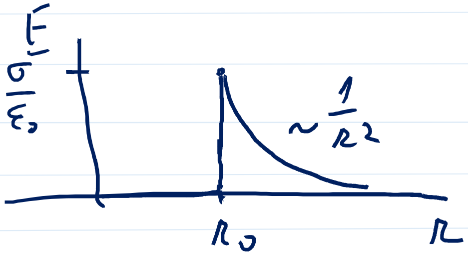
S1.1.19-1.1.20 Určete pomocí Gaussova zákona průběhy Intenzity a potenciálu a pole v okolí následujících homogenně nabitých objektů:

a) kulová slupka o poloměru R_0

ϱ b) koule, DÚ nabitá obj. nábojem hust. ϱ

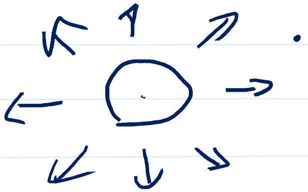
c) válcová plocha a válec DÚ



radiální ze symetrie

a)

Koule

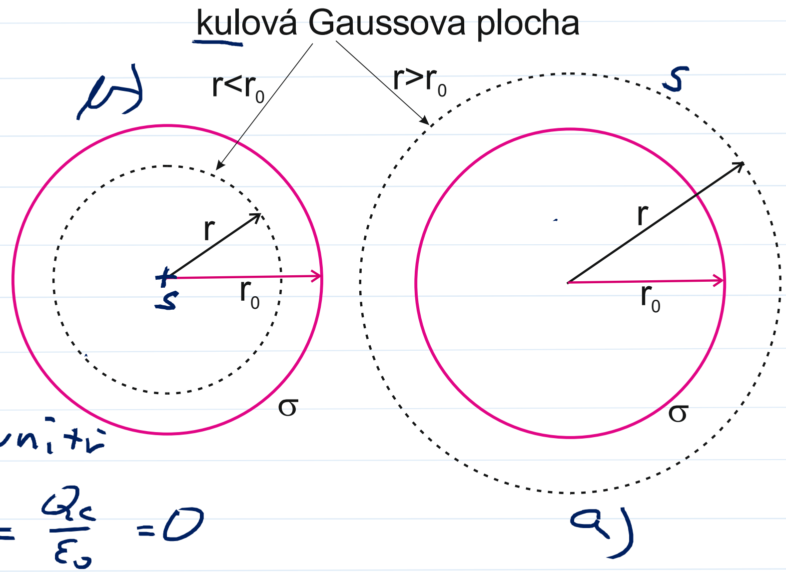


$E = E(R)$ R vzd. od středu

$\vec{E} \perp$ kouli se středem S

$\Downarrow$

$\vec{E} \parallel d\vec{S}$



a)

$R > R_0$ S - kulová plocha o poloměru R

z G. z.

$$\oint_S \vec{E}\, d\vec{S} = \frac{Q_c}{\varepsilon_0}$$ — uzavřený v kouli o pol. r

$\Leftarrow Q_c = \sigma \cdot 4\pi R_0^2$

$$\Phi_S = \int_S E\, dS = E \oint_S dS = E\, 4\pi R^2 = \frac{\sigma\, 4\pi R_0^2}{\varepsilon_0}$$

$$E = \frac{Q_c}{4\pi\varepsilon_0 R^2} \qquad E = \frac{\sigma}{\varepsilon_0} \cdot \frac{R_0^2}{R^2}$$

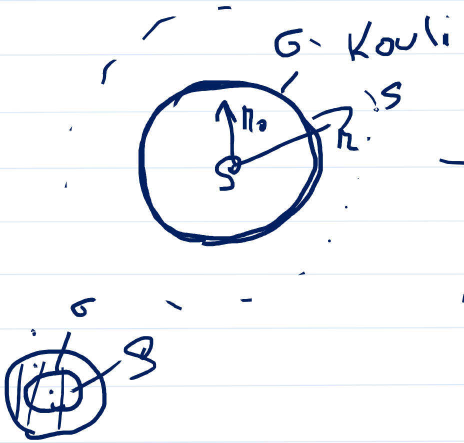
b)

$R < R_0$ - uvnitř tok

$$E\, 4\pi R^2 = \frac{Q_c}{\varepsilon_0} = 0$$

$$\Rightarrow E = 0$$

~~Potencial -rovina~~, drat....



$$\oint_S \vec{E}\cdot d\vec{S} = \frac{Q_c}{\varepsilon_0} \leftarrow Q_c = 4\pi r_0^2\sigma + \frac{4}{3}\pi r_0^3 \rho$$

