Logic of Questions I

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Inquisitive Semantics

- a framework for a logical analysis of questions
- developed around 2009 by Jeroen Groenendijk, Floris Roelofsen, and Ivano Ciardelli
- https://projects.illc.uva.nl/inquisitivesemantics/

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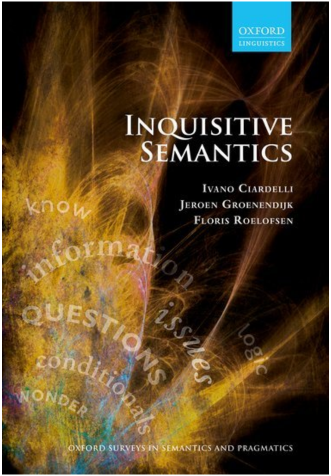
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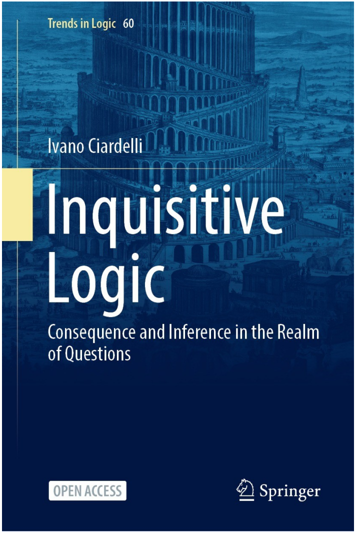
Trends in Logic 60
Ivano Ciardelli
Inquisitive Logic
Consequence and Inference in the Realm of Questions
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The basic logical picture

- logic is concerned with (formal) validity of arguments
- an argument is a structure

  $$A_1, \ldots, A_n / B$$

  where $A_1, \ldots, A_n, B$ are declarative sentences

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- Buy bread and butter! / Buy bread!
- Invite all Peter's friends!, John is a Peter's friend. / Invite John!
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Making sense of arguments with questions I
Inferential Erotetic Logic

P1 Mary is Peter's mother.

P2 If Mary is Peter's mother, then John is Peter's father or George is Peter's father.

C Who is Peter's father: John or George?

Making sense of arguments with questions II
Inquisitive Logic

$S1, S2$ are statements and $Q1, Q2$ questions

an argument	its intended interpretation
$S1/S2$	$S1$ implies $S2$
$Q1/S2$	$Q1$ presupposes $S2$
$S1/Q2$	$S1$ resolves $Q2$
$Q1/Q2$	any information that resolves $Q1$ resolves also $Q2$

Examples

(a) The statements *if Mary is Peter's mother, then John is not Peter's father* and *John is Peter's father* together resolve the question *whether Mary is Peter's mother*.

(b) The question *who is Peter's father: John or George?* pressuposes that *John or Georg is Peter's father*.

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Difference between IEL and InqL

A Who is Peter's father: John or George?

B John or George is Peter's father.

- According to InqL A entails B but B does not entail A
- According to IEL B entails A but A does not entail B

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More complex examples

Assume that *if today is not Monday and Mary is in the pub, then John is in the library* and *if John is in the library and Mary is not in the pub, then it is Monday*. Then any information that resolves the question *whether John is in the library* resolves also the conditional question *whether Mary is in the pub if it is not Monday*.

$(\neg m \wedge p) \rightarrow l, (l \wedge \neg p) \rightarrow m, ?l / \neg m \rightarrow ?p$

More complex examples

Any information that resolves the conditional questions *whether John is in the library, if Mary is in the pub* and *whether Mary is in the pub, if it is not Monday*, resolves also the question *whether John is in the library, if it is not Monday*.

$p \to ?l, \neg m \to ?p / \neg m \to ?l$

Formalization via inquisitive disjunction

- disjunctive questions: *whether A or B*.
- polar questions: *whether A* = *whether A or not A*.

