# Stochastic vs Deterministic Programming in Water Management

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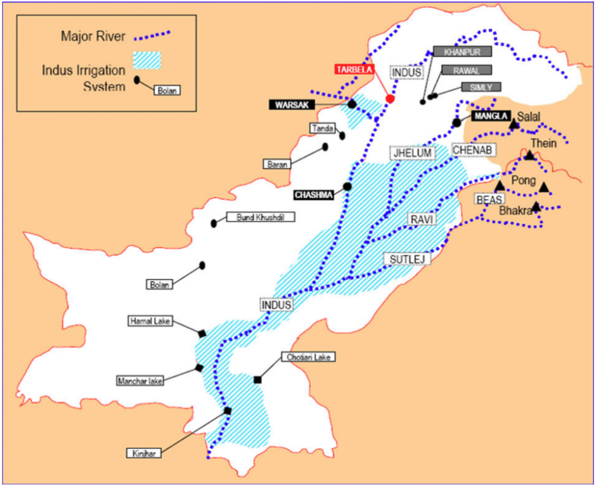
# Outline


1. Introduction
2. Mathematical Model
3. Formulation for IBIS
4. Scenario Generation
5. Modeling conditions
6. Results
7. Conclusion

# Motivation

- Water management in the Indus Basin Irrigation System (IBIS)
- Vast network: 2 major reservoirs, 19 barrages, 44 canals
- Agriculture is vital to Pakistan's economy (90% of food production)

# Motivation




*Indus Irrigation System (Source:WCD, Terbela Dam a Scoping report)*

# Deterministic vs Stochastic Programming

- Deterministic LP assumes perfect knowledge of inputs
- Stochastic LP models uncertainty in inflows and rainfall
- Flexibility improves system efficiency and economic outcomes
- Captures real-world randomness in water availability

# Two-Stage Stochastic Programming

**General form:**

$$\min_{x} \quad c^\top x + \mathbb{E}_\xi[q(\xi)^\top y(\xi)]$$

$$\text{subject to} \quad Ax = b,$$

$$T(\xi)x + Wy(\xi) = h(\xi),$$

$$x \geq 0, \quad y(\xi) \geq 0.$$

# Interpretation in our model

- $x$: area sown for each crop in each canal command (first-stage decision)
- $y(\xi)$: area actually cultivated after rainfall/inflows are realized (second-stage decision)
- $\xi$: random variable that represents the rainfall and water inflow
- $T(\xi), W$: matrices linking sowing and cultivation (identity matrices here)

# Simplified Recourse Constraints

- Second stage constraint:

$$x - y(\xi) - z(\xi) = 0$$

  where $z(\xi)$ = dropped cultivation area.
- Equivalent to:

$$0 \leq y(\xi) \leq x$$

- Only cultivate what has been sown.

# Interpretation in Water Management

- **First stage (here-and-now decisions)**:
  - Decide the area to be sown with each crop before knowing future inflows and rainfall.
- **Second stage (wait-and-see decisions)**:
  - Adjust cultivation areas after actual inflows and rainfall are realized.
  - Some sown areas may be dropped depending on water availability.

# Table: Indexes and Cardinality

| Index | Name | Cardinality |
|---|---|---|
| $c$ | Crop | 13 |
| $i$ | Storage (reservoir) | 2 |
| $k$ | State | 4 |
| $l$ | Canal command | 44 |
| $n$ | Node | 35 |
| $r$ | River | 7 |
| $s$ | Scenario | 200 |
| $t$ | Time-step | 12 (monthly) or 36 (ten-daily) |
| $z$ | Rainfall zone | 3 |

*Note: Node set includes reservoirs, rim stations, and ordinary nodes.*

# Objective Function

**Maximize Expected Revenue:**

$$\max \left[ \sum_{s \in S} \pi_s \left( \sum_{l \in L} \sum_{c \in C} (p_c \lambda_{cl} - \theta_{cl}) Y_{lc}^s \right) - \sum_{l \in L} \sum_{c \in C} \vartheta_{cl} X_{lc} \right]$$

Where:

- $\pi_s$ = scenario probability
- $p_c$ = crop price, $\lambda_{cl}$ = yield
- $\theta_{cl}$ = labor cost, $\vartheta_{cl}$ = sowing cost

