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A planning and scheduling problem for an operating theatre using an open scheduling strategy

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Introduction

- Goal is to design a weekly surgery schedule in the operating theatre
- Objectives:
  - Minimize overtime cost in the operating theatre
  - Minimize the unexpected idle time between surgical cases
  - Maximize utilization of the operating rooms
- Constraints:
  - Operating rooms
  - Recovery places
- Application is in Belgian University Hospital
- Problem is solved in two phases

Approach

First phase

- The planning problem is solved to give the date of surgery for each patient
- How do we solve it?
  - Problem is described as a set-partitioning integer-programming model
  - It is solved by a column-generation-based heuristic procedure (CGBH)

Second phase

- The daily scheduling problem determines the sequence of operations
- How do we solve it?
  - Problem is treated as a two-stage hybrid flow-shop problem
  - Solved by a hybrid genetic algorithm (HGA)

Planning strategies

- **Open scheduling strategy**
  - Surgeons can choose any workday for a case
  - Used in our case
- Block scheduling strategy
  - Surgeons or group of surgeons are assigned to a set of time blocks, where they can arrange their surgical cases
- Modified block scheduling strategy
  - Some of the operating room's are reserved while others are left open
  - Unused time blocks are released at some agreed time (e.g. 72h before the surgery)

First phase

- Goal: each surgical case is assigned with a date of a surgery
- Hypothesis:
  - All operating rooms are multifunctional
  - Open scheduling strategy is used
  - Emergency cases are not taken into consideration
  - Surgical case cannot be interrupted
- Approach:
  - Set-partitioning model for the master problem
  - Generation of feasible plans with auxilary problem
  - Column-generation-based heuristic procedure (CGBH)
- Initial set of feasible plans needed $\Rightarrow$ Best Fit Descending algorithm

$$\min_{x_j} \sum_{j \in J} C_j x_j \tag{1}$$

subject to:

$$\sum_{j \in J} a_{ij} x_j = 1, \quad \forall i \in \Omega, D_i \leq N_D \tag{2}$$

$$\sum_{j \in J} a_{ij} x_j \leq 1, \quad \forall i \in \Omega, D_i > N_D \tag{3}$$

$$\sum_{j \in J} b_j^d e_{kj} x_j \leq 1, \quad k \in \{1, \ldots, M\}, d \in \{1, \ldots, N_D\} \tag{4}$$

$$\sum_{j \in J} b_j^d \left(\sum_{i \in \Omega_l} a_{ij} t_i \right) x_j \leq A_l^d, \quad \forall l, d \tag{5}$$

$$x_j \in \{0, 1\}, \quad \forall j \in J \tag{6}$$

Generation of feasible plans with auxiliary problem

- Each column j of master problem corresponds to a feasible plan for one operating room in one day
- Parameters a_{ij}, b_j^d and e_{kj} must respect the following constrains:

$$\sum_{i \in \Omega} a_{ij} t_i \leq \sum_{d=1}^{N_D} \sum_{k=1}^{M} b_j^d e_{kj} (R_k^d + S_k^d), \quad \forall j \in J \tag{8}$$

$$\sum_{d=1}^{N_D} \sum_{k=1}^{M} b_j^d e_{kj} = 1, \quad \forall j \in J \tag{9}$$

$$\sum_{d=\hat{D}}^{N_D} \sum_{k=1}^{M} b_j^d e_{kj} = 0, \text{ if } N_D > \hat{D} = \min(a_{ij} D_i | i \in \Omega), \quad \forall j \in J \tag{10}$$

Auxiliary Problem Components

Column j: $A_j = \left(a_{1j}, \ldots, a_{|\Omega|j}, b_j^1 e_{1j}, \ldots, b_j^{N_D} e_{Mj}, b_j^1 \sum_{i \in \Omega_{N_S^1}} a_{ij}, \ldots, b_j^{N_D} \sum_{i \in \Omega_{N_S^{N_D}}} a_{ij} \right)^T$

Reduced cost: $\sigma_j = C_j - \sum_{i \in \Omega} a_{ij} \pi I_i - \sum_{d=1}^{N_D} \sum_{k=1}^{M} b_j^d e_{kj} \pi II_k^d - \sum_{d=1}^{N_D} \sum_{l=1}^{N_S} b_j^d e_{kj} \left(\sum_{i \in \Omega_l} a_{ij} t_i \right) \pi III_l^d$

Dual variables:

