\in\mathbb{R}^{m\times n}$.

Problem 6. Prove or disprove whether for all matrices A, B, C and $\mathbf{0}$ of the same order and real numbers $\alpha,\beta\in\mathbb{R}$, it holds that:

(a) $A + (B + C) = (A + B) + C$
(b) $A + B = B + A$
(c) $A + \mathbf{0} = A$
(d) $\alpha(\beta A) = (\alpha\beta)A$
(e) $\alpha(\beta A) = \beta(\alpha A)$
(f) $A + (-1)A = \mathbf{0}$
(g) $1A = A$

Problem 7. Give a non-symmetric matrix A and a symmetric matrix B such that their product does not commute, i.e., such that $AB \neq BA$.

Is the product of symmetric matrices commutative?

Problem 8. Prove or disprove the following statements:

(a) For all $A, B \in \mathbb{R}^{n\times n}$, if A is symmetric and commutes with B then A commutes also with B^T.
(b) For all $A, B \in \mathbb{R}^{n\times n}$, if A commutes with B then A commutes with B^T.