VĚTA NPJE:

G racionální matice

- $\alpha_b = 1$

$\Downarrow$

- G má poly pravý inverz $(\sigma D, D^{-1})$

$\Downarrow$

- uG poly $\Rightarrow$ u poly

$\Uparrow$

$$\Gamma = \begin{pmatrix} \frac{\alpha_1}{\beta_1} & 1 & & & \\ & 1 & 1 & & \\ & & \ddots & 1 & \\ & & & & \frac{\alpha_b}{\beta_b} \end{pmatrix}$$

UKÁŽEME

$\alpha_b \neq 1 \Rightarrow$ najdeme u: uG poly a u není poly

$G \cdot G' = (1 \; 1)$

$G = (\cancel{A} \cdot \cancel{\Gamma} \cdot \cancel{B})$

$G' = \cancel{B}^{-1} \cancel{\Gamma}^{-1} \cdot \cancel{A}^{-1}$

$$\Gamma^{-1} = \begin{pmatrix} \frac{\beta_1}{\alpha_1} & \ddots & \frac{\beta_b}{\alpha_b} \\ & 0 & \end{pmatrix}$$

G racionální Smithova forma $q \cdot G$ polynomiální

$$qG = A \cdot \Gamma \cdot B \qquad \Gamma = \begin{pmatrix} \mu_1 & \ddots & & 0 \\ & & \mu_b & \end{pmatrix}$$

$$G = A \cdot \frac{\Gamma}{q} \cdot B$$

$\frac{\mu_i}{q} = \frac{\alpha_i}{\beta_i}$ $\mu_i | \mu_{i+1}$

$\alpha_i | \alpha_{i+1}$ $\beta_{i+1} | \beta_i$

$$G = A \cdot \begin{pmatrix} \frac{\alpha_1}{\beta_1} & & & \\ & \ddots & & 0 \\ & & \frac{\alpha_b}{\beta_b} & \end{pmatrix} \cdot B$$

$$\left(0, \dots, 0, \frac{\beta_b}{\alpha_b}\right) \cdot \Gamma = (0, \dots, 0, 1, 0 \dots 0)$$

$\overset{?}{=} \frac{\beta_b}{\alpha_b} \cdot e_b$

uG

$$\boxed{u = \frac{\beta_b}{\alpha_b} e_b \cdot \cancel{A}^{-1}} \cdot (\cancel{A} \Gamma B) = B_b \text{ poly}$$

$$u \cdot A = \underbrace{\frac{\beta_b}{\alpha_b}}_{\text{není polynom}} e_b \Rightarrow u \text{ není polynom} \quad \square$$

$\alpha_b \neq 1$ $\frac{\beta_b}{\alpha_b}$ $\frac{p}{q} = \dots$

$\alpha_b \neq D^k$ $\frac{1}{1+D} = 1 - \dots$

$\frac{1}{D^k} = D^{-k}$ $\mathbb{F}[D, D^{-1}]$ $\not\ni \frac{1}{p}$

$\mathbb{F}$ inv. D, D^{-1} $p \neq D, D^{-1}, a_0 D^0$