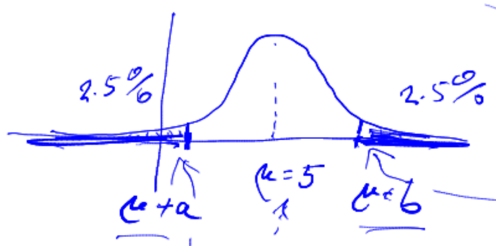
# 9. cvičenie

① a) $X \sim N(\mu, 1)$ --- $F_X$  $X - \mu \sim N(0,1)$

$97.5\%$ / $2.5\%$ $= P(X - \mu \le z) = \phi(z)$




nájdeme $\phi^{-1}\left(1 - \frac{0.05}{2}\right)$ a $\phi^{-1}\left(\frac{0.05}{2}\right)$

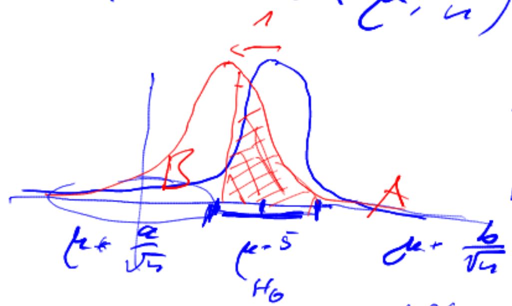
$b = 1.96$  $a = -1.96$

$3.04 = F_X^{-1}(0.025)$  $H_0$  $5.96 = F_X^{-1}(0.975)$

$(a = -b)$

Podľa defin. $P(X < \mu + a \text{ alebo } X > \mu + b) = \frac{0.05}{2} + \frac{0.05}{2}$

b) $\bar{X}_n = \frac{X_1 + \dots + X_n}{n} \sim N\left(\mu, \frac{1}{n}\right)$, $\sigma = \frac{1}{\sqrt{n}}$  $\frac{X-\mu}{\sigma}$




$P(Z \le a) = 2.5\%$  $Z = \frac{\bar{X}_n - \mu}{1/\sqrt{n}} \sim N(0,1)$

$P\left(\bar{X}_n \le \mu + \frac{a}{\sqrt{n}}\right)$

$100 \xrightarrow{4\times} 400$

$5 - \frac{1.96}{\sqrt{n}}$  $5 + \frac{1.96}{\sqrt{n}}$  $\frac{2 \cdot 1.96}{10} \xrightarrow{2\times} \frac{2 \cdot 1.96}{20}$

v časti d: $\doteq \phi^{-1}(0.975)$

$3.2$ $\doteq 0$

c) $\bar{X}_n = X \sim N\left(4, \frac{1}{n}\right)$  $\frac{X-4}{1/\sqrt{n}} \sim N(0,1)$

$P\left(X > 5 + \frac{b}{\sqrt{n}}\right) = P\left(\frac{X-4}{1/\sqrt{n}} > \sqrt{n} + b\right) = 1 - \Phi(\sqrt{10} + 1.96) = A$

$P\left(X < 5 + \frac{a}{\sqrt{n}}\right) = P\left(\frac{X-4}{1/\sqrt{n}} < \sqrt{n} + a\right) \doteq \phi(-1.96 + \sqrt{10})$

$\doteq \phi(1.20) \doteq 0.88 = B$

P(nezamietneme) $= 1 - A - B \doteq 0.1$ --- závisí na skutočnom $\mu$

$\doteq \phi(\sqrt{10} + 1.96) - \phi(\sqrt{10} - 1.96)$ chyba 2. druhu

d) aj tu CLV  $0.88$ --- sila testa

② a) $\hat{\mu} = \frac{X_1 + \dots + X_n}{n} = \frac{8.47 + \dots + 10.40}{5} = \underline{\underline{10.022}}$

$\hat{\sigma}^2 = \frac{1}{n-1}\left((X_1 - \hat{\mu})^2 + (X_2 - \hat{\mu})^2 + \dots\right) = \underline{1.09\dots}$

b) $N(9, \frac{\hat{\sigma}^2}{n})$ : $\underline{9 \pm 1.96 \cdot \frac{\hat{\sigma}}{\sqrt{n}}}$ : $\underline{[8.08, 9.92]}$

c) t-rozd. s $4 = n-1$ st. vol. $\underline{\underline{[7.70, 10.29]}}$ .... $\underline{\text{nezamítneme}}$!

$\boxed{\frac{\bar{X}_n - \mu}{\hat{S}_n/\sqrt{n}}} \sim$ t-rozd. s $(n-1)$ st. vol.

$\underline{P\left(\frac{\bar{X}_n - \mu}{\hat{S}_n/\sqrt{n}} < z\right)} = \Psi_{n-1}(z) = \overset{0.975}{\cancel{0.05}}$

$z = \Psi_{n-1}^{-1}(0.05)$

$\Psi_{n-1}^{-1}(0.975)$

$\bar{X}_n < \overset{9}{\mu} + z \cdot \frac{\hat{S}_n}{\sqrt{n}}$

$\Psi_4^{-1}$

④ $X_i \sim Pois(\lambda)$ $H_0: \underline{\lambda = 35}$

ⓐ $\bar{X}_n \overset{CLV}{\sim} N(\mu, \sigma^2)$ přibližně $\mu = \lambda \Rightarrow \underline{\underline{35}}$

$\underline{\sigma^2} = \frac{\lambda}{n} = \frac{35}{5} = 7$

$\bar{X}_n \overset{?}{\in} \left[\mu - 1.96\sqrt{7}, \mu + 1.96\sqrt{7}\right) = [29.8, 40.2]$

ⓑ $X_1 + \dots + X_5 \sim Pois(5\lambda)$ .... distr. fce pro $\lambda = 35$ $F = F_{Pois(5 \cdot 33)}$

$\quad\quad 165$

$X_1 + \dots + X_5 \in \left[\underline{F^{-1}(0.025)}, \underline{F^{-1}(0.975)}\right]$

$\underline{150}$ $\quad\quad \underline{201}$

$\underline{\underline{P(X_1 + \dots + X_5 = k) = \frac{(5\lambda)^k}{k!} e^{-5\lambda}}}$

naměřeno: 223 ✓ 159 ✓