Spočtěte $\int_M f(x,y,z) \, d\lambda^3(x,y,z)$ kde

$f(x,y,z) = \frac{1}{1+x+y}$, $M = \{(x,y,z) \in \mathbb{R}^3 ;\ x,y,z \geq 0,$

$x + y + z \leq 1\}$.

M je uz. $\Rightarrow$ M je měř.

f je spoj. na M $\Rightarrow$ f je měř.

$$\int_{\Delta} f \, d\lambda^3 \overset{\text{Fubini, } f \geq 0 \text{ msb}}{=} \int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{1}{1+x+y} \, dz \, dy \, dx$$

$$= \int_0^1 \int_0^{1-x} \frac{1-x-y}{1+x+y} \, dy \, dx = \int_0^1 \int_0^{1-x} \frac{2}{1+x+y} - 1 \, dy \, dx$$

$$= \int_0^1 \Big[2 \log(1+x+y) - y \Big]_0^{1-x} dx =$$

$$= \int_0^1 2\log(2) - 2\log(1+x) - 1 + x \, dx =$$

$$= \left[2\log(2)x - x + \frac{x^2}{2} - 2\left((1+x)\log(1+x) - (1+x)\right)\right]_0^1$$

$$= 2\log(2) - 1 + \frac{1}{2} - 2\left(2\log(2) - 2\right)$$

$$- \left(0 - 0 + 0 - 2\left(1 \cdot 0 - 1\right)\right) = -2\log(2) + \frac{3}{2}.$$