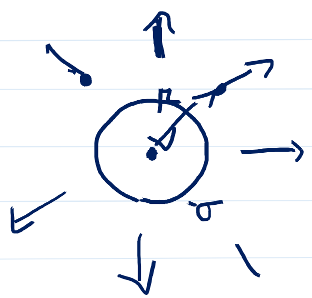
$r > r_0$

$$4\pi r^2 E(r) = \frac{1}{\varepsilon_0}\left(4\pi r_0 \sigma + \frac{4}{3}\pi r_0^3 \rho\right)$$

$r < r_0$

$$4\pi r^2 E(r) = \frac{1}{\varepsilon_0} \frac{4}{3}\pi r^3 \rho$$

analogie s drátem



$\vec{E} = E(r)$

vzd. od osy

G.z.

$\vec{E} \perp d\vec{S}$

$r > r_0$ vně

$$\Phi_S = \underbrace{\oint \vec{E}\,d\vec{S}}_{\text{válec}} = \underbrace{\int E(r)\,dS}_{\text{plášť}} + \underbrace{\int \vec{E}\cdot d\vec{S}}_{\text{podstavy}}^{=0} = E(r)\,2\pi r l$$

$$\varphi = -\frac{\tau}{2\pi\varepsilon_0}\ln r + C$$

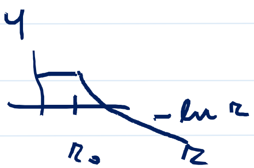
$$\Phi_S = \frac{Q_c}{\varepsilon_0} \leftarrow Q_c = 2\pi r_0 l \cdot \sigma$$

S – válec

$$E(r)\,2\pi r l = \frac{2\pi r_0 l \sigma}{\varepsilon_0}$$

délková hustota $\tau = 2\pi r_0 \sigma$

$$E(r) = \frac{\sigma}{\varepsilon_0}\frac{r_0}{r} = \frac{2\pi r_0 \sigma}{2\pi\varepsilon_0 r} = \frac{\tau}{2\pi\varepsilon_0 r}$$ – stejné jako pro drát



