

14) $X_1 \dots X_m \sim P_0(\lambda)$

i) $T_m = \frac{L_m(\lambda_1)}{L_m(\lambda_0)} = \frac{e^{-m\lambda_1} \lambda_1^{\sum x_i}}{e^{-m\lambda_0} \lambda_0^{\sum x_i}} = e^{-m(\lambda_1 - \lambda_0)} \left(\frac{\lambda_1}{\lambda_0}\right)^{\sum x_i} \geq c$

$-m(\lambda_1 - \lambda_0) + \sum x_i \log(\lambda_1/\lambda_0) \geq \log c$ $\lambda_1 > \lambda_0$
 $\log \lambda_1/\lambda_0 > 0$

$\sum x_i \geq [\log c + m(\lambda_1 - \lambda_0)] / \log(\lambda_1/\lambda_0) =: x$

$x: \text{wgl } \alpha \geq P_{H_0}(\sum x_i \geq x) \Rightarrow x = \text{quantil } (1-\alpha) P_0(m\lambda_0)$
 $\sum x_i \stackrel{H_0}{\sim} P_0(m\lambda_0)$

ii) $\sum x_i \leq x \Rightarrow x = \alpha\text{-quantil } P_0(m\lambda_0)$ purvisch

iii) $\hat{\lambda} = \bar{X}$ (69.59) $\sqrt{m}(\bar{X} - \lambda) \xrightarrow{d} N(0, \lambda)$

$\sqrt{m} \frac{\bar{X} - \lambda_0}{\sqrt{\lambda_0}} \stackrel{H_0}{\sim} N(0,1)$ test: $\sqrt{m} |\bar{X} - \lambda_0| \leq \mu_{1-\alpha/2}$

resp $\sqrt{m} \frac{|\bar{X} - \lambda_0|}{\sqrt{\bar{X}}} \geq \mu_{1-\alpha/2}$ resp $\frac{m(\bar{X} - \lambda_0)^2}{\bar{X}} \geq \chi_{1-\alpha}^2$ asympt. test.

iv) $LR_m \Rightarrow \frac{L_m(\bar{X})}{L_m(\lambda_0)} \Rightarrow LR_m = 2[-m(\bar{X} - \lambda_0) + \sum x_i \log(\bar{X}/\lambda_0)] \geq \chi_{1-\alpha}^2$

17) $X_1 \dots X_m \sim N(\mu_x, \sigma_x^2)$

i) MV: $\hat{\mu} = \bar{X}, \hat{\sigma}^2 = \frac{1}{m} \sum (x_i - \bar{X})^2$
 $\ell_m(\hat{\mu}, \hat{\sigma}^2) = c - \frac{m}{2} \log \hat{\sigma}^2 - \frac{1}{2\hat{\sigma}^2} \sum (x_i - \bar{X})^2 = c - \frac{m}{2} \log \hat{\sigma}^2 - \frac{m}{2}$

$m H_0: \mu_x = \mu_0$

$\tilde{\ell}_m(\sigma^2) = c - \frac{m}{2} \log \sigma^2 - \frac{1}{2\sigma^2} \sum (x_i - \mu_0)^2$

$\tilde{U}_m(\sigma^2) = \frac{m}{2\sigma^2} + \frac{\sum (x_i - \mu_0)^2}{2\sigma^4} = 0 \Rightarrow \tilde{\sigma}^2 = \frac{1}{m} \sum (x_i - \mu_0)^2$

$\tilde{\ell}(\tilde{\sigma}^2) = c - \frac{m}{2} \log \tilde{\sigma}^2 - \frac{1}{2\tilde{\sigma}^2} \sum (x_i - \mu_0)^2 = c - \frac{m}{2} \log \tilde{\sigma}^2 - \frac{m}{2}$

$LR_m = 2[-\frac{m}{2} \log \hat{\sigma}^2 + \frac{m}{2} \log \tilde{\sigma}^2] = m \log \left[\frac{\tilde{\sigma}^2}{\hat{\sigma}^2} \right] \geq \chi_{1-\alpha}^2$

t-test: $\sqrt{m}(\bar{X} - \mu) \xrightarrow{d} N(0, \sigma^2)$ $\sqrt{m} \frac{\bar{X} - \mu_0}{\sqrt{S^2}} \stackrel{H_0}{\sim} t_{m-1}$

ii) $m H_0: \sigma_x^2 = \sigma_0^2$

$\tilde{\ell}_m(\mu) = c - \frac{m}{2} \log \sigma_0^2 - \frac{1}{2\sigma_0^2} \sum (x_i - \mu)^2$

$\tilde{U}_m(\mu) = \frac{1}{\sigma_0^2} \sum (x_i - \mu) = 0 \Rightarrow \tilde{\mu} = \frac{1}{m} \sum x_i$

$\tilde{\ell}_m(\tilde{\mu}) = c - \frac{m}{2} \log \sigma_0^2 - \frac{1}{2\sigma_0^2} \sum (x_i - \bar{X})^2$