# Constraint: Crop Area Limits

- Land occupation:

$$\sum_{c \in C} a_{clt} \leq \text{Land}_l \quad \forall l \in L, t \in T$$

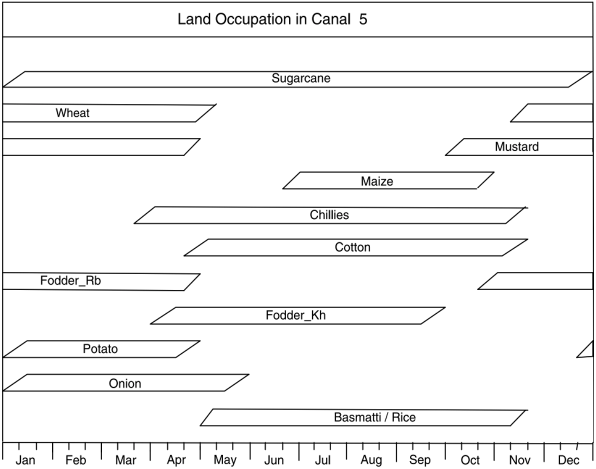
- $$X_{lc} - Y_{lc}^{s} \geq 0 \quad \forall l \in L, c \in C, s \in S$$

- Maximum permitted crop area by zone:

$$\sum_{l:z(l)=z} X_{lc} \leq \text{CropArea}_{zc} \quad \forall z \in Z, c \in C$$

(represents agronomic and policy limits)

# Land occupation




*Land occupation* $a_{clt}$

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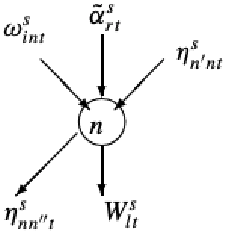
# Constraint: System network




*System network*

$$\sum_{n':(n',n)\in\mathcal{N}} \eta^s_{n'nt} + \sum_{i:(i,n)\in\mathcal{N}} \omega^s_{int} + \tilde{\alpha}^s_{r(n)t}$$

$$- \sum_{n'':(n,n'')\in\mathcal{N}} \eta^s_{nn''t} - \sum_{l:n(l)=n} W^s_{lt} \geq 0, \quad \forall n \in N,\ t \in T,\ s \in S$$

# Constraint: Reservoir Storage Dynamics

- Water linkage:

$$\Delta^s_{i(t-1)} + \tilde{\alpha}^s_{r(i)t} - \Delta^s_{it} - \sum_{(i,n)\in N} \omega^s_{int} - \epsilon_{it} = 0; \quad \forall i \in N_S, t \in T, s \in S$$

- Water consumption balance:

$$\delta_l W^s_{lt} + \tau_{lt} - \sum_{c\in C} \max(v_{clt} - \tilde{\sigma}^s_{z(l)t}, 0) Y^s_{lt} \geq 0; \quad \forall l \in L, t \in T, s \in S$$

- Water availability

$$\sum_{r:n\in NR} \tilde{\alpha}^s_{r(n)t} + \sum_{i:(i,n)\in N} \sum_{n:(i,n)\in N} \omega^s_{int} - \sum_{l\in L} W^s_{lt} \geq 0; \quad \forall t \in T, s \in S$$

# Constraint: Flood Aversion and Political Water Sharing

- Flood aversion:

$$\sum_{r:n\in NR} \tilde{\alpha}_s^{r(n)t} + \sum_{i:(i,n)\in N} \sum_{n:(i,n)\in N} \omega_s^{int} - \sum_{l\in L} W_{lt}^s \leq H; \quad \forall t \in T, s \in S$$

- Political water sharing:

$$\sum_{l:k(l)=k} \sum_{t\in T} W_{lt}^s = \text{Share}_k \sum_{l\in L} \sum_{t\in T} W_{lt}^s \quad \forall k \in K, s \in S$$

# Constraint: Labor Availability

- Labor constraint per rainfall zone:

$$\sum_{l:z(l)=z} \sum_{c \in C} \beta_{clt} Y_{lc}^{s} \leq \text{Labor}_z \quad \forall z \in Z, t \in T, s \in S$$

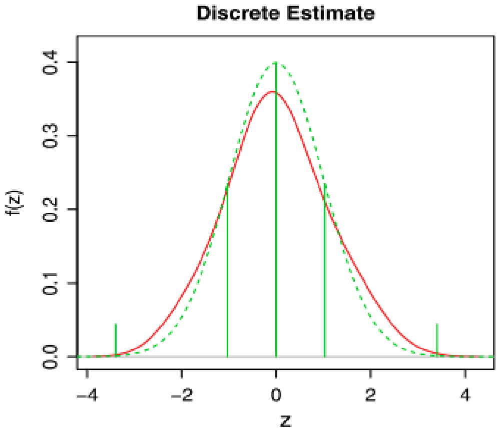
- Non negativity

$$X_{lc}, Y_{lc}^{s}, \Delta_{it}^{s}, W_{lt}^{s} \geq 0$$

# Scenario Generation

- Random parameters: inflows (2 seasons), rainfall (3 zones)
- Historical data fitted to Normal distributions
- $$d_1(G, \tilde{G}) = \sup\left\{ \int f(u)\, dG(u) - \int f(u)\, d\tilde{G}(u) : L_1(f) \leq 1 \right\}$$

