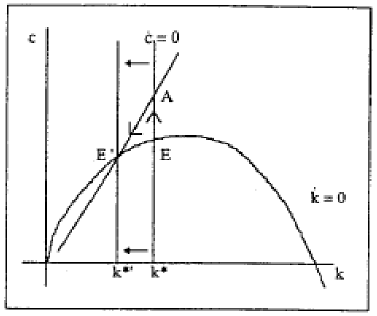
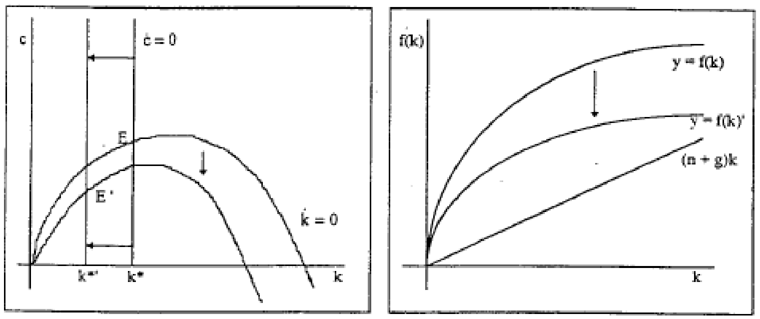
a) A rise in $\theta$. This is a fall in the elasticity of substitution -- $1/\theta$ --which means that people become less willing to substitute consumption between periods. It also means that the marginal utility of consumption falls off more rapidly as consumption rises. If the economy is growing, this tends to make households value present consumption more than future consumption.

The capital-accumulation equation is unaffected. The condition required for $\dot{c} = 0$ is given by $f'(k) = \rho + \theta g$. Since $f''(k) < 0$, the $f'(k)$ that makes $\dot{c} = 0$ is now higher. Thus the value of k that satisfies $\dot{c} = 0$ is lower. The $\dot{c} = 0$ locus shifts to the left. The economy moves up to point A on the new saddle path -- people consume more now. Movement is then down along the new saddle path until the economy reaches point E'. At that point, $c^*$ and $k^*$ are lower than their original values.



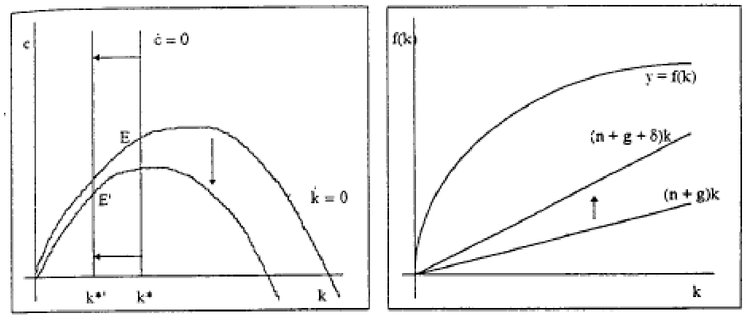

b) A downward shift of the production function.
We can assume that this means for any given k, both f(k) and f'(k) are lower than before.




The condition required for $\dot{k} = 0$ is given by $c = f(k) - (n + g)k$. We can see from the right diagram that the $\dot{k} = 0$ locus will shift down more at higher levels of k. Also, since for a given k, $f'(k)$ is lower now, the golden-rule k will be lower than before. Thus the $\dot{k} = 0$ locus shifts as depicted in the diagram.
The condition required for $\dot{c} = 0$ is given by $f'(k) = \rho + \theta g$. For a given k, $f'(k)$ is now lower. Thus we need a lower k to keep $f'(k)$ the same and satisfy the $\dot{c} = 0$ equation. Thus the $\dot{c} = 0$ locus shifts left. The economy will eventually reach point E' with lower $c^*$ and lower $k^*$. Whether c initially jumps up or down depends upon whether the new saddle path passes above or below point E.



c) A positive rate of depreciation.




The new capital-accumulation equation is:

(3) $\dot{k}(t) = f(k(t)) - c(t) - (n + g + \delta)k(t)$

The level of saving and investment required just to keep any given k constant is now higher -- and thus the amount of consumption possible is now lower -- than in the case with no depreciation. The level of extra investment required is also higher at higher levels of k. Thus the $\dot{k} = 0$ locus shifts down more at higher levels of k.

In addition, the real return on capital is now $f'(k(t)) - \delta$ and so the household's maximization will yield :

(4) $\dfrac{\dot{c}(t)}{c(t)} = \dfrac{f'(k(t)) - \delta - \rho - \theta g}{\theta}$

The condition required for $\dot{c} = 0$ is $f'(k) = \delta + \rho + \theta g$. Compared to the case with no depreciation, $f'(k)$ must be higher and k lower in order for $\dot{c} = 0$. Thus the $\dot{c} = 0$ locus shifts to the left. The economy will eventually wind up at point E' with lower levels of $c^*$ and $k^*$. Again, whether c jumps up or down initially depends upon whether the new saddle path passes above or below the original equilibrium point of E.

**Problem 2.7**

With a positive depreciation rate -- $\delta > 0$ -- the Euler equation and the capital-accumulation equation are given by:

(1) $\dfrac{\dot{c}(t)}{c(t)} = \dfrac{f'(k(t)) - \delta - \rho - \theta g}{\theta}$ and (2) $\dot{k}(t) = f(k(t)) - c(t) - (n + g + \delta)k(t)$

We begin by taking first-order Taylor approximations to (1) and (2) around $k = k^*$ and $c = c^*$. That is, we can write:

(3) $\dot{c} \cong \dfrac{\partial \dot{c}}{\partial k}[k - k^*] + \dfrac{\partial \dot{c}}{\partial c}[c - c^*]$ and (4) $\dot{k} \cong \dfrac{\partial \dot{k}}{\partial k}[k - k^*] + \dfrac{\partial \dot{k}}{\partial c}[c - c^*]$

where $\partial \dot{c}/\partial k$, $\partial \dot{c}/\partial c$, $\partial \dot{k}/\partial k$ and $\partial \dot{k}/\partial c$ are all evaluated at $k = k^*$ and $c = c^*$.

Since $c^*$ and $k^*$ are constants, $\dot{c}$ and $\dot{k}$ are equivalent to $c - c^*$ and $k - k^*$ respectively. We can therefore rewrite (3) and (4) as: