3. Consider a Markov random field on a lattice L with respect to the relation $i \sim j$. If $i \in L$ has no neighbours, i.e. $\partial i = \emptyset$, does that imply that Z_i and Z_{-i} are independent?

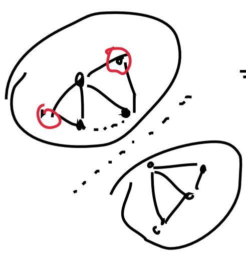
4. Consider a Markov random field on a lattice L with respect to the relation $i \sim j$. If $i, j \in L$ are not neighbours, i.e. $i \nsim j$, does that imply that Z_i and Z_j are independent?

3) Z_i , $Z_{-i} = (Z_1, .., Z_{i-1}, Z_{i+1}, ..., Z_n)$

$\partial i = \emptyset \overset{?}{\Rightarrow} Z_i, Z_{-i}$ indep.

$p(z_i) = p(z_i | z_{-i}) = \frac{p(\ldots}{p\ldots}$

assume: MRF ... $p(z_i | z_{-i}) = p(z_i | \underbrace{z_{\partial i}}_{\emptyset}) = p(z_i)$

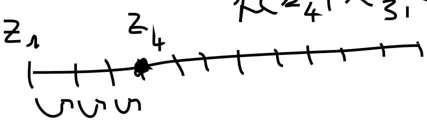


$\Rightarrow p(z_i, z_{-i}) = p(z_i) \cdot p(z_{-i}) \Rightarrow$ independence.

4) NO, depencies propagate in the graph

↳ Ising model

↳ Markov chains ...



$p(z_4 | z_3, z_2, z_1) = p(z_4 \ldots$

$\Downarrow$

does not i…
independen…
of Z_1, Z_4

$p(z_4 | z_1) \neq p(z_4)$