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Safe Dike Heights at Minimal Costs: An Integer Programming Approach

2025

Motivation

- Almost 60% of the Netherlands is under threat of flooding from sea, rivers, or lakes.
- More than 3,500 km of dikes and dunes protect the country.
- Annual maintenance and investment costs exceed €1 billion.
- The heightening of the dike is essential, but must be cost-effective.

Flood Safety Standards in the Netherlands

Figure: Flood safety standards in the Netherlands: protection ranges from 1-in-1,250 inland to 1-in-10,000 per year along the coast. © 2009 Verkeer en Waterstaat [3]

Historical and Policy Background

- **1953 flood:** 1835 deaths, 67 dike breaches, 10% GDP loss $\rightarrow$ formation of the Delta Committee.
- **Van Dantzig (1956):** Introduced first economic model for optimal dike height.
- Resulted in statutory flood safety standards based on dike rings (up to 1 in 10,000 per year).
- **1995 high water levels:** 200,000 evacuated renewed interest in economic flood protection.
- **Delta Program (2012):** Re-assessment of safety levels due to rising risks and values.
- **New CBA:** Based on the Brekelmans et al. (2012) model adopted as legal standard in 2016.

Previous Approaches

- **Brekelmans et al. (2012)** — *Safe Dike Heights at Minimal Costs: The Nonhomogeneous Case*
  - Non-homogeneous case
  - Mixed-Integer Nonlinear Programming (MINLP) approach
- **Eijgenraam et al. (2016)** — *Optimal Strategies for Flood Prevention*
  - Homogeneous case
  - Derived explicit expression for global solution

Research Objectives

- Develop an integer linear programming model to determine optimal dike heightening strategies.
- Perform a cost-benefit analysis that minimizes the total expected costs (investment + expected flood damage).
- Allow for greater flexibility in input parameters
- Provide an efficient, easy-to-implement algorithm usable in practice.
- Compare the model with the approach of Brekelmans et al. (2012).

Common Formulas for Flooding and Damage Costs

- **Flood probability:**

$$P_t = P_0 e^{\beta \alpha t} e^{-\beta (h_t - h_0)}$$

- **Damage:**

$$V_t = V_0 e^{\gamma t} e^{\eta (h_t - h_0)}$$

- **Investment costs:**

$$I_t = \begin{cases} (a + b(h_t - h_{t-1}))e^{\lambda (h_t - h_0)} & \text{if } h_t > h_{t-1} \\ 0 & \text{if } h_t = h_{t-1} \end{cases}$$

Model Specifications and Notation

- **Non-homogeneous dike rings:** Each dike ring consists of multiple segments with potentially different characteristics. Let L be the set of segments.
- **Flood probability:** Determined by the weakest segment:

$$P_t^{\text{dike}} = \max_{l \in L} P_t^l$$

- **Discretization:**
  - $T = \{0, 1, \ldots, |T| - 1\}$ – discrete time periods.
  - $H = \{0, 1, \ldots, |H| - 1\}$ – discrete safety levels.
  - $H^l \subseteq H$ – allowed safety levels for segment $l \in L$.

Decision Variables and Parameters

Decision Variables:

- $\mathrm{CY}(t, l, h_1, h_2) = 1$: Segment l is upgraded from level h_1 to h_2 at time t; 0 otherwise.
- $\mathrm{DY}(t, l, h) = 1$: Segment l at level h is the weakest (determines flood) at time t; 0 otherwise.

Input Parameters:

- $\mathrm{cost}(t, l, h_1, h_2)$: Cost for upgrading (or maintaining) segment l at time t.
- $\mathrm{prob}(t, l, h)$: Flood probability for segment l at level h and time t.
- $\mathrm{damage}(t, l, h)$: Expected damage in case of flooding at time t.

$$\min \sum_{t\in T}\sum_{l\in L}\sum_{h_1,h_2\in H^l} \text{cost}(t,l,h_1,h_2)\cdot CY(t,l,h_1,h_2) + \sum_{t\in T}\sum_{l\in L}\sum_{h\in H^l} \text{prob}(t,l,h)\cdot \text{damage}(t,l,h)\cdot DY(t,l,h) \tag{2}$$

$$CY(0,l,0,0)=1;\quad CY(0,l,h_1,h_2)=0\quad \forall l\in L,\ h_1>0,\ h_2\geq h_1 \tag{3}$$

$$\sum_{h_1\in H^l h_1\leq h_2} CY(t-1,l,h_1,h_2) = \sum_{h_3\in H^l : h_3\geq h_2} CY(t,l,h_2,h_3) \quad \forall t\in T\setminus\{0\},\ l\in L,\ h_2\in H^l \tag{4}$$