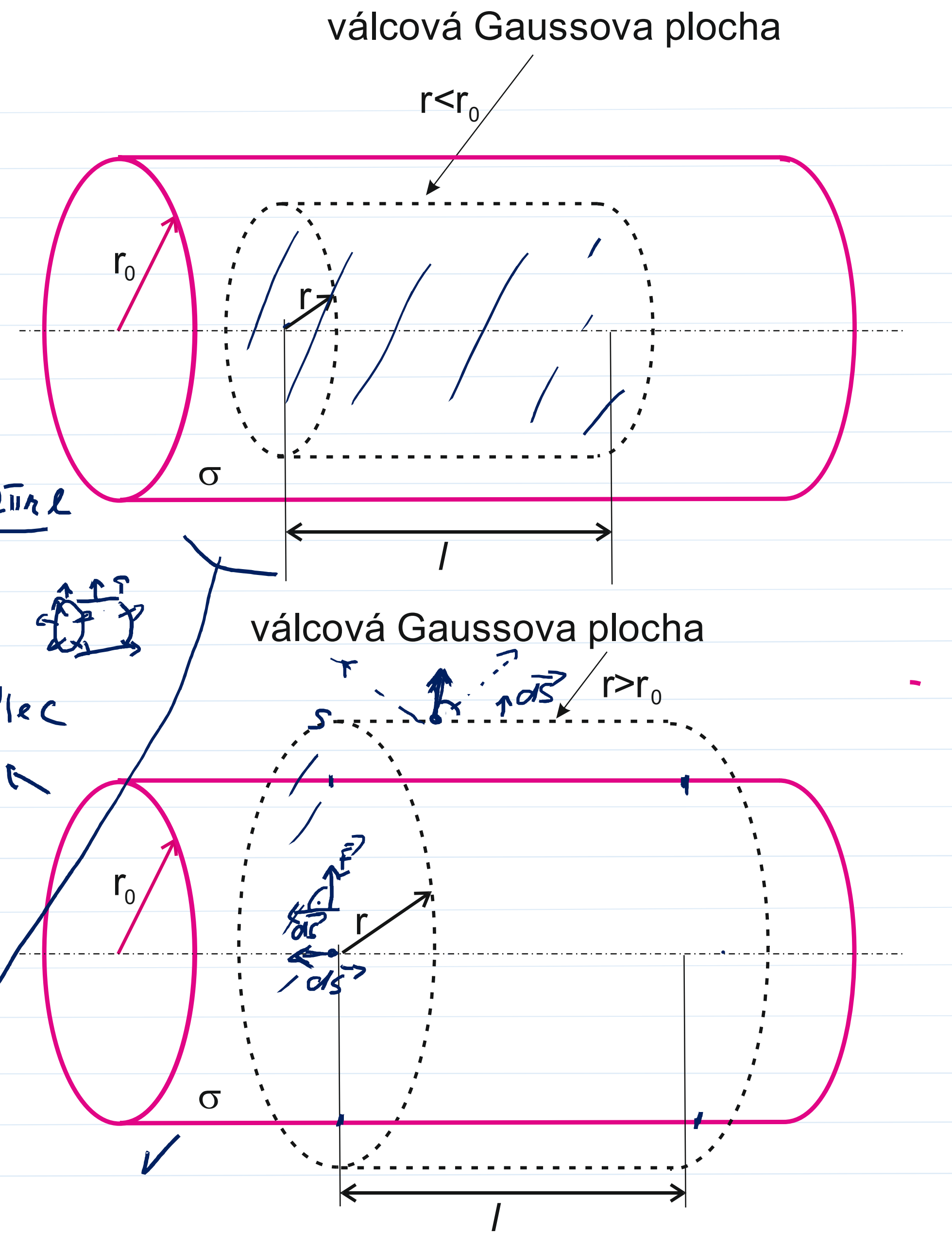
uvnitř

$r < r_0$

$\varphi =$ konst.

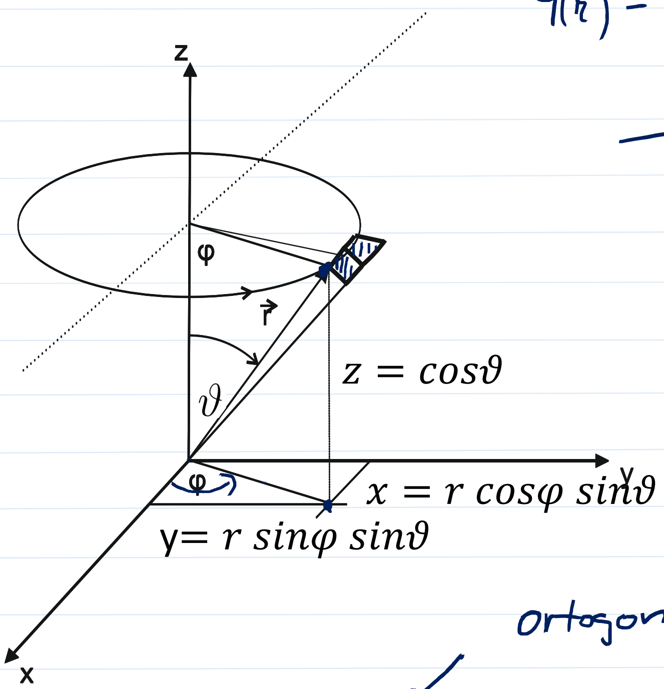
$$E(r)\,2\pi r l = \frac{Q_c}{\varepsilon_0} = 0 \quad Q_c = 0$$

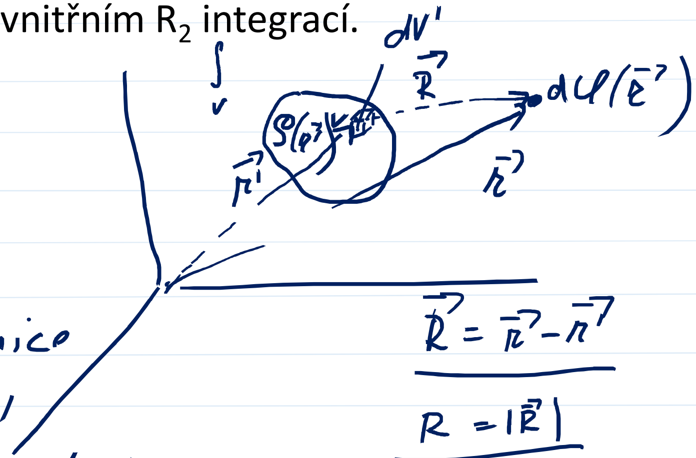
$\Rightarrow E(r) = 0$ pro $r < r_0$



S.1.1.18 Určete potenciál pole nabité kulové vrstvy o vnějším poloměru R_1 a vnitřním R_2 integrací. Hustota náboje je konstantní .