$LR_m = 2[-\frac{m}{2} \log \hat{\sigma}^2 + \frac{m}{2} \log \sigma_0^2 - \frac{m}{2} + \frac{1}{2\sigma_0^2} \sum (x_i - \bar{X})^2] = m \log \left[\frac{\sigma_0^2}{\hat{\sigma}^2} \right] + m \left[\frac{\hat{\sigma}^2}{\sigma_0^2} - 1 \right] \geq \chi_{1-\alpha}^2$

oproti tomu $\frac{(n-1)S_m^2}{\sigma_0^2} \stackrel{H_0}{\sim} \chi_{n-1}^2$ $\frac{(n-1)S_m^2}{\sigma_0^2} = \frac{\sum (X_i - \bar{X})^2}{\sigma_0^2} = \frac{n \hat{\sigma}^2}{\sigma_0^2}$

iii) na H_0 : $L_m(\hat{\mu}, \hat{\sigma}^2) = c - \frac{n}{2} \log \hat{\sigma}^2 - \frac{1}{2\hat{\sigma}^2} \sum (X_i - \mu_0)^2$
 $LR_m = 2 \left[-\frac{n}{2} (\log \hat{\sigma}^2 - \log \sigma_0^2) - \frac{n}{2} + \frac{1}{2\sigma_0^2} \sum (X_i - \mu_0)^2 \right] = n \log \left[\frac{\sigma_0^2}{\hat{\sigma}^2} \right] - n + \frac{\sum (X_i - \mu_0)^2}{\sigma_0^2}$
 $\geq \chi_2^2(1-\alpha)$

iv) 2 Bn 63: $\Gamma_m \left(\begin{pmatrix} \bar{X} \\ \hat{\sigma}^2 \end{pmatrix} - \begin{pmatrix} \mu \\ \sigma^2 \end{pmatrix} \right) \xrightarrow{d} N_2 \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2 & 0 \\ 0 & 2\sigma^4 \end{pmatrix} \right)$
Anděl Vzh 4.15: $X \sim N_2(\mu, V)$, $h(V) = 2 \Rightarrow (X - \mu)' V^{-1} (X - \mu) \sim \chi_2^2$
 obecně $X \sim N_m(\mu, V)$, $h(V) = r \Rightarrow (X - \mu)' V^{-1} (X - \mu) \sim \chi_r^2$

$V = J^{-1}(\mu, \sigma^2) / m$ $V^{-1} = m J(\mu, \sigma^2)$
 $m \left(\begin{pmatrix} \bar{X} \\ \hat{\sigma}^2 \end{pmatrix} - \begin{pmatrix} \mu_0 \\ \sigma_0^2 \end{pmatrix} \right)' J(\mu_0, \sigma_0^2) \left(\begin{pmatrix} \bar{X} \\ \hat{\sigma}^2 \end{pmatrix} - \begin{pmatrix} \mu_0 \\ \sigma_0^2 \end{pmatrix} \right) \stackrel{H_0}{\sim} \chi_2^2$
 odhad $J(\hat{\mu}, \hat{\sigma}^2) \Rightarrow W_m = m \left[\frac{(\bar{X} - \mu_0)^2}{\hat{\sigma}^2} + \frac{(\hat{\sigma}^2 - \sigma_0^2)^2}{2\hat{\sigma}^4} \right] \geq \chi_2^2(1-\alpha)$

78) $X \sim \text{Mult}(n; p_1, p_2, p_3, p_4)$ $H_0: p_1 = p_2 = p_3 = p_4 = 1/4$ $H_1: \neq H_0$
 $L_m(p) = c p_1^{x_1} p_2^{x_2} p_3^{x_3} p_4^{x_4}$ MV: P. 68 $p_j = \frac{\sum_{i=1}^n X_{ij}}{n}$ $p = \frac{\sum x_i}{n}$
 $L_m(p_0) = c (1/4)^n$
 $LR_m = 2 \left[\sum_{j=1}^4 x_j \log \left(\frac{x_j}{n} \right) - n \log 1/4 \right] = 2 \sum_{j=1}^4 x_j \log \left[\frac{4x_j}{n} \right] \stackrel{H_0}{\sim} \chi_3^2(1-\alpha)$
 $= 324,15 > \chi_3^2(1-\alpha) = 7,8$ namíche H_0

80) $L_m(\alpha, \beta) = \prod_{i=1}^n \frac{\lambda(x_i)^{y_i}}{y_i!} e^{-\lambda(x_i)}$ $f_X(x_i) = \frac{\pi \lambda(x_i)^{y_i}}{y_i!} e^{-\lambda(x_i)}$ $\frac{\partial \lambda}{\partial \alpha} = \lambda$ $\frac{\partial \lambda}{\partial \beta} = \lambda x$
 $L_m(\alpha, \beta) = \sum_{i=1}^n y_i \log \lambda(x_i) - \sum_{i=1}^n \lambda(x_i) + c$ $\lambda(x) = \alpha + \beta x$
 $\frac{\partial L_m}{\partial \alpha} = \sum \frac{y_i}{\lambda(x_i)} - \sum y_i = 0$ $\frac{\partial L_m}{\partial \beta} = \sum \frac{y_i x_i}{\lambda(x_i)} - \sum y_i x_i = 0$ $\frac{\partial \tilde{L}}{\partial \alpha} = \sum y_i - m \tilde{\alpha} = 0$
 $\tilde{L} = \log \tilde{L}$
 $LR_m = 2 \left[\sum y_i \log(\hat{\alpha} + \hat{\beta} x_i) - \sum (\hat{\alpha} + \hat{\beta} x_i) - \sum y_i \log(\tilde{y}) + m \tilde{y} \right] \geq \chi_q^2(1-\alpha)$

$\hat{\alpha} = -1,185$ $\hat{\beta}_1 = 0,51$ $\hat{\beta}_2 = 0,33$
 $\hat{\alpha} = -0,52$ $\hat{\beta}_1 = -0,51$ $\hat{\beta}_2 = 1,87$

$LR_m = 11,21 > \chi_2^2(0,95) = 5,99$ namíche H_0 : $(\beta_1, \beta_2)' = (0, 0)$

$\chi_1^2(1-\alpha)$