

QUANTUM INFORMATION

MFJ UK

- (0) Compute the dot product of vectors $|2\rangle$ and $|3\rangle$ from $\mathbb{H}_{16} \cong \mathbb{H}_2^{\otimes 4}$.
- (1) Let M be an observable and $|\varphi\rangle$ be a state. Show that $\mathbb{E}(M)$ on $|\varphi\rangle$ can indeed be computed as $\langle\varphi|M|\varphi\rangle$.

Recall the Pauli matrices

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

and the Hadamard matrix

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

- (3) Decide which of the above matrices represents observable values. For those which are, compute their expected measurement value on states $|0\rangle, |1\rangle, |+\rangle$ and $|-\rangle$.
- (4) Show that the operator $P_{|\varphi\rangle} = |\varphi\rangle\langle\varphi|$ is indeed the projection operator onto the linear space generated by $|\varphi\rangle$.
- (5) Show that for unitary operators A and B their tensor product $A \otimes B$ is also unitary. Is it true that the tensor product of observables is observable?
- (6) Recall that for matrices A and B of shapes (m, n) and (p, q) , respectively, their tensor product $A \otimes B$ is defined as the matrix of shape (mp, nq) equal to $(a_{i,j}B)_{i,j}$. Show that such a definition corresponds to the abstract definition of the tensor product of two linear operators (don't forget to pick the correct basis).
- (7) Let H_2 be $H \otimes H$. Compute H_2 and then write it as a linear combination of projection operators (a.k.a. *spectral decomposition*).
- (8) Let $|\phi\rangle$ be first measured in basis $\{|0\rangle, |1\rangle\}$, then in basis $\{|+\rangle, |-\rangle\}$ and again in basis $\{|0\rangle, |1\rangle\}$. Compute the probability of the last measurement being equal to $|1\rangle$. Compute the conditional probability of the last measurement being equal to $|1\rangle$ assuming the first measurement equals $|1\rangle$. How do these numbers change if we drop the middle measurement and leave only the first and the last measurements?