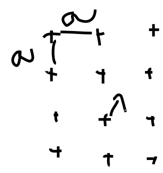


3. Consider independent random variables U_1 a U_2 with uniform distribution on the interval $[0, a]$, $a > 0$, and the point process Φ in $\mathbb{R}^2$defined as

$$\Phi = \sum_{m,n \in \mathbb{Z}} \delta_{(U_1 + ma, U_2 + na)}.$$

Determine the Palm distribution and the reduced second-order moment measure of the process. Express its contact distribution function and the nearest-neighbour distribution function.

$w_1 = x$

Palm distribution P_x : degenerate ... $\Phi = \varphi_1^x = \sum_{m,n \in \mathbb{Z}} \delta(x_1 + ma, \dots$

$P_x^!$: $\Phi = \varphi_2^x = \sum_{(m,n) \in \mathbb{Z}^2 \setminus \{(0,0)\}} \delta(x_1 + ma, x_2 + na) = \varphi_1^x - \delta_{(x_1, x_2)}$

Reduced 2nd order moment measure:

$\lambda \mathcal{K}(B) = \mathbb{E}_0^! \Phi(B)$, $B \in \mathcal{B}(\mathbb{R}^2)$

stationary point processes (Remark 37)

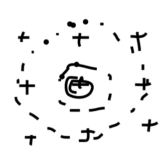
$= \varphi_2^0(B) = \sum_{(m,n) \in \mathbb{Z}^2 \setminus \{(0,0)\}} \mathbb{1}((ma, na) \in B)$

$\lambda = \frac{1}{a^2}$ ... from earlier exercise $\rightarrow$ $\mathcal{K}(B) = a^2 \cdot \sum_{(m,n) \in \mathbb{Z}} \dots$

Nearest neighbour distance distribution function:

$r > 0$ $G(r) = P_0^! (\{ \nu \in \mathcal{N} : \nu(b(o,r)) > 0 \})$

$G(r) = \mathbb{1}(a \leq r)$



$U_1 = 0$, $U_2 = 0$

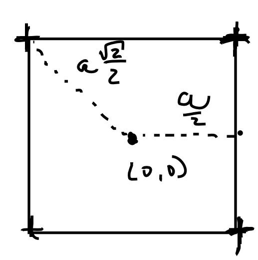
„event-to-event"
nearest distance d.f."

$F(r) = \mathbb{P}(\Phi(\underbrace{b(o,r)}_{\text{ball}}) > 0) = 1 - \mathbb{P}(\Phi(b(o,r)) = 0)$ „point-to-event"
locatation-to-point

ball => „spherical contact dist. f."

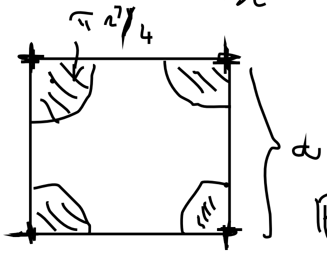
$F(r) = 1$, $r \geq a \frac{\sqrt{2}}{2}$

$= 0$, $r \leq 0$



$r \in \left(\frac{a}{2}, a\frac{\sqrt{2}}{2}\right)$ ... more than 1 point can be in $b(\sigma, r)$

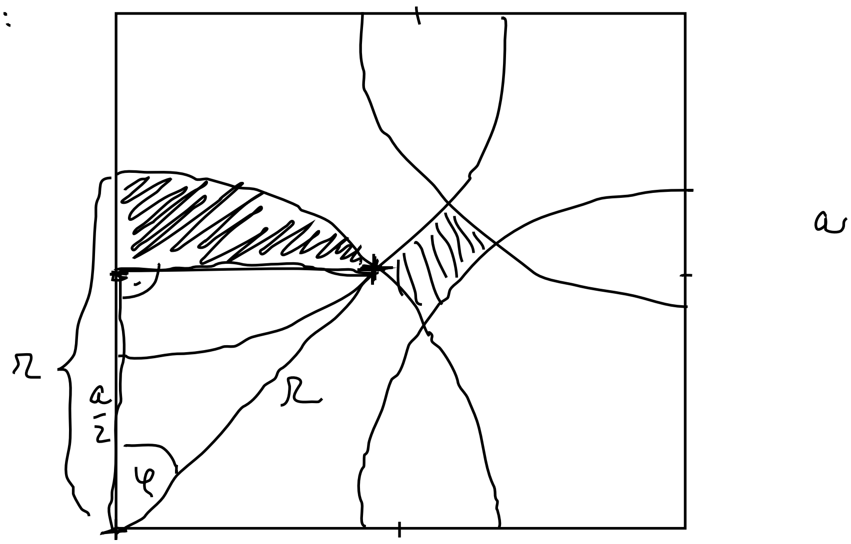
$\rightarrow r \in \left(0, \frac{a}{2}\right)$ ... at most 1 point can be in $b(\sigma, r)$



„randomly placing center of the disc into the fixed grid"

$$P(\Phi(b(\sigma, r)) > 0) = \frac{\pi r^2}{a^2}$$

$r \in \left(\frac{a}{2}, a\frac{\sqrt{2}}{2}\right)$:



$$P(\Phi(b(\sigma, r)) > 0) =$$
$$= \frac{1}{a^2}\left[\pi r^2 - 8 \cdot |\square|\right]$$

$$|\triangleright| = \pi r^2 \cdot \frac{\varphi}{2\pi}$$
$$|\triangleright| = \frac{1}{2} \cdot \frac{a}{2}\sqrt{r^2 - \frac{a^2}{4}}$$

$$\Rightarrow |\square| = \pi r^2 \cdot \frac{\varphi}{2\pi} - \frac{a}{4}\sqrt{r^2 - \frac{a^2}{4}}$$

$$\cos\varphi = \left(\frac{a}{2}\right)/r = \frac{a}{2r}, \quad \varphi = \arccos\left(\right.$$