QUANTUM INFORMATION

MFF UK

(0) Compute inner product of the vectors $|2\rangle$ and $|3\rangle$ from $\mathbb{H}_{16} \cong \mathbb{H}_2^{\otimes^4}$.

(1) Let M be an observable and $|\varphi\rangle$ be a state. Show that $\mathbb{E}(M)$ on $|\varphi\rangle$ can indeed be computed as $\langle\varphi|M|\varphi\rangle$.

Recall Pauli matrices

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

and the Hadamard matrix

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

(3) Decide which of the above matrices are observables. For those who are, compute their expected measurement value on states $|0\rangle, |1\rangle, |+\rangle, |-\rangle$.

(4) Show that the operator $P_{|\varphi\rangle} = |\varphi\rangle\langle\varphi|$ is indeed a projection operator onto the linear space generated by $|\varphi\rangle$.

(5) Show that for unitary operators A and B their tensor product $A \otimes B$ is also unitary. What about observables?

(6) Recall that for matrices A and B of shapes (m, n) and (p, q), respectively, their tensor product $A \otimes B$ is defined as a matrix of shape (mp, nq) equal to $(a_{i,j}B)$. Show that such definition does indeed correspond to the abstract definition of tensor product of two linear operators (don't forget to pick the correct basis).

(7) Let H_2 be $H \otimes H$. Compute H_2 and then write it as a linear combination of projection operators (a.k.a. *spectral decomposition*).

(8) Let $|\phi\rangle$ be first measured in basis $\{|0\rangle, |1\rangle\}$, then in basis $\{|+\rangle, |-\rangle\}$ and again in basis $\{|0\rangle, |1\rangle\}$. Compute the probability of the last measurement being equal to $|1\rangle$. Compute the conditional probability of the last measurement being equal to $|1\rangle$ assuming the first measurement is equal to $|1\rangle$. How do these numbers change if we drop the middle measurement and leave only the first and the last measurements?

(9) * * Come up with the *correct* interpretation of the quantum mechanics.