A nonstandard propositional language

- InqB is a logic for a basic propositional language with one additional operator: inquisitive disjunction $\mathbb{V}$;
- $\mathbb{V}$ is a question-generating operator: $\varphi \mathbb{V} \psi$ is interpreted as the question *whether* φ *or* ψ;
- $?\varphi =_{def} \varphi \mathbb{V} \neg\varphi$ (the question *whether* φ)

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The standard language of propositional (intuitionistic) logic

$$\varphi := p \mid \bot \mid \varphi \to \varphi \mid \varphi \wedge \varphi \mid \varphi \mathbin{\rotatebox[origin=c]{180}{$\mathbb{\wedge}$}} \varphi.$$

$$\neg\varphi =_{def} \varphi \to \bot$$

$$\varphi \vee \psi =_{def} \neg(\neg\varphi \wedge \neg\psi)$$

$$\varphi \leftrightarrow \psi =_{def} (\varphi \to \psi) \wedge (\psi \to \varphi)$$

$$?\varphi =_{def} \varphi \mathbin{\rotatebox[origin=c]{180}{$\mathbb{\wedge}$}} \neg\varphi$$

$$!\varphi =_{def} \neg\neg\varphi$$

Formalization

(a) Valid: The statements *if today is Monday then it is not Tuesday* and *today is Tuesday* together resolve the question *whether today is Monday*.

$m \to \neg t, t / ?m$,

(i.e. $m \to \neg t, t / m \vee\!\!\!\vee \neg m$)

Formalization

(b) Invalid: Any information that resolves the (conditional) question *whether John will go swimming today, if it is Monday* resolves also the (unconditional) question *whether John will go swimming today*.

$m \rightarrow ?s / ?s$

(i.e. $m \rightarrow (s \vee\!\!\!\vee \neg s) / s \vee\!\!\!\vee \neg s$)

Basic Inquisitive Logic InqB

Intuitionistic logic plus

split $(\alpha \to (\psi \mathbin{\vee\!\!\!\vee} \chi)) \to ((\alpha \to \psi) \mathbin{\vee\!\!\!\vee} (\alpha \to \chi))$,

rdn $\neg\neg\alpha \to \alpha$,

where α ranges over $\mathbin{\vee\!\!\!\vee}$-free formulas.

Propositions expressed by questions

Frege claimed in *The Thought* that a statement (like *Mary is drinking beer*) and the corresponding yes-no question (*Is Mary drinking beer?*) have the same content and differ only in something that is not a part of the content itself.

Frege on questions

An interrogative sentence and an indicative one contain the same thought; but the indicative contains something else as well, namely, the assertion. The interrogative sentence contains something more too, namely a request. Therefore two things must be distinguished in an indicative sentence: the content, which it has in common with the corresponding sentence-question, and the assertion.

A problem for Frege's approach

This view seems to be limited to yes-no question.

When we take a disjunctive question (*Is Mary drinking red wine or white wine?*), we cannot identify its content in the same style with the content of any single declarative sentence.

A different approach

Some authors suggest that one can identify the semantic content of a question with the content of a declarative sentence that describes the epistemic presuppositions of the question.

Peliš, M., Majer, O.: Logic of Questions from the Viewpoint of Dynamic Epistemic Logic, in: *The Logica Yearbook 2009*, Peliš, M. (ed.), College Publications, London 2010, pp. 157-172.

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A different approach

According to this view, there is also no substantial difference between declarative and interogative propositions, though we need a rich language, namely a language of a modal epistemic logic, to capture properly the semantic content of questions.

Questions express propositions

In inquisitive semantics questions are regarded as expressing a special kind of propositions.

The meaning of a sentence = its truth conditions

"To understand a proposition means to know what is the case if it is true."

L. Wittgenstein, TLP, 4.024

The sentential meaning of declarative sentences.

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Informative and inquisitive content of sentences

- Inquisitive semantics introduces a richer notion of sentential meaning that is applicable to declarative sentences as well as to questions.
- In inquisitive semantics, sentential meaning is modelled as consisting of an informative part and an inquisitive part.

Informative and inquisitive content of sentences

- Inquisitive semantics introduces a richer notion of sentential meaning that is applicable to declarative sentences as well as to questions.
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Localization of the actual world

- The informative content *info*(A) of a given sentence A can be represented as a set of possible worlds and the sentence provides the information that the actual world is located somewhere in the set.
- The inquisitive content *inq*(A) can be understood as a *request* to locate the actual world more precisely. The request *inq*(A) can be modeled as a set of those nonempty subsets of *info*(A) that contain enough information to settle the request.