$$\varphi(\vec r) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\vec r\,')\,dV'}{R}$$





Kulová symetrie
sférické souřadnice

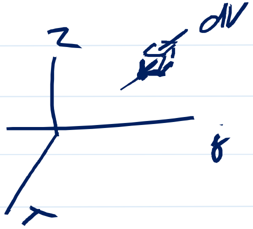
r, φ, ϑ — místo působení

r', φ', ϑ' — zdroje

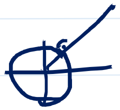
$\vec R = \vec r - \vec r\,'$

$R = |\vec R|$

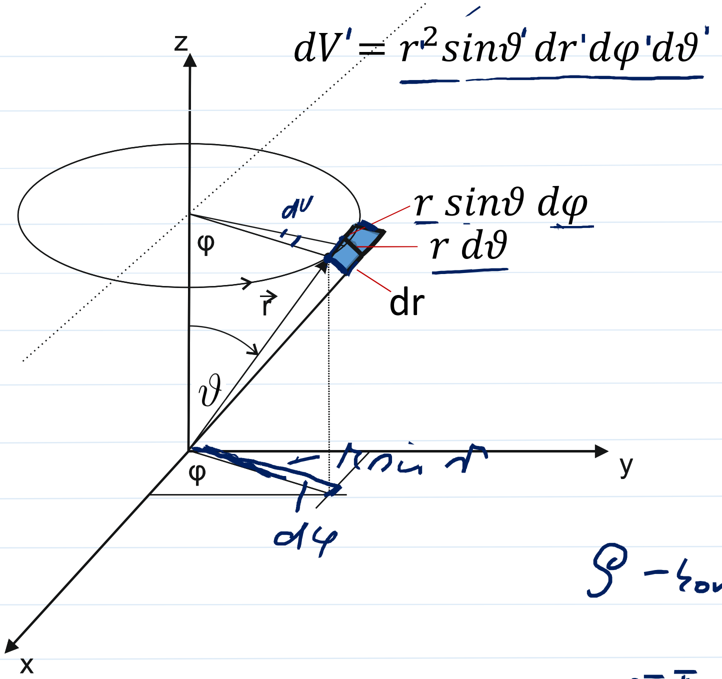
$dV = dx\,dy\,dz$



ortogonální

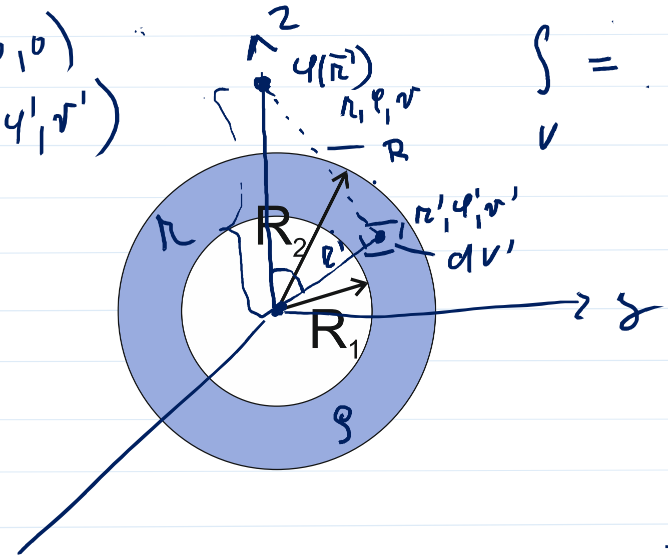


S.1.1.18 Určete potenciál pole nabité kulové vrstvy o vnějším poloměru R_2 a vnitřním R_1 integrací. Hustota náboje je konstantní .



$\vec{r} = (r, 0, 0)$

$\vec{r}\,' = (r', \varphi', \vartheta')$



$$\int_V = \int_0^{2\pi}\int_0^{\pi}\int_{R_1}^{R_2}$$

$\varphi(\vec{r}) = \varphi(r)$

ρ – konst

$R = |\vec{r} - \vec{r}\,'| =$

$$R = \sqrt{r'^2 + r^2 - 2r'r\cos\vartheta'}$$

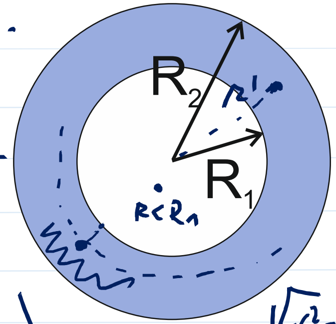
$$\varphi(\vec{r}) = \varphi(r) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\vec{r}\,')\,dV'}{R} = \frac{\rho}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^{\pi}\int_{R_1}^{R_2} \frac{r'^2 \sin\vartheta' \, dr' \, d\varphi' \, d\vartheta'}{\sqrt{r'^2 + r^2 - 2r'r\cos\vartheta'}}$$

