2nd exam NMAI059 Probability and Statistics 1 – June 22, 2021

Write the number of the problem and your surname on each paper.

Do not write more than one problem on the same sheet of paper!

You have **150 minutes.**

No calculators, cell phones, ... are allowed during the exam. (Please silence your cell phones in advance.)

If the result contains expressions that are difficult to evaluate without a calculator, don't evaluate them (137×173 is as good, if not better, than 23701, you may leave $\Phi^{-1}(0.975)$ unevaluated as well).

Explain in detail all calculations.

You may use one (handwritten) A4 cheat sheet.

1. (10 points) (Figures at the end.) (a) Decide, which of the figures show probability density function of some random variable. For Figures 5 and 6 choose an appropriate value of b, c to make the function a pdf, if possible. Do the next two parts only for those figures, that show a pdf.

(b) Estimate expectation of the distribution.

(c) Order the distributions by their variance. Do this part only for Figures 3–6, more precisely for those among 3–6 that show a pdf.

2. (10 points) There are one hundred balls in the box with numbers 1, 2, ..., 100. We pull out three of them (we do not return them back).

(a) What is the probability that they all have a number at most equal to 40?

(b) What is the expected value of the sum of the numbers on the drawn balls?

(c) What is the expected value of the number of balls drawn whose number is at most equal to 40?

3. (10 points) Oral exam takes time that follows exponential distribution with expected value 20 minutes. Two students are scheduled: one for 10:00, another for 10:20. If the first student examination takes longer than 20 minutes, the second one starts just after the first one finishes. Otherwise, the second one starts at 10:20 sharp.

What is the expected time when the second student finishes?

(The problems continue on the other side.)

4. (10 points) (a) Define independent events.
(b) Define conditional expectation of a discrete random variable.

5. (10 points) Explain hypothesis testing. (In particular explain what Type I and Type II errors are.)

6. (10 points) State and prove the theorem about convolution formula for the sum of independent random variables, the case of discrete random variables.

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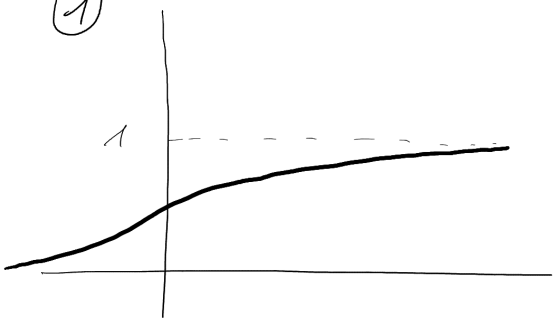

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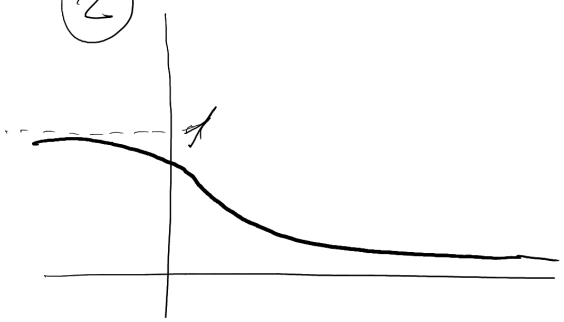

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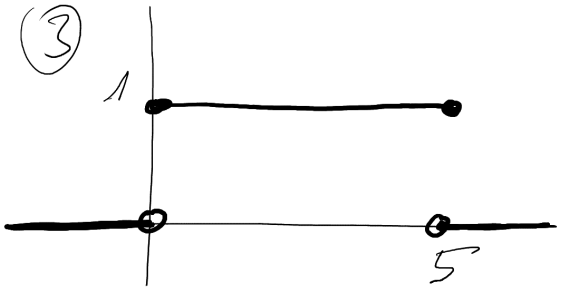

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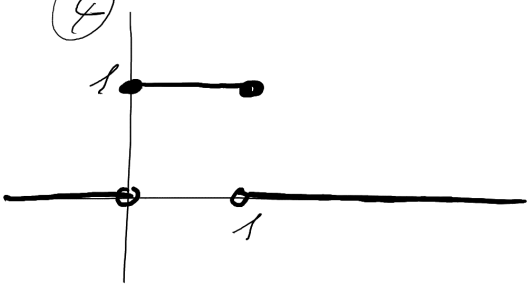

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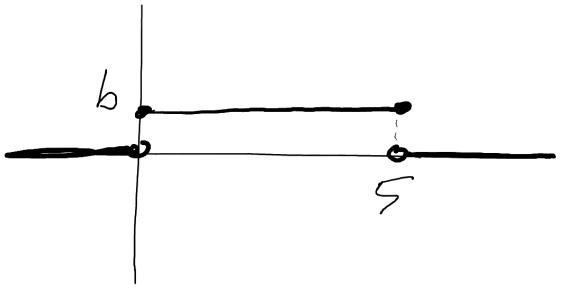

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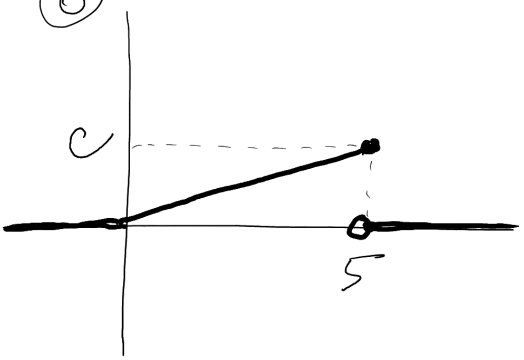

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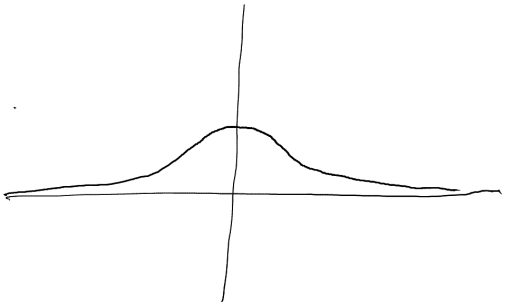

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