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Info(A) can be retrieved from *Inq*(A).

- The request to locate the actual world more precisely in *info*(A) should not a priori exclude any world of *info*(A).
- As a consequence, *info*(A) has to be the union of *inq*(A).

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Propositions as sets of information states

- A proposition is not just a set of possible worlds but a set of sets of possible worlds (i.e. a set of information states).
- Propositions are downward closed.

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Truth-functional semantics for classical logic

A truth-functional model: $\mathcal{M} = \langle W, V \rangle$.

The relation of truth:

- p is true in w iff $w \in V(p)$,
- $\perp$ is not true in w,
- $\alpha \to \beta$ is true in w iff α is not true in w or β is true in w
- $\alpha \wedge \beta$ is true in w iff α is true in w and β is true in w

Inquisitive semantics

An inquisitive model: $\mathcal{N} = \langle \mathcal{P}(W), V \rangle$.

The support relation:

$s \vDash p$ iff $s \subseteq V(p)$,

$s \vDash \bot$ iff $s = \emptyset$,

$s \vDash \neg\varphi$ iff for any nonempty $t \subseteq s$, $t \nvDash \varphi$,

$s \vDash \varphi \to \psi$ iff for any $t \subseteq s$, if $t \vDash \varphi$ then $t \vDash \psi$,

$s \vDash \varphi \wedge \psi$ iff $s \vDash \varphi$ and $s \vDash \psi$,

$s \vDash \varphi \vee\!\!\!\vee\, \psi$ iff $s \vDash \varphi$ or $s \vDash \psi$.

Theorem

In every inquisitive model:

(a) *every formula is supported by the empty state,*
(b) *support is downward persistent for all formulas,*
(c) *support of declarative formulas is closed under arbitrary unions,*
(d) *every formula is equivalent to the inquisitive disjunction of a finite set of declarative formulas.*

Truth-support bridge

- As regards the declarative language the two semantics are equivalent:

  universal truth = universal support
  preservation of truth = preservation of support
- The standard framework is based on ontic objects (possible worlds) and an ontic relation of truth;
- The inquisitive framework is based on informational objects (information states = partial representations of possible worlds) and an informational relation of support.

C. I. Lewis: Implication and the algebra of logic, 1912

One of the important practical uses of implication is the testing of hypotheses whose truth or falsity is problematic. The algebraic [truth-table] implication has no use here. If the hypothesis happens to be false, it implies anything you please... In other words, no proposition could be verified by its logical consequences. If the proposition be false, it has these "consequences" anyway.

Examples

a) Jane is in the cinema.
b) Is Peter in the cinema?
c) Is Jane also in the cinema like Peter?
d) Peter or Jane is in the cinema.
e) Is Peter or Jane in the cinema?
f) Who is in the cinema: Peter or Jane?
g) If Peter is in the cinema, Jane is also there.
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An example due to Ivano Ciardelli

- a certain disease may give rise to two symptoms: S_1, S_2
- hospital's protocol:

  if a patient presents symptom S_2, the treatment is always prescribed; if the patient only presents symptom S_1, the treatment is prescribed just in case the patient is in good physical condition; if not, the risk associated with the treatment outweigh the benefits, and the treatment is not prescribed

A formalization of the protocol

The protocol:

- $t \leftrightarrow s_2 \vee (s_1 \wedge g)$

where

- s_1: the patient has symptom S_1
- s_2: the patient has symptom S_2
- g: the patient is in good physical condtion
- t: the treatment is prescribed

Types of information

Examples of types of information:

- patient's symptoms ($S_1, S_2, \ldots$)
- patient's conditions (good, bad)
- treatment (prescribed, not prescribed)

Types of information

Types of information correspond to questions:

- what are the patient's symptoms: $?s_1 \wedge ?s_2$
- whether the patient is in good physical conditions: $?g$
- whether the treatment is prescribed: $?t$

Dependencies among information types correspond to logical relations among questions

$$t \leftrightarrow s_2 \vee (s_1 \wedge g), ?s_1 \wedge ?s_2, ?g \models ?t$$

The language $\mathcal{L}_{IEL}$

$$\varphi := p \mid \perp \mid \varphi \to \varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid K_a\varphi \mid E_a\varphi$$