Optimization Model (Model C) (Continuation)

$$\sum_{h_1 \in H^{l_0}} \sum_{\substack{h_2 \geq h_1 \\ \text{prob}(t,l_0,h_2) > \text{prob}(t,l^*,h^*)}} CY(t, l_0, h_1, h_2) + \sum_{l \in L} \sum_{\substack{h \in H^l \\ \text{prob}(t,l,h) \leq \text{prob}(t,l^*,h^*)}} DY(t, l, h) \leq 1$$

$$\forall t \in T,\ l_0, l^* \in L,\ h^* \in H^{l^*} \tag{5}$$

$$\sum_{l \in L} \sum_{h \in H^l} DY(t, l, h) = 1 \quad \forall t \in T \tag{6}$$

$$CY(t, l, h_1, h_2) \in \{0, 1\} \quad \forall t \in T,\ l \in L,\ h_1, h_2 \in H^l,\ h_2 \geq h_1 \tag{7}$$

$$DY(t, l, h) \in \{0, 1\} \quad \forall t \in T,\ l \in L,\ h \in H^l \tag{8}$$

Additional constraints:

$$\sum_{t^*=t+1,\ldots,t+\text{up}(l)} \sum_{h_1 \in H^l} \sum_{\substack{h_2 \in H^l \\ h_2 > h_1}} CY(t^*, l, h_1, h_2) \leq 1 \quad \forall l \in L,\ t = 0, \ldots, |T| - \text{up}(l) \tag{9}$$

Discretization Schemes

- Heightenings:
  - In steps of 10 centimeters up to 100 cm,
  - In steps of 20 centimeters from 100 to 200 cm,
  - In steps of 30 centimeters for heights above 200 cm.
- Time periods (planning horizon is 300 years):
  - 5-year periods are used up to the year 2100,
  - 10-year periods are used after 2100.
- Empirically tested: the optimal solution hardly depends on the discretization scheme.

Preprocessing Techniques

- **Technique 1:** Is identical heightening one period later always better?
- **Technique 2:** Late heightening not optimal?
- **Technique 3:** Is heightening in two steps ever better than in one step?

Technique 1

A heightening at time t can be postponed to time $t+1$ if it improves or maintains the objective value.

Additional assumption:

$$\text{cost}(t, l, h_1, h_2) + \text{cost}(t+1, l, h_2, h_3) \geq \text{cost}(t, l, h_1, h_3) \quad \forall t \in T,\ l \in L,\ h_1 < h_2 < h_3 \in H^l$$

Objective difference:

$$\text{MxChngT}(t, l, h_1, h_2) = \text{cost}(t, l, h_1, h_2) - \text{cost}(t+1, l, h_1, h_2)$$

$$+ (\text{prob}(t, l, h_2) - \text{prob}(t, l, h_1)) \cdot \text{damage}(t)$$

$$+ \text{cost}(t+1, l, h_2, h_2) - \text{cost}(t, l, h_1, h_1)$$

If $\text{MxChngT}(t, l, h_1, h_2) \geq 0$, then variable $CY(t, l, h_1, h_2)$ can be removed.

Technique 2

This technique searches for heightenings of a segment in one of the latest time periods that are not efficient: the total costs of the heightening (including maintenance and flooding costs) are larger than the total costs of not heightening.

For a specific time period t^* and segment l, this happens for a heightening from h_1 to $h_2 > h_1$ if:

$$\text{cost}(t^*, l, h_1, h_2) + \sum_{t > t^*} \text{cost}(t, l, h_2, h_2) + \sum_{t \geq t^*} \text{prob}(t, l, h_2) \cdot \text{damage}(t)$$

$$> \sum_{t \geq t^*} \text{cost}(t, l, h_1, h_1) + \sum_{t \geq t^*} \text{prob}(t, l, h_1) \cdot \text{damage}(t)$$

If the condition also holds for all later periods $t > t^*$ and all possible heightenings $h_1, h_2 \in H^l$ with $h_2 > h_1$, then we can remove all such variables:

$$CY(t^*, l, h_1, h_2) := 0.$$

Technique 3: Two-Step Heightening More Efficient

This technique identifies large heightenings that are so expensive it is always better to heighten a segment in two consecutive steps instead of one.

Let $t_1, t_2 \in T$, with $t_1 < t_2$, $l \in L$, and $h_1 < h_2 < h_3 \in H^l$.
Let also mxh_l be the height with the lowest flood probability for segment l.

If:

$$\begin{aligned} &\mathrm{cost}(t_1, l, h_1, h_3) + \sum_{t_2 \geq t > t_1} \mathrm{cost}(t, l, h_3, h_3) \\ &- \mathrm{cost}(t_1, l, h_1, h_2) - \sum_{t_2 > t > t_1} \mathrm{cost}(t, l, h_2, h_2) \\ &- \mathrm{cost}(t_2, l, h_2, h_3) \\ &+ \sum_{t_2 > t \geq t_1} [\mathrm{prob}(t, l, \mathrm{mxh}_l) - \mathrm{prob}(t, l, h_2)] \cdot \mathrm{damage}(t) > 0 \end{aligned}$$

Then set $\mathrm{CY}(t_1, l, h_1, h_3) := 0$

Technical Details

- The same data are used as in Brekelmans et al. (2012).
- Computing times are measured on a Windows Server 2003-based computer with Intel Xeon E5-2670 processors.
- Only the CPLEX-based branch-and-cut procedure is used.
- CPLEX is instructed to first branch on the variable `DY`; all other settings remain at their default values.

