Tvrzení:

Nechť P je k-dimenzionální nadrovina v $\mathbb{R}^n$, $k < n$, pak P má prázdný vnitřek.

VoIF:

$f(x,y) = e^{xy} + \sin(y) + y^2 = 1$, $(2,0)$

1) $f \in C^\infty(\mathbb{R}^2)$

2) $f(2,0) = 1$

3) $\frac{\partial f}{\partial y}(x,y) = e^{xy}x + \cos(y) + 2y$

$\frac{\partial f}{\partial y}(2,0) = 2 + 1 + 0 = 3 \neq 0$

VoIF

$\Rightarrow \exists \varepsilon > 0$ že $\forall x \in (2-\varepsilon, 2+\varepsilon)\ \exists \varphi(x) \in (-\varepsilon, \varepsilon)$, $\varphi \in C^\infty$

Chceme $\varphi'(2)$. $\frac{\partial F}{\partial x}(x,y) = e^{xy} y$, $\frac{\partial F}{\partial x}(2,0) = 0$

Podle vzorečku

$$\varphi'(x) = - \frac{\overbrace{e^{x\varphi(x)} \varphi(x)}^{B(x)}}{\underbrace{e^{x\varphi(x)} x + \cos(\varphi(x)) + 2\varphi(x)}_{A(x)}}$$

$$\varphi'(2) = \frac{0}{3} = 0$$

$$\varphi''(x) = -\frac{A(x)\cdot\left[\varphi'(x)\, e^{x\varphi(x)} + \varphi(x)\, e^{x\varphi(x)}\left(\varphi(x) + x\varphi'(x)\right)\right]}{\left(A(x)\right)^2}$$

$$+ \frac{B(x)\cdot\left[e^{x\varphi(x)} + x\, e^{x\varphi(x)}\left(\varphi(x) + x\varphi'(x)\right) + \dots - \sin(\varphi(x))\,\varphi'(x) + 2\varphi'(x)\right]}{\left(A(x)\right)^2}$$

$$\varphi''(2) = -\frac{3\cdot[0+0]+0}{9} = 0$$

$$e^{x\varphi(x)} \cdot \left(\overbrace{\varphi(x) + x\,\varphi'(x)}^{C(x)}\right) + \cos(\varphi(x)) \cdot \varphi'(x) +$$

$$+ 2\varphi(x)\,\varphi'(x) = 0$$

$x = 2$, $\varphi(2) = 0$:

$$e^{0} \cdot \left(0 + 2\varphi'(2)\right) + \cos(0) \cdot \varphi'(2) + 0 = 0$$

$$\Rightarrow \varphi'(2) = 0$$

$$e^{x\varphi(x)} \cdot C^2(x) + e^{x\varphi(x)}\Big(\varphi'(x) + \varphi'(x) + x\,\varphi''(x)\Big)$$

$$+ \big(-\sin(\varphi(x))\big)\varphi'(x)^2 + \cos(\varphi(x))\,\varphi''(x) +$$

$$+ 2\varphi'(x)^2 + 2\varphi\varphi'' = 0$$

$$x = 2,\quad \varphi(2) = 0,\quad \varphi'(2) = 0,\quad C(2) = 0$$

$$1 \cdot 0 + 1 \cdot \big(0 + 0 + 2\varphi''(2)\big) + 0 + \varphi''(x) +$$

$$+ 0 + 0 = 0 \Rightarrow 3\varphi''(2) = 0 \Rightarrow \varphi''(2) = 0$$

$$f(x,y) = x^y + y^x = 3 \quad , (1,2)$$

1) $f \in C^\infty(\{x>0, y>0\})$, $(1,2) \in \{x>0, y>0\}$

$\{x>0, y>0\}$ je ot.

2) $f(1,2) = 3$

3) $$\frac{\partial f}{\partial y}(x,y) = x^y \log(x) + x\, y^{x-1}$$

$$\frac{\partial f}{\partial y}(1,2) = 1$$

Podle věty ... etc.

Chceme $\varphi'(1)$, $\varphi''(1)$.

$$x^{\varphi(x)} \cdot \Big(\overbrace{\varphi'(x)\log(x) + \frac{\varphi(x)}{x}}^{A(x)} \Big) + \varphi(x)^{x} \cdot \Big(\overbrace{\log(\varphi(x)) + \frac{x}{\varphi(x)}\varphi'(x)}^{B(x)} \Big) = 0$$

$x = 1, \ \varphi(x) = 2$:

$$1^2 \cdot 2 + 2 \cdot \left(\log(2) + \frac{1}{2} \varphi'(x) \right) = 0$$

$$\Rightarrow \varphi'(x) = -2 - 2\log(2)$$

$$x^{\varphi(x)} A^2(x) + x^{\varphi(x)} \left(\varphi''(x) \log(x) + \frac{\varphi'(x)}{x} + \frac{\varphi'(x)}{x} - \frac{\varphi(x)}{x^2} \right) + \varphi(x)^x \cdot B(x)^2 +$$

$$\varphi(x)^x \cdot \left(\frac{1}{\varphi(x)} \varphi'(x) + \frac{\varphi'(x)}{\varphi(x)} + x \left(\frac{\varphi(x)\varphi''(x) - \varphi'(x)^2}{\varphi(x)^2} \right) \right)$$

$$= 0$$

$$x = 1, \quad \varphi(x) = 2, \quad \varphi'(x) = -2 - 2\log(2) = C$$

$$1^2 \cdot 2^2 + C + C - 2 + 2\left(\log(2) + \frac{C}{2}\right)^2 +$$

$$+ 2 \cdot \left(\frac{1}{2} C + \frac{1}{2} C + \frac{2\varphi''(x) - C^2}{4} \right) = 0$$

$$2 + 4C + \varphi''(x) - \frac{C^2}{2} + 2\log^2(2) + 2\frac{C^2}{4} + 4\log(2)\frac{C}{2}$$

$$= 0$$

$$c^2 = 4 + 8\log(2) + 4\log^2(2)$$

$$-\varphi''(x) = -8 - 12\log(2) - 2\log^2(2)$$

$$\varphi''(x) = 10 + 12\log(2) + 2\log^2(2)$$