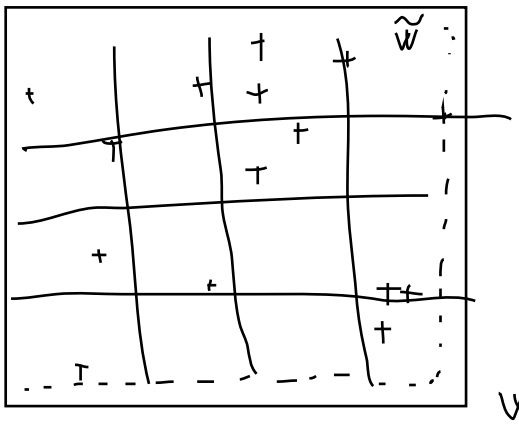
5. Consider the point pattern $\{x_1, \ldots, x_n\}$ observed in a compact observation window $W \subset \mathbb{R}^2$ and assume it is a realization of a stationary point process. How to estimate its intensity? How to estimate the values $F(r)$ and $G(r), r > 0$?



$\lambda(W) \equiv \lambda =$ „expected # points in a set of unit area"

$\Lambda(B) = \mathbb{E}\Phi(B) = \lambda \cdot |B|$

disjoint

$W_1, \ldots, W_m \qquad |W_i| = 1$

$\Phi(W_1), \ldots, \Phi(W_m)$

$$\frac{1}{m}\sum_{i=1}^{m}\Phi(W_i) = \frac{1}{m}\Phi\left(\bigcup_{i=1}^{m} W_i\right) =$$

$$\frac{1}{|W|}\cdot\Phi(W) = \hat{\lambda} \quad \Rightarrow \quad = \frac{1}{\left|\bigcup_{i=1}^{m} W_i\right|}\Phi\left(\underbrace{\bigcup_{i=1}^{m} W_i}_{\widetilde{W}}\right) = \frac{1}{|\widetilde{W}|}\cdot\Phi($$

$\lambda =$ „expected # of points per unit area"

$\mathbb{E}\hat{\lambda} = \frac{1}{|W|}\cdot\mathbb{E}\Phi(W) = \frac{1}{|W|}\Lambda(W) = \frac{1}{|W|}\lambda|W| = \lambda$ $\Rightarrow$ unbiased estimator

$\left[\ \widehat{\Lambda(B)} = \Phi(B)\ \right]$