3. Consider two independent Poisson point processes Φ_1 and Φ_2 with the intensity measures Λ_1 and Λ_2. Show that $\Phi = \Phi_1 + \Phi_2$ is a Poisson process and determine its intensity measure.

1) $\Phi(B) \sim Po(\Lambda(B))$ $\qquad \Lambda(B) = \Lambda_1(B) + \Lambda_2(B)$

$B \in \mathcal{B}_0$ $\qquad \Lambda = \Lambda_1 + \Lambda_2$

$$\Phi(B) = \underbrace{\underset{\sim Po(\Lambda_1(B))}{\Phi_1(B)} + \underset{\sim Po(\Lambda_2(B))}{\Phi_2(B)}}_{\text{indep.}} \sim Po(\Lambda_1(B) +$$

2) $B_1, \dots, B_\ell \sim \mathcal{B}_0$, disjoint

$\Phi(B_1), \dots, \Phi(B_\ell)$ indep. ✓

$$\left.\begin{matrix} \boxed{\begin{matrix}\Phi_1(B_1) \\ + \\ \Phi_2(B_1)\end{matrix}} & \begin{matrix}, \dots, \\ \\ , \dots, \end{matrix} & \boxed{\begin{matrix}\Phi_1(B_\ell) \\ + \\ \Phi_2(B_\ell)\end{matrix}} \end{matrix}\right\} \text{all indep.}$$

sums of non-overlapping pairs of indep. r.v.s are independent