$\pi I_i (i \in \Omega, D_i \leq N_D)$ corresponds to the part $\left(a_{1j}, \ldots, a_{|\Omega|j} \right)$

$\pi II_k^d (d = 1, \ldots, N_D, k = 1, \ldots, M)$ corresponds to the part $\left(b_j^1 e_{1j}, \ldots, b_j^{N_D} e_{Mj} \right)$

$\pi III_l^d (l = 1, \ldots, N_S^d, d = 1, \ldots, N_D)$ corresponds to the last part of the column j

Column-Generation-Based Heuristic (CGBH)

- Two-phase hybrid approach:
  - LP relaxation + column generation
  - Integer feasibility restoration
- Combines:
  - Exact method (column generation)
  - Heuristic (MaxXMinc rule)

CGBH Algorithm Steps

Step 0: Initialize with master problem as current problem

Step 1: Solve linear relaxation of the current problem by an explicit Column-Generation procedure

- If infeasible $\Rightarrow$ no feasible plan by this procedure

Step 2: Check solution:

- If solution respects all integer constraints:
  - First iteration $\Rightarrow$ optimal solution found
  - Later iteration $\Rightarrow$ acceptable solution found

Step 3: If solution violates integer constraints:

- Select one plan using MaxXMinC criterion (x_j)

Step 4: Remove from current problem:

- Surgical cases from selected plan
- Operating room from selected plan

CBGH Algorithm Steps

Step 5: When all surgical cases have been assigned or all operating rooms have been planned, procedure is complete

- If all those surgical cases with deadline earlier than the end of the planning duration assigned $\Rightarrow$ feasible solution
- If not $\Rightarrow$ no feasible solution obtained
- If there are still some unassigned cases and unallocated rooms, the reduced problem becomes the current problem $\Rightarrow$ continue with Step 1

Second phase

- Goal: For each surgical case decide the room and the time of surgery while we already know the date
- Hypothesis:
  - No surgeon can operate on more than one patient at the same time
  - No recovery bed can be occupied by more than one patient at the same time
  - All operating rooms open simultaneously and all recovery beds are empty at the beginning
  - All the scheduled patients are ready for their surgery on the given day
  - Once started, an operation cannot be interrupted until it is finished, same with recovery beds
  - The induction for each operation and the clean-up time before leaving the operating room are included in operating time
  - Recovery time can be shared between operating room and recovery room

Second phase

- Approach: Hybrid genetic algorithm
- N patients are ready to enter the operating theatre
- Operating theatre constiting of M_1 multifunctional operating rooms and M_2 recovery places
- the operation duration of patient i is pre-estimated as $t_i^{(1)}$
- the recovery duration of patient i is pre-estimated as $t_i^{(2)}$
- Aim is to minimize the daily operating cost:

$$f = \omega C_{max}^{(1)} + C_{max}^{(2)}$$

- More methods combined in this algorithm (Tabu search procedures)

Genetic Approach

- Constituent = feasible daily surgery schedule
- Algorithm simulates a natural evolution
  - Parents = already existing constituents
  - Children = New constituens
    - Crossover operators: OX crossover, two-point crossover
    - Mutation operators
  - We are trying to create better schedules in every generation

Coding Scheme

- Constituent is coded as an integer, consisting of four parts:
  - V_1 : a vector of size N, recording the order of patients passing through operating rooms
  - V_2 : a vector of size N, containing the indices of the recovery beds in the same order as the patients
  - V_3 : a vector of size $(M_1 - 1)$, indicating the delimitation positions at which the patients in V_1 are assigned to the different operating rooms
  - V_4 : vector of size N, containing the order of patients in the recovery beds

Mutation Operators

- Modify solutions locally to maintain genetic diversity

A. For V_1 (Patient sequence):

- Randomly selects two elements in V_1 and exchange them
- Modify V_4 if necessary

B. For V_2 (Bed assignment):

- Select two elements with different values in V_2 and exchange them
- Modify V_4 if necessary

C. For V_3 (OR partitioning):

- Randomly generate an integer $k \in \{1, \ldots, M_1\}$
- Delete the kth element in V_3 and make a vector of size $M_1 - 2$
- Randomly generate integer $j \in \{1, \ldots, N+1\}$ and insert it into V_3 so that the elements of V_3 are in ascending order of size
- Modify V_4 if necessary