Branch and Cut Method

1. **Relaxation:** Solve the LP relaxation of the integer program.
   - The LP solution provides a **lower bound** on the objective.
   - Initialize the **upper bound** as ∞ (best known integer solution).
2. **Termination:** If no nodes remain to explore, return the best known integer solution (upper bound).
3. **Branching:** Choose a fractional variable x_i and split into subproblems:

$$x_i \leq \lfloor x_i \rfloor \quad \text{and} \quad x_i \geq \lceil x_i \rceil$$

4. **Bounding and Cutting:** Solve the subproblems:
   - If integer: update the upper bound if it's better.
   - If fractional:
     - **Prune** if infeasible or worse than current upper bound.
     - **Else:** Add valid cutting planes to tighten the LP and continue.

Comparison of models

Table 1
Comparison of MINLP-approach of Brekelmans et al. (2012) and model C.

Dike ring	Number of segments	MINLP approach of Brekelmans et al. (2012)		Model C		Difference in true objective (%)
		MINLP objective (M)	True objective (M)	Model C objective (M)	True objective (M)	
10	4	107.51	107.51	108.26	108.26	0.69
13	4	10.38	10.38	10.33	10.33	–0.48
14	2	94.04	94.04	94.57	94.57	0.56
16	8	1044.45	1046.08	1064.65	1064.80	1.79
17	6	377.05	377.05	380.65	380.66	0.96
21	10	217.40	217.71	221.62	221.62	1.80
22	5	373.98	374.08	378.67	378.68	1.23
36	6	395.65	395.65	395.34	395.34	–0.08
38	3	136.26	136.29	136.75	136.76	0.34
43	8	486.72	488.10	492.66	492.66	0.93
47	2	16.57	16.57	16.63	16.63	0.36
48	3	42.92	42.92	43.37	43.37	1.05

Effect of the preprocessing techniques

Table 2
Effect of the preprocessing techniques on the solution of branch-and-cut algorithm of model C.

Dike ring	$\|L\|$	Total number of variables before pre-processing	Reduction in the number of variables: Technique 1 (%)	Technique 2 (%)	Technique 3 (%)	Total (%)	Solution time (minutes): With pre-processing	Without pre-processing
10	4	42,988	44	0	11	56	0.06	0.12
13	4	42,988	64	0	3	67	0.03	0.11
14	2	21,494	35	0	10	45	0.03	0.04
16	8	85,976	33	0	18	51	0.36	0.55
17	6	64,482	49	0	14	63	0.10	0.27
21	10	107,470	34	0	16	50	0.89	1.34
22	5	53,735	36	0	15	51	0.16	0.50
36	6	64,482	25	0	15	40	0.10	0.12
38	3	32,241	29	0	12	41	0.05	0.08
43	8	85,976	53	1	15	68	0.48	0.70
47	2	21,494	63	1	10	74	0.03	0.03
48	3	32,241	45	0	18	64	0.04	0.07

$|L|$: number of segments.

Anti piping measures

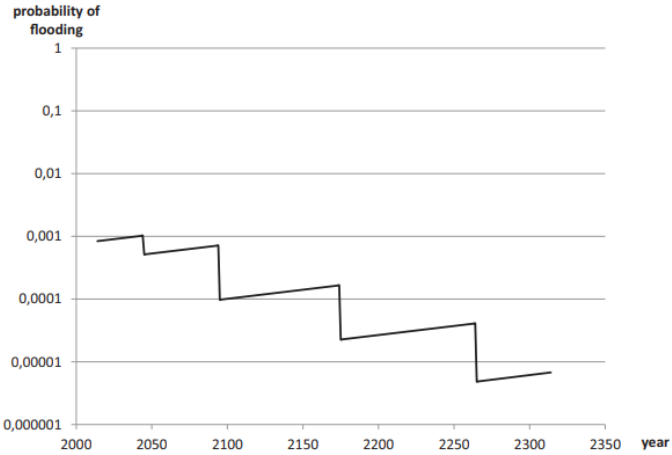

Figure: Flood probabilities for dike ring 10 according to the optimal investment strategy, with the option of constructing anti piping measures only.

Conclusion

- Flexible and efficient model for dike height optimization.
- Proven optimal solutions with fast computation using preprocessing.
- Applicable in both Dutch and international flood risk settings.

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Thank you for your attention!