$$\int_A \frac{y^3}{x^3}\, d\lambda^2(x,y)\,, \quad A = \{(x,y)\in\mathbb{R}^2;\ 1\le xy\le 3,\ x\le y\le 2x\}$$

A je uz. $\Rightarrow$ A je měř

f je spoj. na A $\Rightarrow$ f je měř.

Použijeme substituci $u = xy$, $v = \frac{y}{x}$

Pozn: $x\le y\le 2x$ $\Rightarrow x\ge 0$, $1\le \frac{y}{x}\le 2$

z $x\ge 0$ plyne $y\ge 0$

(*) $\varphi : (0,\infty) \times (0,\infty) \mapsto \mathbb{R}^2$ | $\varphi(\Omega) = \{(u,v) \in \mathbb{R}^2;\ 1 \le u \le 3,\ 1 \le v \le 2\}$

$\varphi(x,y) = \left(xy, \frac{y}{x}\right)$

$$|J\varphi(x,y)| = \begin{vmatrix} y & x \\ -\frac{y}{x^2} & \frac{1}{x} \end{vmatrix} = 2\frac{y}{x} > 0 \text{ na } (0,\infty)\times(0,\infty)$$

φ je prosté a na $(0,\infty)\times(0,\infty)$:

$$\left.\begin{aligned} xy &= a \\ \frac{x}{y} &= b \end{aligned}\right\} \Rightarrow \begin{aligned} y &= \sqrt{\frac{a}{b}} \\ x &= \sqrt{ab} \end{aligned}$$

Tedy pro $g(u,v) = v^2$ je

$$\int_M \frac{y^3}{x^3} \, d\lambda^2(x,y) = \int_{\varphi^{-1}(\varphi(M))} g(\varphi(x,y)) \cdot |J\varphi(x,y)| \cdot \frac{1}{2} \, d\lambda^2(x,y)$$

(*) V. o Substituci

$$\stackrel{(*)}{=} \int_{\varphi(M)} g(u,v) \frac{1}{2} \, d\lambda^2(u,v) \stackrel{\text{Fubini, g} \geq 0}{=}$$

$$= \int_1^3 \int_1^2 \frac{v^2}{2} \, dv \, du = \frac{7}{3}$$