Hybrid genetic algorithm steps

Step 1: Construct an initial population

- Given that patients are sorted randomly or in ascending order of their indices, they will be first successively scheduled into a random-chosen operating rooms available on the given day
- They are re-sorted by their completion time in ascending order
- Afterwards each of them is scheduled into a randomly chosen place in the recovery room

Step 2: Calculate the fitness value of constituent s of the current population as

$$F(S) = \sqrt{\frac{f_w}{f_s}}$$

Step 3: Select two constituents by applying a roulette-wheel selection and use them as parents in following steps

Hybrid genetic algorithm steps

Step 4: Recombine the two parents selected in Step 3

- Generate two new constituents (children)
- One of two crossover operators is randomly applied

Step 5: Randomly select of of the children generated in Step 4 and then mutate it

- One of three mutation operators is randomly applied

Step 6: Tabu search procedure is used for another child obtained in Step 4 to generate another new constituent

Hybrid genetic algorithm steps

Step 7: Select constituents in the current generation, according to their fitness value, in order to regenerate the initial population

- Elitism mechanism is applied to ensure that the constituent with the best fitness value will be always selected

Step 8: Return to Step 2 and repeat the procedure until one of the termination condition is satisfied

Experimental Dataset

- 1st November 2006 to 31st October 2007 (52 weeks)
- The University Hospital of Amrboise Paré; nine surgical specialties
- In practice, a variation of block scheduling strategy is used
- Operating theatre consists of:
  - 6 operating rooms (8:00 a.m. to 4:00 p.m.)
  - One recovery room with 10 recovery places
- Experiments are based on 6321 records
- The data mainly consists of:
  - Date of the surgery, induction time, the start and the end of a surgery
  - Time of the patient‘s leaving operating room, surgeon, specialty and personnel info

Experimental Dataset

- After eliminating the urgent cases and the surgical cases with incomplete data, finally we have 5427 records from 49 weeks
- Number of surgical cases in one week varies from 53 to 131
- In the experiment a operation deadline is set as its surgery date
- No data about recovery time of each patient
  - Assume that recovery time is generated from lognormal distribution
  - ranges from 30 to 60 minutes, with the mean equal to the operation duration minus 10 minutes, standard deviation of 15 minutes
- If one surgeon is assigned to 1 day, he is supposed to be available for the whole day

Performance Indicators

- **Occupancy Rate of Operating Room (OROR)**
  - ratio of the number of those operating rooms with at least one surgical case assigned in to the total number of operating rooms available in 1 week
- **Utilisation Rate of Operating Rooms (UROR)**
  - ratio of the number of open hours that are occupied by patients in the operating rooms with at least one surgical case assigned in to their regular open hours in 1 week
- **Percentage of Scheduled Patients (PPS)**
  - ratio of the number of patients scheduled into the involved week to the number of patients awaiting the assignments in 1 week
- **Overtime (OT)** = Total number of overtime hours in one week
- **Idle Time (IT)** = Total number of idle hours during the regular open period of operating rooms in 1 week

Results (53 cases)

Metric	Actual	CGBH
Idle Time (min)	114.44	5.71
OR Utilization (%)	89.69	84.97
Overtime (min)	50.00	3.57
OR Occupancy (%)	36.67	23.33
Scheduled Patients (%)	100	100
Operating Cost	46,422.5	32,794.5

Scenario: 53 surgical cases, 13 surgeons

Results (101 Cases)

Metric	Actual	CGBH
Idle Time (min)	71.17	6.58
OR Utilization (%)	89.90	111.90
Overtime (min)	40.22	90.26
OR Occupancy (%)	73.33	83.33
Scheduled Patients (%)	91.09	100
Operating Cost	114,399.5	112,342.5

Scenario: surgical cases, 27 surgeons

Results (131 Cases)

Metric	Actual	CGBH
Idle Time (min)	73.63	2.50
OR Utilization (%)	95.20	107.86
Overtime (min)	39.37	54.32
OR Occupancy (%)	90.00	73.33
Scheduled Patients (%)	98.47	100
Operating Cost	138,194.2	127,406.0

Scenario: 131 surgical cases, 31 surgeons

Conclusion

- Two phase approach
- Heuristic approach in both parts
- **Schedules 100% patients** vs 91-98% manually
- **Cost reduction** up to 29%

Open scheduling improves OR efficiency

Reference

Fei, H., Meskens, N., & Chu, C. (2010). **A planning and scheduling problem for an operating theatre using an open scheduling strategy**. *Computers & Industrial Engineering*, 58(2), 221-230.