# Scenario Generation




*Dicrete estimate for inflow distribution*

# Scenario Generation

- If G is the standard normal N(0, 1) distribution, the optimal location of approximating mass points are -3.4, -1.029, 0, 1.029, 3.4
- If $\tilde{G}$ sits on two points, the optimal values are -0.7979, 0.7979
- Discretized into 5 (inflow) and 2 (rainfall) support points
- Total scenarios: $5 \times 5 \times 2 \times 2 \times 2 = 200$

# Models

- Monthly horizon, single-stage deterministic model (average inflow and rainfall).
- Monthly horizon, two-stage stochastic model with 200 scenarios.
- Ten-daily horizon, two-stage stochastic model with 200 scenarios.
- Solved using dual-simplex algorithm

# Model dimensions

| Model | Rows | Columns | Non-zeros | Memory Used |
|---|---|---|---|---|
| Monthly deterministic | 2,773 | 2,437 | 20,405 | <1 MB |
| Monthly stochastic | 572,374 | 487,861 | 3,080,807 | 253 MB |
| Ten-daily stochastic | 767,830 | 1,191,061 | 7,751,040 | 611 MB |

Table: Model statistics summary

# Cropping Patterns and Revenue

- Deterministic: 5.318 bn USD
- Monthly Stochastic: 6.130 bn USD
- Ten-daily Stochastic: 7.175 bn USD

# Cropping Policy and Revenue

| Crop | Deterministic | Stochastic (monthly) | Stochastic (ten-daily) | Actual (2003–2004) |
|---|---|---|---|---|
| Basmatti | 577 | 697 | 519 | – |
| Irri | 758 | 758 | 838 | 2503 |
| Maize | 81 | 81 | 87 | 896 |
| Mustard | 310 | 310 | 331 | 244 |
| Sugarcane | 924 | 924 | 1026 | 947 |
| Fodder-kharif | 1313 | 1313 | 1403 | Not available |
| Fodder-rabi | 1352 | 1352 | 1445 | Not available |
| Cotton | 2881 | 2881 | 3028 | 3221 |
| Gram | 880 | 880 | 468 | 1038 |
| Wheat | 2296 | 3624 | 4403 | 8330 |
| Potato | 32 | 32 | 77 | 111 |
| Onion | 56 | 56 | 60 | 122 |
| Chilies | 78 | 69 | 85 | 39 |
| **Total Area** | **11538** | **12977** | **13770** | – |
| **Exp. Revenue (bn USD)** | **5.318** | **6.130** | **7.175** | Not available |

# The Value of Stochastic Solution (VSS)

- **VSS** = RP − EEV
- Expected result with perfect recourse (RP): 6.130 bn USD
- Expected result under deterministic solution (EEV): 5.473 bn USD
- **VSS** = 0.657 bn USD

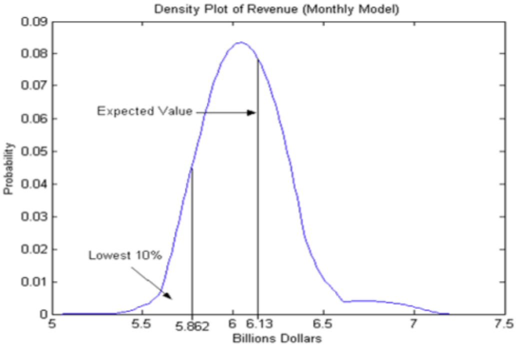
# Expected Value of Perfect Information (EVPI)

- **EVPI** = PI − RP
- Perfect Information (PI): 6.241 bn USD
- Recourse Problem (RP): 6.130 bn USD
- **EVPI** = 0.111 bn USD

# Storage and Power Generation

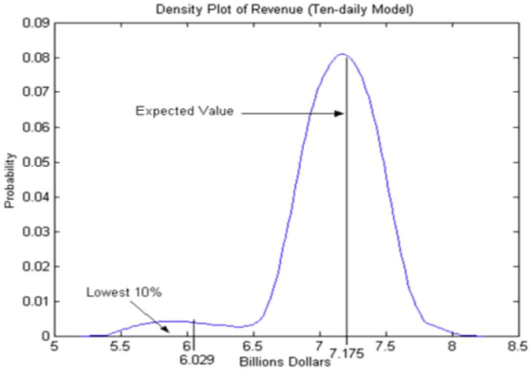
- Storage levels maintained within min-max bounds
- Power generation (hydro): varies between 10,316 to 29,352 GWH
- Ten-daily model allows better adjustment to extreme inflow scenarios

# Revenue plot




*Monthly model*

# Revenue plot




*Ten-daily model*

# Summary

- Stochastic programming significantly improves revenues
- Shorter time horizons (ten-daily) add even more flexibility
- Captures uncertainty in inflow and rainfall efficiently
- Scenario-based stochastic optimization is crucial in water management

Thank you for your attention