# 1st exam NMAI059 Probability and Statistics 1 – June 16, 2021

Write the number of the problem and your surname on each paper.

Do not write more than one problem on the same sheet of paper!

You have **150 minutes.**

No calculators, cell phones, ... are allowed during the exam. (Please silence your cell phones in advance.)

If the result contains expressions that are difficult to evaluate without a calculator, don't evaluate them ($137 \times 173$ is as good, if not better, than 23701, you may leave $\Phi^{-1}(0.975)$ unevaluated as well).

**Explain in detail** all calculations.

You may use one (handwritten) A4 cheat sheet.

**1.** (10 points) (Figures at the end.) (a) Decide, which of the figures describe a cdf of some random variable. Do the next two parts just for those, that show a cdf.

(b) Estimate the expectation.

(c) Order the variables by their variance.

**2.** (10 points) (a) Two players, Adam and Beatrix, roll the dice repeatedly, in order ABABAB... What is the probability that Adam will roll a six first?

(b) Cecil joins the game, they now roll in the order ABCABCABC... The probability that Adam will get the six first, then Beatrix, and only after that Cecil is 216/1001. Explain. (If Adam gets a six more than once, and only then Beatrix, that's fine too, we're just concerned with the order of the first time they roll a six.)

**3.** (10 points) Pareto distribution with parameter $\alpha > 1$ has pdf $f(x) = \frac{\alpha}{x^{\alpha+1}}$ for $x \in [1, \infty)$ (and zero elsewhere).

(a) Verify that it is a pdf.

(b) We sample values 5, 2, 3 from this distribution. Derive a point estimate $\hat{\alpha}$ using the maximal likelihood method.

(c) Let $X$ follow the Pareto distribution, that is let $f_X = f$. Calculate $\mathbb{E}(X)$.

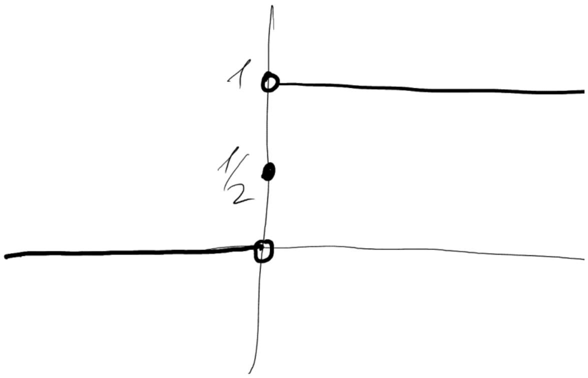
(d) Find the point estimate $\hat{\alpha}$ by the moment method.

(The problems continue on the other side.)

**4.** (10 points) (a) Define the notion joint cumulative distribution function.
(b) Describe, how to compute the empirical cumulative distribution function.

**5.** (10 points) State the Central limit theorem. Explain what is it good for.

**6.** (10 points) State and prove the theorem about expectation of a sum of random variables. (Only prove it for the case of discrete random variables.)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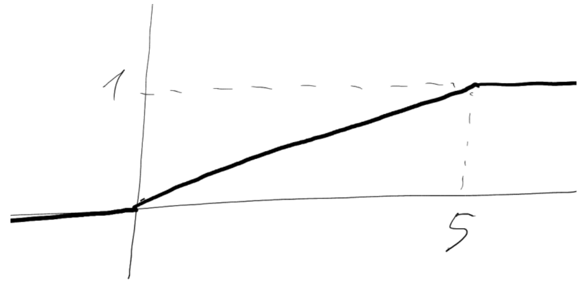
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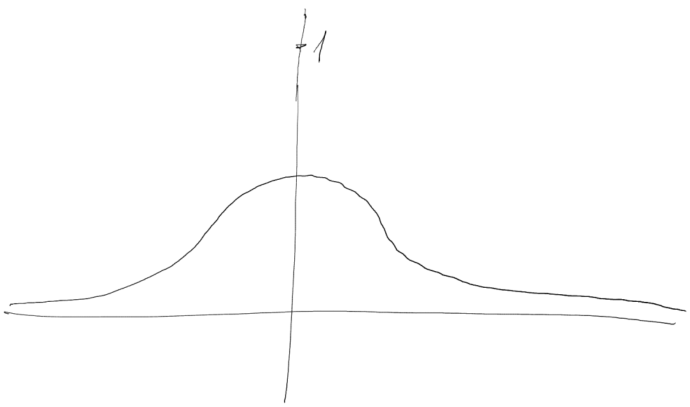
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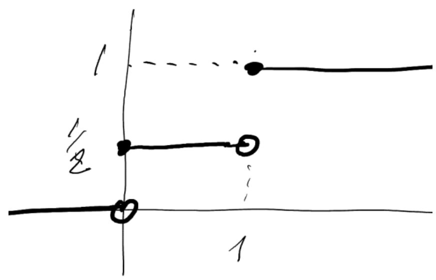
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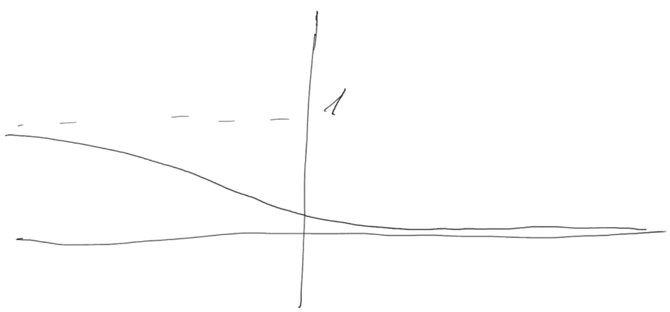
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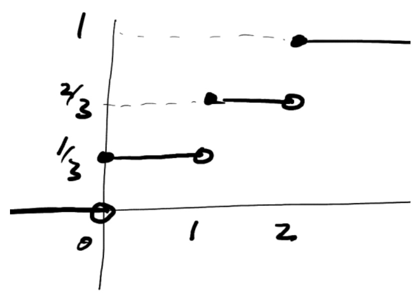
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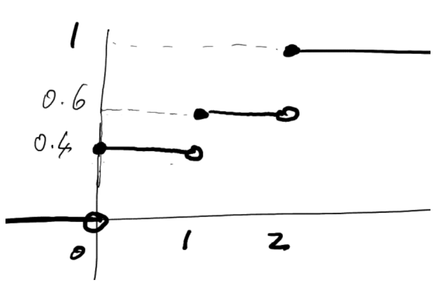
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