QUANTUM INFORMATION

MFF UK

(0) Compute norm of the complex vector (0) in the one-dimensional arithmetic vector space. Compute norm of the complex vector $|0\rangle \in \mathbb{H}_2$.

Let

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \qquad\qquad |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}\,.$$

(1) Show that $|+\rangle$, $|-\rangle$ is an orthonormal basis of $\mathbb{H}_2$.

(2) Express $|0\rangle$ and $|1\rangle$ in terms of $|+\rangle$ and $|-\rangle$ and compute their transition matrix.

Pauli matrices are defined as

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

and the Hadamard matrix as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

(3) Verify whether the above matrices are normal.

(4) Find inverses of X, Y, Z and H.

(5) Verify whether Pauli matrices commute.

(6) Find eigen-values, eigen-vectors, diagonal form, and spectral decomposition of X, Y, Z and H.

(7) * Prove that $A^\dagger$ really equals $\varphi_A^\dagger$ when scalar product is the standard one. How would $\varphi_A^\dagger$ look like for general scalar product?