$$= \frac{\rho}{4\pi\varepsilon_0} \overbrace{2\pi}^{\int d\varphi'} \int_0^\pi \int_{R_1}^{R_2} \frac{2r\, r'^2 \sin\vartheta'\, dr'\, d\vartheta'}{2r \underbrace{\sqrt{r^2+r'^2-2rr'\cos\vartheta'}}_{t}} = \frac{\rho}{2\varepsilon_0} \int_t \int_{R_1}^{R_2} \frac{r'\,dr'\,dt}{2r\sqrt{t}} = \frac{\rho}{2\varepsilon_0} \int_{R_1}^{R_2} \frac{r'}{2r} 2\left[t^{1/2}\right] dr' =$$

$$dt = 2rr'\sin\vartheta'\, d\vartheta'$$

$$\int t^{-1/2} dt = 2t^{1/2}$$

$$\Big[\;\Big]_0^\pi$$



$$= \frac{\rho}{2\varepsilon_0} \int_{R_1}^{R_2} \frac{r'}{r}\Big[\underbrace{\sqrt{r^2+r'^2+2rr'}}_{\sqrt{(r+r')^2}} - \underbrace{\sqrt{r^2+r'^2-2rr'}}_{\sqrt{|r-r'|^2}}\Big] dr' = \frac{\rho}{2\varepsilon_0} \int_{R_1}^{R_2} \frac{r'}{r}\big(\underbrace{|r+r'|}_{(r+r')} - |r-r'|\big)\, dr' \to \text{větvení}$$

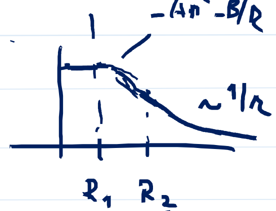
$\sqrt{x^2} = |x|$

$$Q_c = \frac{4}{3}\pi(R_2^3-R_1^3)\rho$$

$r > R_2 \Rightarrow r > r' \Rightarrow |r-r'| = r - r'$

$$\varphi(r) = \frac{\rho}{2\varepsilon_0} \int_{R_1}^{R_2} \frac{r'}{r}\big(r+r'-(r-r')\big)\, dr' = \frac{\rho}{2\varepsilon_0 r} \int_{R_1}^{R_2} r' 2r'\, dr' = \frac{\rho}{\cancel{2}\varepsilon_0 r} \cancel{2} \left(R_2^3 - R_1^3\right)\frac{1}{3} = \frac{\rho}{3\varepsilon_0}\left(R_2^3-R_1^3\right)\frac{1}{r}$$

$$= \frac{Q_c}{4\pi\varepsilon_0 r}$$



$r < R_1 \Rightarrow r' > r \to |r-r'| = r'-r$

$$\varphi(r) = \frac{\rho}{2\varepsilon_0 r} \int_{R_1}^{R_2} r'\, 2r\, dr' = \frac{\rho}{2\varepsilon_0}\left(R_2^2-R_1^2\right) = \underline{\text{konst}}$$

$R_1 \le r < R_2$ – kombinace předchozí

$$\varphi(r) = \frac{\rho}{2\varepsilon_0 r}\left[\int_{R_1}^{r} 2r'^2\, dr' + \int_r^{R_2} 2rr'\, dr'\right] = \frac{\rho}{2\varepsilon_0 r}\left[\frac{2}{3}\left(r^3-R_1^3\right) + r\left(R_2^2-r^2\right)\right] =$$

$$\cancel{-\frac{\rho}{6\varepsilon_0} R^2 - \frac{\rho}{3\varepsilon_0} R_1^3 \frac{1}{R} + \frac{\rho}{2\varepsilon_0} R}$$

$$-A r^2 - B\frac{1}{r} + K$$

DÚ

1.1.22. Dva dlouhé tenké vodiče, vložené rovnoběžně ve vzdálenosti *d* od sebe jsou nabity s lineární hustotou +λ a -λ (λ=konst.). Určete intenzitu pole ***E*** v bodě, který leží v rovině symetrie ve vzdálenosti *x* od roviny v níž leží vodiče.