1.12.

$$3.\ \lim_{x\to 0}\frac{e^{\sin 2x}-e^{\arcsin x}}{x}=\lim_{x\to 0}\left(\frac{e^{\sin 2x}-1}{x}+\frac{1-e^{\arcsin x}}{x}\right)$$

$$=\lim_{x\to 0}\left(\frac{e^{\sin 2x}-1}{\sin 2x}\cdot\underbrace{\frac{\sin 2x}{2x}}_{\xrightarrow[x\to 0]{}1}\cdot 2+\frac{1-e^{\arcsin x}}{\arcsin x}\cdot\frac{\arcsin x}{x}\right)$$

$$\overset{AL}{=}\lim_{x\to 0}\left(\frac{e^{\sin 2x}-1}{\sin 2x}\cdot\frac{\sin 2x}{2x}\cdot 2\right)+\lim_{x\to 0}\left(\frac{1-e^{\arcsin x}}{\arcsin x}\cdot\frac{\arcsin y}{x}\right)$$

$$=2\cdot\lim_{x\to 0}\frac{e^{\sin 2x}-1}{\sin 2x}+\lim_{x\to 0}\frac{1-e^{\arcsin x}}{\arcsin x}\cdot\frac{\arcsin x}{x}$$

!

$$=2+(-1)\cdot 1=1$$

$$\lim_{x\to 0}\frac{\arcsin x}{x}\overset{?}{=}\lim_{t\to 0}\frac{\arcsin(\sin t)}{\sin t}=\lim_{t\to 0}\frac{t}{\sin t}=1$$

vnější: $\frac{t}{\sin t}\xrightarrow[t\to A=0]{}1$

vnitřní: $\arcsin x\xrightarrow[x\to 0]{}0=:A$

(P) $\arcsin x$ je prostá

$$\left.\right\}\ \lim_{x\to 0}\frac{\arcsin x}{x}=1$$

6. $$\lim_{x\to+\infty} x\left(\frac{\pi}{4} - \operatorname{arctg}\frac{x}{x+1}\right) = L$$

$$\lim_{y\to 1} \frac{\frac{\pi}{4} - \operatorname{arctg} y}{y-1} = -\frac{1}{2}$$

vnější, vnitřní: $\operatorname{arctg} y \xrightarrow{y\to 1} \frac{\pi}{4}$

$$\lim_{t\to\frac{\pi}{4}} \frac{\frac{\pi}{4} - t}{\operatorname{tg} t - 1} = \lim_{t\to\frac{\pi}{4}} \frac{-1}{\frac{\operatorname{tg} t - 1}{t-\frac{\pi}{4}}} = \lim_{t\to\frac{\pi}{4}} \frac{-1}{\frac{\sin t - \cos t}{t-\frac{\pi}{4}}}\cdot\cos t$$

($\cancel{t = \operatorname{tg}}$ $\operatorname{tg} t = y$)

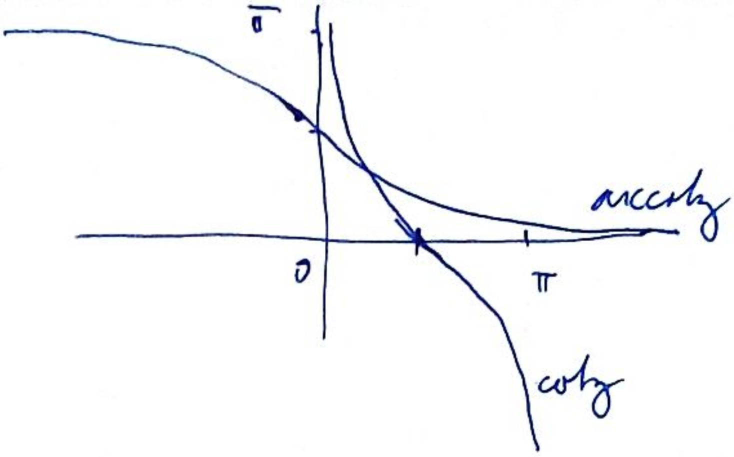
$$\overset{AL}{=} -\frac{\sqrt{2}}{2}\lim_{t\to\frac{\pi}{4}} \frac{t-\frac{\pi}{4}}{\sin\left(t-\frac{\pi}{4}+\frac{\pi}{4}\right) - \cos\left(t-\frac{\pi}{4}+\frac{\pi}{4}\right)} =$$

