NMAI057 – Linear algebra 1

Tutorial 3

Date: October 13, 2021 TA: Denys Bulavka

Problem 1. Complete the chain of elementary transformations such that the matrices contain only integers: $\begin{pmatrix} 6 & 2 & 4 & 2 \\ 7 & -1 & 2 & 2 \\ -8 & 4 & 0 & -2 \end{pmatrix} \sim \begin{pmatrix} 3 & . & . & . \\ 7 & . & . & . \\ -8 & . & . & . \end{pmatrix} \sim \begin{pmatrix} 3 & . & . & . \\ 7 & . & . & . \\ 4 & . & . & . \end{pmatrix} \sim \begin{pmatrix} 3 & . & . & . \\ 4 & . & . & . \\ 4 & . & . & . \end{pmatrix} \sim \begin{pmatrix} 3 & . & . & . \\ 4 & . & . & . \\ 0 & . & . & . \end{pmatrix} \sim \begin{pmatrix} 3 & . & . & . \\ 1 & . & . & . \\ 0 & . & . & . \end{pmatrix} \sim \begin{pmatrix} 0 & . & . & . \\ 1 & . & . & . \\ 0 & . & . & . \end{pmatrix} \sim \begin{pmatrix} 1 & . & . & . \\ 0 & . & . & . \\ 0 & . & . & . \end{pmatrix}$.

Problem 2. By use of elementary transformations, reduce the following matrix to the echelon form and find at least one non-trivial solution to the system $\mathbf{A}x = 0$, if it exists.

$$a)\begin{pmatrix} 2 & 3 & 4 & 5 \\ 3 & 0 & 2 & -3 \\ 7 & 6 & 10 & 7 \end{pmatrix},\ b)\begin{pmatrix} 2 & -3 & 13 & 18 \\ 6 & -9 & 7 & 10 \\ 2 & -3 & -3 & -4 \end{pmatrix},\ c)\begin{pmatrix} 1 & 2 & -1 & 1 \\ 2 & -1 & 4 & 10 \\ 1 & 0 & 3 & -5 \\ 2 & 5 & 2 & 2 \end{pmatrix},\ d)\begin{pmatrix} 1 & -1 & 1 & 2 \\ 1 & 8 & 7 & -7 \\ 1 & 2 & 3 & -1 \\ 1 & 5 & 5 & -4 \end{pmatrix}.$$

Problem 3. Solve the following system of linear equations and verify the solution:

1. $$\begin{aligned} 3x_1 + 2x_2 + x_3 &= 5 \\ 2x_1 + 3x_2 + x_3 &= 1 \\ 2x_1 + x_2 + 3x_3 &= 11 \\ 5x_1 + 5x_2 + 2x_3 &= 6 \end{aligned}$$

2. $$\begin{aligned} -x_1 + x_2 + 3x_3 &= -2 \\ 2x_1 - x_2 - 6x_3 + x_4 &= 2 \\ -x_1 + x_2 + 4x_3 &= -2 \\ x_2 + 2x_3 + 2x_4 &= 0 \end{aligned}$$

3. $$\begin{aligned} -x_1 - x_2 + 2x_3 &= 1 \\ 2x_1 + 3x_2 - 4x_3 + 3x_4 &= 1 \\ x_1 + x_2 - x_3 + 2x_4 &= 2 \\ - x_2 + 2x_3 + 2x_4 &= 5 \end{aligned}$$

4. $$\begin{aligned} 2x_1 + 2x_2 + 8x_3 - 3x_4 + 9x_5 &= 2 \\ 2x_1 + 2x_2 + 4x_3 - x_4 + 3x_5 &= 2 \\ x_1 + x_2 + 3x_3 - 2x_4 + 3x_5 &= 1 \\ 3x_1 + 3x_2 + 5x_3 - 2x_4 + 3x_5 &= 1 \end{aligned}$$

Problem 4. Show that for any two solutions $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$ and $\mathbf{x}' = (x_1', x_2', \dots, x_n')^T$ of a given system we get also a solution $\alpha\mathbf{x} + (1-\alpha)\mathbf{x}' = (\alpha x_1 + (1-\alpha)x_1', \alpha x_2 + (1-\alpha)x_2', \dots, \alpha x_n + (1-\alpha)x_n')^T$ for an arbitrary real α.

Generalize your argument also for more solutions $\mathbf{x}, \mathbf{x}', \dots, \mathbf{x}^{(k)}$.

Problem 5. Interpolate a circle through points $A = (2, 1)$, $B = (4, 3)$ and $C = (0, 7)$.