NMAI057 – Linear algebra 1

Tutorial 3

Date: October 13, 2020 TA: Pavel Hubáček

Example 1: Compute the following expressions:

(a) $2A$
(b) $A + B$
(c) $A - B$
(d) C^T
(e) Cv
(f) AB
(g) BC

for

$$A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix},\ B = \begin{pmatrix} -1 & -1 \\ 0 & 3 \end{pmatrix},\ C = \begin{pmatrix} 3 & 0 & 1 \\ 2 & -2 & 0 \end{pmatrix},\ v = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

Example 2: Prove or disprove the following:

(a) For all matrices $A \in \mathbb{R}^{m\times n}$, $A + A = 2A$.
(b) For all square matrices $A \in \mathbb{R}^{m\times m}$, $A = A^T$.

Problem 1. Compute $(-1)A + 2BC$ for matrices

$$A = \begin{pmatrix} 3 & 1 \\ 4 & 1 \end{pmatrix}, B = \begin{pmatrix} 5 & 9 \\ 2 & 7 \end{pmatrix}, C = \begin{pmatrix} 1 & -1 \\ 2 & 2 \end{pmatrix}.$$

Problem 2. Solve the systems of linear equations $(A \mid b)$ and $(B \mid c)$ given by

$A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$ a $b = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, and

$B = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 2 & 0 \end{pmatrix}$ a $c = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$.

Verify the correctness of your result x (resp. y) by computing the matrix product $Ax = b$ (resp. $By = c$).

Problem 3. Express the elementary row operations as matrix products, i.e., for each elementary row operation, find a matrix $E \in \mathbb{R}^{m\times m}$ such that EA is the result of applying the operation to matrix A for all matrices $A \in \mathbb{R}^{m\times n}$.

Problem 4. Prove or disprove whether for all matrices A, B, C and $\mathbf{0}$ of the same order and real numbers $\alpha, \beta \in \mathbb{R}$, it holds that:

(a) $A + (B + C) = (A + B) + C$

(b) $A + B = B + A$

(c) $A + \mathbf{0} = A$

(d) $\alpha(\beta A) = (\alpha\beta)A$

(e) $\alpha(\beta A) = \beta(\alpha A)$

(f) $A + (-1)A = \mathbf{0}$

(g) $1A = A$

(h) $A(B + C) = AB + AC$

(i) $\alpha(A + B) = \alpha A + \alpha B$

(j) $(\alpha + \beta)A = \alpha A + \beta A$

(k) $\alpha A + \beta B = (\alpha + \beta)(A + B)$

(l) $(A^T)^T = A$

(m) $A^T A$ is symmetric

(n) $(A + B)^T = A^T + B^T$

(o) $(\alpha A)^T = \alpha(A^T)$

(p) $AI_n = A$

Problem 5. Give a non-symmetric matrix A and a symmetric matrix B such that their product does not commute, i.e., such that $AB \neq BA$.

Is the product of symmetric matrices commutative?

Problem 6. Prove or disprove the following statements:

(a) For all $A, B \in \mathbb{R}^{n\times n}$, if A is symmetric and commutes with B then A commutes also with B^T.

(b) For all $A, B \in \mathbb{R}^{n\times n}$, if A commutes with B then A commutes with B^T.