QUANTUM INFORMATION

MFF UK

(0) Compute the norm of the complex vector (0) in the one-dimensional arithmetic vector space over $\mathbb{C}$. Compute the norm of the complex vector $|0\rangle \in \mathbb{H}_2$.

Let

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \qquad\qquad |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}.$$

(1) Show that $\{|+\rangle, |-\rangle\}$ forms an orthonormal basis of $\mathbb{H}_2$.

(2) Express $|0\rangle$ and $|1\rangle$ as linear combinations of $|+\rangle$ and $|-\rangle$. Then, compute the corresponding transition matrix.

Pauli matrices are defined as

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

and the Hadamard matrix as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

(3) Which of the above four matrices is normal?

(4) Compute $X^{-1}, Y^{-1}, Z^{-1}, H^{-1}$.

(5) Do the Pauli matrices commute with each other?

(6) Find eigenvalues, eigenvectors, diagonal form, and spectral decompositions of X, Y, Z, and H.

(7) * Let U, V be complex vector spaces endowed with scalar product $\cdot_U, \cdot_V$. Let φ be a linear operator from U to V. Recall that the adjoint operator $\varphi^\dagger$ is defined as a linear map from V to U satisfying $\varphi^\dagger(u) \cdot_V v = u \cdot_U \varphi(v)$.

Let U be $\mathbb{C}^n$ and V be $\mathbb{C}^m$ both with the standard dot product. Let φ be given by the matrix A. Show that $A^\dagger$ is the matrix representation of $\varphi^\dagger$.