## QUANTUM INFORMATION

MFF UK

(0) For $g, h \in \mathbb{Z}_{r_{1}} \times \ldots \times \mathbb{Z}_{r_{m}}$ show that $\chi^{-1}(g)=\chi(g)^{*}$ and $\chi_{g}(h)=\chi_{h}(g)$.
(1) Show that DFT and $\mathrm{DFT}^{-1}$ are complex conjugate.
(2) Describe the canonical basis and the basis of characters of the space of functions from $G$ to $\mathbb{C}$, where:

- $G=\mathbb{Z}_{2} \times \mathbb{Z}_{3}$,
- $G=\mathbb{Z}_{9}$,
- $G=\mathbb{Z}_{3} \times \mathbb{Z}_{3}$.

Compute DFT and IFT matrices for all the cases described above.
(3) Are there some special relations between DFT and IFT matrices for $\mathbb{Z}_{2}^{m}$ ?
(4) Construct a circuit computing IFT for $\mathbb{Z}_{8}$ explicitly.

