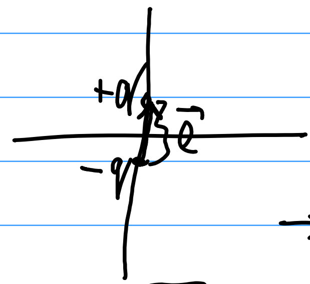
DIPÓL 2. TAYLORŮV ROZVOJ



$$f(x) = \underbrace{f(0)} + f'(0)\,x + \frac{1}{2} f''(0)\,x^2 + \dots$$

$$\rightarrow \quad \varphi = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2+y^2+\underbrace{\left(z-\frac{l}{2}\right)^2}}} - \frac{1}{\sqrt{x^2+y^2+\left(z+\frac{l}{2}\right)^2}} \right)$$

$$\boxed{\begin{aligned} &l \to 0 \\ &q\vec{l} \to \vec{p} \end{aligned}}$$

$$\left(z \mp \frac{l}{2}\right)^2 \doteq z^2 \mp (z)\,l + O(l^2)$$

$$z^2 - lz + \frac{l^2}{4}$$

$$\varphi = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{r^2 - zl}} - \frac{1}{\sqrt{r^2 + zl}} \right)$$

$$\left(r^2 \mp zl\right)^{-\frac{1}{2}} \doteq \frac{1}{r} \pm \frac{1}{2}\left(r^2\right)^{-\frac{3}{2}} \cdot z \cdot l + O(l^2)$$

$$= \frac{1}{r} \pm \frac{1}{2}\,\frac{zl}{r^3}$$

$$\varphi = \frac{q}{4\pi\varepsilon_0}\left(\cancel{\frac{1}{r}} + \frac{1}{2}\,\frac{zl}{r^3} - \cancel{\frac{1}{r}} + \frac{1}{2}\,\frac{zl}{r^3} \right)$$

$$= \frac{q}{4\pi\varepsilon_0}\,\frac{zl}{r^3} = \frac{1}{4\pi\varepsilon_0}\,\frac{\vec{r}\cdot\vec{p}}{r^3}$$

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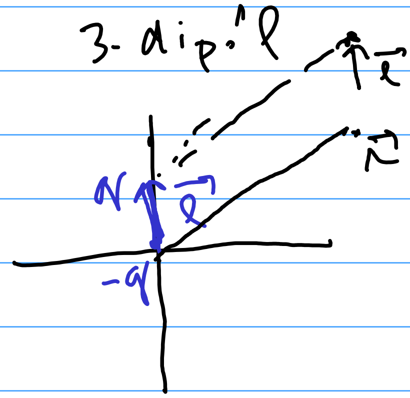
3. dipól



$$\varphi^-(\vec{r}) = -\varphi^+\left(\vec{r} + \vec{l}\right) = \frac{-q}{4\pi\varepsilon_0 r}$$

$$\varphi^+(\vec{r}) = \nabla\varphi^+(\vec{r}) \cdot \left(-\vec{l}\right) + \varphi^+\left(\vec{r} + \vec{l}\right)$$

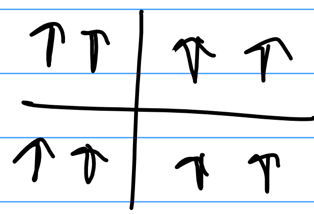
$$= -\frac{q\,\vec{r}}{4\pi\varepsilon_0 r^3} \cdot \left(-\vec{l}\right)$$

$$\varphi^-(\vec r) + \varphi^+(\vec r) = \cancel{-\frac{q}{4\pi\varepsilon_0 r}} + \cancel{\frac{q}{4\pi\varepsilon_0 r}} + \frac{q\vec r}{4\pi\varepsilon_0 r^3}\cdot\vec l = \frac{\vec p\cdot\vec r}{4\pi\varepsilon_0 r^3}$$

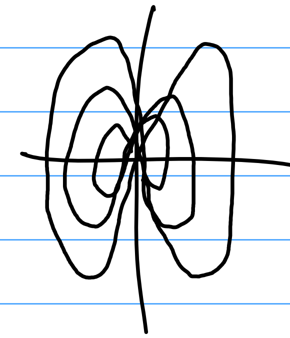
$\boxed{\cancel{1.1.12} + 1.1.13}$

$\vec E = (0, 0, E_0)$

$\vec p = (0, 0, p)$



+



1) $\varphi(\vec r) = 0$ TVAR?