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Epistemic modalities

the formula	represents
$K_a p$	The agent *a* knows that *p*.
$K_a ?p$	The agent *a* knows whether *p*.
$E_a ?p$	The agent *a* entertains whether *p*.
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- $K_a(\text{question}) = \text{statement}$
- $E_a(\text{question}) = \text{statement}$
- $W_a(\text{question}) = \text{statement}$

Declarative formulas

Definition

The set of declarative $\mathcal{L}_{IEL}$-formulas is the least set that contains all atomic formulas, $\perp$, $K_a\varphi$ and $E_a\varphi$, for any $\mathcal{L}_{IEL}$-formula φ, and is closed under $\wedge$ and $\rightarrow$.

Models

Definition

A concrete inquisitive epistemic model (CIE-model) is a triple $\langle W, \Sigma_{\mathcal{A}}, V \rangle$, where

- W is a nonempty set of possible worlds
- $\Sigma_{\mathcal{A}} = \{\Sigma_a \mid a \in \mathcal{A}\}$ is a set of inquisitive state maps
- V is a valuation assigning subsets of W to atomic formulas

Inquisitive state maps

- Σ_a assigns to every world w the issue of the agent a in the world w
- every issue is represented by a set of information states (those states that resolve the issue)
- every information state is represented by a set of possible worlds (those worlds that are compatible with the information, i.e. that are not excluded by the information)
- the information state of the agent in a world determines the boundaries for the issue of the agent in the world

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Inquisitive state maps

$\Sigma_a : W \to \mathcal{P}(\mathcal{P}(W))$, $\sigma_a :\to \mathcal{P}(W)$ satisfying:

- $\Sigma_a(w)$ is nonempty downward closed,
- $\sigma_a(w) = \bigcup \Sigma_a(w)$,
- for any $w \in W$, $w \in \sigma_a(w)$ (factivity),
- for any $w, v \in W$, if $v \in \sigma_a(w)$, then $\Sigma_a(v) = \Sigma_a(w)$ (introspection).

Support conditions

- $s \vDash K_a\varphi$ iff $\forall w \in s$: $\sigma_a(w) \vDash \varphi$,
- $s \vDash E_a\varphi$ iff $\forall w \in s\ \forall t \in \Sigma_a(w)$: $t \vDash \varphi$.

Theorem

In every inquisitive epistemic model:

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Axiomatization of IEL

INT	Axioms of intuitionistic logic and modus ponens
split	$(\alpha \to (\varphi \vee\!\!\!\vee \psi)) \to ((\alpha \to \varphi) \vee\!\!\!\vee (\alpha \to \psi))$
rdn	$\neg\neg\alpha \to \alpha$

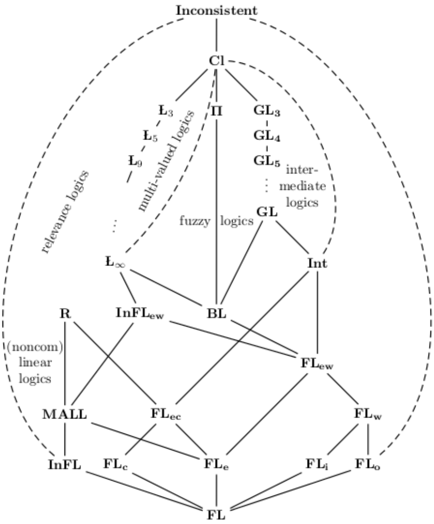
S5	S5-axioms and necessitation for K_a and E_a
K2	$K_a(\varphi \vee\!\!\!\vee \psi) \leftrightarrow (K_a\varphi \vee K_a\psi)$
KE	$E_a\alpha \leftrightarrow K_a\alpha$

(α ranges over declarative formulas)

Is inquisitive logic a non-classical logic?

Two alternative approaches:

- inquisitive logic as a superintuitionistic logic in the standard propositional language
- inquisitive logic as a conservative extension of classical logic in an enriched language



Picture taken from Galatos, N. Jipsen, P. Kowalski, T., Ono, H. (2007) *Residuated Lattices: An Algebraic Glimpse at Substructural Logics*. Elsevier Science.