Problem 2.5

a) The equation describing the dynamics of the capital stock per unit of effective labor is:

(1) $\dot{k}(t) = f(k(t)) - c(t) - (n+g)k(t)$

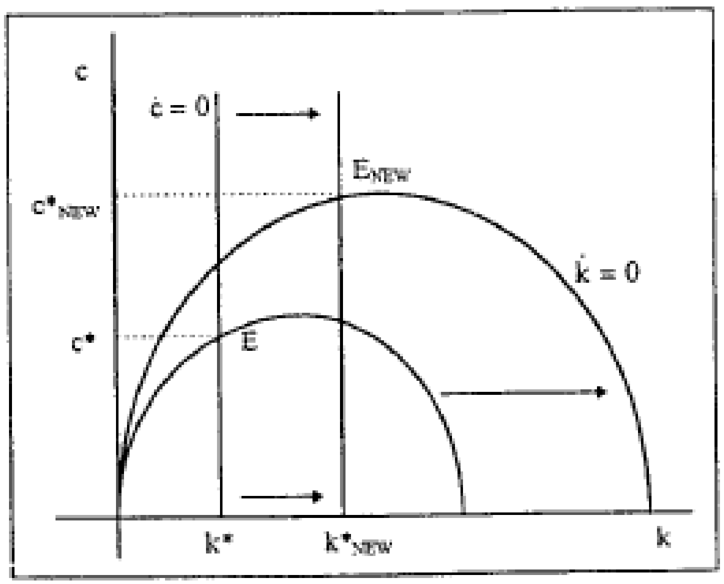
For a given k, the level of c that implies $\dot{k} = 0$ is given by c = f(k) - (n + g)k. Thus a fall in g makes the level of c consistent with $\dot{k} = 0$ higher for a given k. That is, the $\dot{k} = 0$ curve shifts up. Intuitively, a lower g makes break-even investment lower at any given k and thus allows for more resources to be devoted to consumption and still maintain a given k. Since (n + g)k falls proportionately more at higher levels of k, the $\dot{k} = 0$ curve shifts up more at higher levels of k. See the diagram.

b) The equation describing the dynamics of consumption per unit of effective labor is given by:

(2) $\dfrac{\dot{c}(t)}{c(t)} = \dfrac{f'(k(t)) - \rho - \theta g}{\theta}$

Thus the condition required for $\dot{c} = 0$ is given by f ' (k) = ρ + θg. After the fall in g, f ' (k) must be lower in order for $\dot{c} = 0$. Since f " (k) is negative this means that the k needed for $\dot{c} = 0$ therefore rises. Thus the $\dot{c} = 0$ curve shifts to the right.

c) At the time of the change in g, the value of k -- the stock of capital per unit of effective labor -- is given by the history of the economy, and it cannot change discontinuously. It remains equal to the k* on the old balanced growth path.



In contrast, c -- the rate at which households are consuming in units of effective labor -- can jump at the time of the shock. In order for the economy to reach the new balanced growth path, c must jump at the instant of the change so that the economy is on the new saddle path.

However, we cannot tell whether the new saddle path passes above or below the original point E. Thus we cannot tell whether c jumps up or down and in fact -- if the new saddle path passes right through point E -- c might even remain the same at the instant that g falls. Thereafter, c and k rise gradually to their new balanced-growth-path values; these are higher than their values on the original balanced growth path.