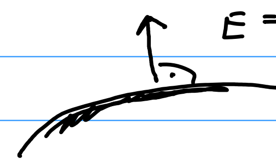
$$\varphi_{dip} = \frac{\vec r\cdot\vec p}{4\pi\varepsilon_0 r^3} \qquad \varphi_H = -E_0\cdot z \quad (!)$$

$$\varphi = \frac{zp}{4\pi\varepsilon_0 r^3} - E_0 z = 0$$

$$E_0 = \frac{p}{4\pi\varepsilon_0 r^3} \to r_0^3 = \frac{p}{4\pi\varepsilon_0 E_0}$$

→ KULOVÁ PLOCHA

2) N=



$E = \frac{\sigma}{\varepsilon_0}$

3) σ NA KULOVÉ SLUPCE?

$\vec E(r_0)$ RADIÁLNÍ?

$$\vec E_D + \vec E_H \qquad E_D = -\nabla\varphi_D = -\frac{1}{4\pi\varepsilon_0}\underbrace{\nabla\frac{\vec p\cdot\vec r}{r^3}}_{\frac{\vec p}{r^3} - \frac{3\vec r}{r^5}\vec p\cdot\vec r}$$

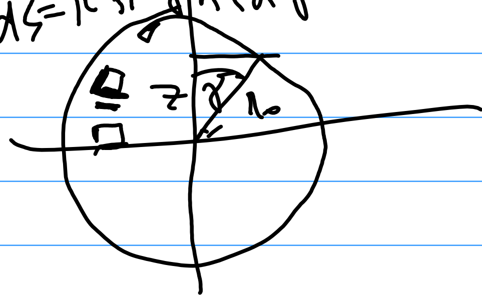
$$\vec E = \frac{1}{4\pi\varepsilon_0}\left(-\frac{(0,0,p)}{r^3} + \frac{3(x,y,z)}{\cancel{r^3}}\frac{pz}{r^2}\right) + (0,0,E_0)$$

$$\vec E(r_0) = \frac{1}{\cancel{4\pi\varepsilon_0}}\left(-\frac{(0,0,\cancel{p})\,\cancel{4\pi\varepsilon_0}E_0}{\cancel{p}} + \frac{3(x,y,z)\,\cancel{4\pi\varepsilon_0}E_0}{\cancel{p}}\frac{\cancel{p}z}{r_0^2}\right.$$

$$+ (0,0,E_0) = \cancel{-(0,0,E_0)} + \frac{3\overbrace{(x,y,z)}^{\vec{r}_0} z E_0}{r_0^2} \cancel{+ (0,0,E_0)}$$

$$|\vec{E}(r_0)| = \frac{3 r_0 z E_0}{r_0^2} = 3\frac{z E_0}{r_0} = 3E_0 \cdot \cos\gamma$$

$dS = r^2 \sin\gamma\, d\varphi\, d\gamma$



$$\sigma = 3E_0 \varepsilon_0 \cos\gamma$$

<u>DÚ:</u> 1) VYKRESLIŤ $\vec{E} = \vec{E}_H + \vec{E}_{DIP}$

$\varphi = \varphi_H + \varphi_{DIP}$

2) CELKOVÝ NÁBOJ NA SLUPKE

$$\int \sigma \cdot dS = \int_0^{2\pi}\int_0^{\pi} \sigma\, d\varphi \cdot r_0^2\, d\gamma \cdot \sin\gamma$$

3) DIPÓLOVÝ MOMENT NA SLUPKE

$$\vec{p} = \int \sigma \vec{r}\, dS$$

$p_x = p_y = 0$ (OVERIŤ)

$$p_z = \int_0^{2\pi}\int_0^{\pi} \sigma \cdot z \cdot r_0^2\, d\varphi\, d\gamma \sin\gamma$$

$\downarrow$

$z = r_0 \cdot \cos\gamma$

$$p_z = \int_0^{2\pi}\int_0^{\pi} 3\varepsilon_0 \varepsilon_0 \cos\gamma \cdot r_0^3 \cos\gamma \cdot \sin\gamma \cdot d\varphi\, d\gamma$$