Concentration risk

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1 Concentration risk definition

Concentration risks in credit portfolios arise from unequal distribution of loans to single borrowers (name concentration) or different industry or regional sectors (sector or country concentration).

The credit risk in a portfolio can be increased also by **default contagion** - when the default of one borrower can cause the default of a dependent borrower.

The standard formula of Solvency 2 recognizes **market risk concentration** in the framework of the market risks module.

SCR according to the standard formula $\longrightarrow$ BSCR $\longrightarrow$ market risk module $\longrightarrow$ submodules:

- interest risk
- equity risk
- property risk (real estate)
- credit spread risk
- currency risk
- market risk concentration: additional risks (to an insurance or reinsurance undertaking) stemmimg either from the lack of diversification in the assets portfolio or from a large exposure to default risk by a single issuer of securities or a group of related issuers. It considers all risk exposures with a loss potential which is large enough to threaten the solvency or the financial position of an insurance or reinsurance undertaking.

2 Concentration risk in credit portfolio

We describe some methods of measurement and modelling of concentration risk in a framework of a portfolio of risky loans. We use the following notation:

We consider losses in a fixed time horizon (one year) T.

EAD (exposure at default) - the portion of the exposure to an obligor which is lost in case of default.

LGD (loss given default) - the extent of the loss incurred in the event of default.

For quantification of concentration risk we consider a portfolio of N loans indexed by $n = 1, \ldots, N$.

We assume that the exposures have been aggregated so that there is a unique obligor for each position. We denote by EAD_n the exposure at default of obligor n. The share of obligor n on the total exposure is given by

$$s_n = \frac{\mathrm{EAD}_n}{\sum_{k=1}^{N} \mathrm{EAD}_k}. \tag{1}$$

Random variable D_n is the indicator of default of obligor n in $[0, T]$. The distribution of D_n is given by the default probability of obligor n, PD_n:

$$\mathrm{P}\left(D_n = 1\right) = \mathrm{PD}_n$$

$$\mathrm{P}\left(D_n = 0\right) = 1 - \mathrm{PD}_n\,.$$

The **portfolio loss** is the random variable

$$L = \sum_{n=1}^{N} \mathrm{EAD}_n\ \mathrm{LGD}_n\ D_n. \tag{2}$$

We can also express the loss in percentage of total exposure as the **portfolio**

loss ratio

$$L = \sum_{n=1}^{N} D_n \ \mathrm{LGD}_n \ s_n. \tag{3}$$

3 Simple measures of concentration

Concentration ratio

Consider a portfolio with exposure shares $s_1 \geq s_2 \geq \cdots \geq s_N$ and such that

$$\sum_{n=1}^{N} s_n = 1.$$

Then for given $1 \leq k \leq N$ the **concentration ratio** is given by

$$CR_k = \sum_{i=1}^{k} s_i. \tag{4}$$

(It is the ratio of the sum of the k biggest exposures to the total sum of exposures in the portfolio.)

We can construct a **concentration curve** consisting of points (k, CR_k), $k = 1, \ldots, N$. If the concentration curve of a portfolio A lies entirely below the concentration curve of a portfolio B, then we say that B is more concentrated than A (frequently, concentration curves will intersect).

Lorenz Curve

The (empirical) Lorenz curve for an ordered data set $A_1 \leq A_2 \leq \cdots \leq A_N$ is defined for $q \in (0, 1)$ as the piecewise linear interpolation with breakpoints

$L(0) = 0$ and

$$L(n/N) = \frac{\sum_{i=1}^{n} A_i}{\sum_{i=1}^{N} A_i}, \; n = 1, \dots, N.$$

For a distribution function F with finite mean, the (theoretical) Lorenz curve is defined by

$$L(q) = \frac{\int_0^q F^{-1}(t) \, \mathrm{d}\, t}{\int_{-\infty}^{+\infty} t \; \mathrm{d}\, F(t)}, \; q \in (0, 1). \tag{5}$$

If F has a density f we obtain by substitution

$$L(F(x)) = \frac{\int_0^{F(x)} F^{-1}(t) \, \mathrm{d}\, t}{\int_{-\infty}^{+\infty} t \, f(t) \, \mathrm{d}\, t} = \frac{\int_{-\infty}^{x} t \, f(t) \, \mathrm{d}\, t}{\int_{-\infty}^{+\infty} t \, f(t) \, \mathrm{d}\, t}. \tag{6}$$

$L(q)$ assigns to every percentage q of the total loan number the cumulative percentage of loan sizes. The case $L(q) = q$ represents the line of perfect equality (it corresponds to the case when $A_1 = \cdots = A_N$ in the empirical setting or to the case with d.f. F concentrated in one point). The perfect inequality is the limit case of a Dirac function with weight 1 at $x = 1$ (the case with $N = 1$).

Remark. For uniform distribution with the d.f. $F(x) = x$, $x \in (0, 1)$, we have

$$L(F(x)) = L(x) = x^2.$$

The comparison of concentration of two portfolios is possible when the Lorenz curves do not intersect (the lower curve means higher concentration).

Lorenz curve measures inequality rather than concentration (two loans of the same size will be considered as well diversified, 100 loans of different sizes as more concentrated).

Gini coefficient Gini coefficient is defined as the ratio of the area between the line of perfect equality and the Lorenz curve (A) to the area under the line of perfect equality (A+B),

$$G = \frac{A}{A+B}.$$

From $A + B = 1/2$ it follows $G = 2\,A = 1 - 2\,B$, i.e.

$$G = 1 - 2\int_0^1 L(q)\,\mathrm{d}\,q. \tag{7}$$

The empirical version for a portfolio of N loans with shares $s_1, \ldots, s_N$ is

$$G = \frac{\sum_{n=1}^{N}(2\,n-1)\,s_n}{N} - 1. \tag{8}$$

Gini coefficient measures the deviation from equality - small values of G correspond to well diversified portfolios.

Herfindahl-Hirschman Index

$$HHI = \sum_{n=1}^{N} s_n^2. \tag{9}$$

$HHI = 0$ corresponds to the fully granular case (each obligor has an infinitesimal share).

$HHI = 1$ corresponds to the case with only one obligor.

The higher the HHI, the more concentrated portfolio.

References: E.Lutkebohmert, Concentration Risk in Credit Portfolios. Springer, 2009.