$$= -\cancel{\frac{\sqrt{2}}{2}}\lim_{t\to\frac{\pi}{4}} \frac{t-\frac{\pi}{4}}{\sin\left(t-\frac{\pi}{4}\right)\cancel{\frac{\sqrt{2}}{2}} + \cancel{\cos\left(t-\frac{\pi}{4}\right)\cdot\frac{\sqrt{2}}{2}} - \left[\cancel{\cos\left(t-\frac{\pi}{4}\right)\frac{\sqrt{2}}{2}} - \sin\left(t-\frac{\pi}{4}\right)\frac{\sqrt{2}}{2}\right]}$$

$$= -\lim_{t\to\frac{\pi}{4}} \frac{t-\frac{\pi}{4}}{2\sin\left(t-\frac{\pi}{4}\right)} = -\frac{1}{2} = \left(-\frac{1}{\operatorname{tg}'\left(\frac{\pi}{4}\right)} = -\frac{1}{\frac{1}{\cos^2\left(\frac{\pi}{4}\right)}} = -\frac{1}{\frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2}} = -\frac{1}{2}\right)$$

$$L = \lim_{x\to+\infty} \overbrace{\frac{\frac{\pi}{4} - \operatorname{arctg}\frac{x}{x+1}}{\frac{x}{x+1} - 1}}^{\to -\frac{1}{2}} \cdot \frac{-1}{x+1}\cdot x = \frac{1}{2}$$

$$\lim_{x\to+\infty} \frac{\operatorname{arccotg}(x)}{\frac{1}{x}}$$



$x = \operatorname{cotg} t$, $t \to 0+$

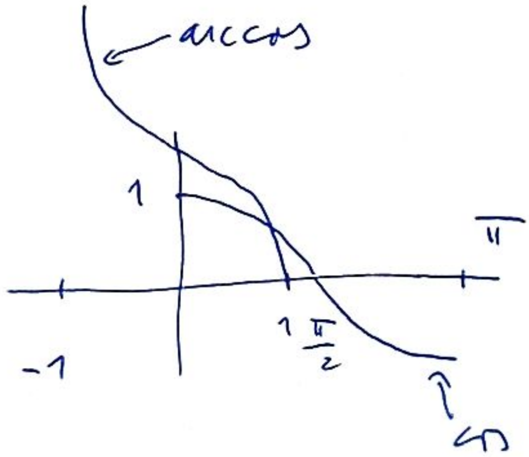
$$\lim_{t\to0+} \underbrace{\frac{t}{\frac{1}{\operatorname{cotg} t}}}_{\text{vnější}} = \lim_{t\to0+} t\cdot\frac{\cos t}{\sin t} = 1$$

vnitřní: $\operatorname{arccotg} x \xrightarrow[x\to+\infty]{} 0+$

(P) arccotg je prostá

$$\Rightarrow \lim_{x\to+\infty} \frac{\operatorname{arccotg} x}{\frac{1}{x}} = 1$$

$$\lim_{x\to-1+} \frac{(\arccos x - \pi)^2}{x+1} = 2$$



"$x = \cos t$", $t \to \pi-$"

$$\lim_{t\to\pi-} \frac{(t-\pi)^2}{\cos t + 1} = \lim_{t\to\pi-} \frac{(t-\pi)^2}{\cos(t-\pi+\pi)+1} =$$

$$= \lim_{t\to\pi-} \frac{(t-\pi)^2}{\cos(t-\pi)(-1)+1} = \lim_{t\to\pi-} \frac{1}{\frac{1-\cos(t-\pi)}{(t-\pi)^2}} = 2$$