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A planning and scheduling problem for an operating theatre using an open scheduling strategy

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ABSTRACT

The objective of this paper is to design a weekly surgery schedule in an operating theatre where time blocks are reserved for surgeons rather than specialities. Both operating rooms and places in the recovery room are assumed to be multifunctional, and the objectives are to maximise the utilisation of the operating rooms, to minimise the overtime cost in the operating theatre, and to minimise the unexpected idle time between surgical cases. This weekly operating theatre planning and scheduling problem is solved in two phases. First, the planning problem is solved to give the date of surgery for each patient, allowing for the availability of operating rooms and surgeons. Then a daily scheduling problem is devised to determine the sequence of operations in each operating room in each day, taking into account the availability of recovery beds. The planning problem is described as a set-partitioning integer-programming model and is solved by a column-generation-based heuristic (CGBH) procedure. The daily scheduling problem, based on the results obtained in the planning phase, is treated as a two-stage hybrid flow-shop problem and solved by a hybrid genetic algorithm (HGA). Our results are compared with several actual surgery schedules in a Belgian university hospital, where time blocks have been assigned to either specific surgeons or specialities several months in advance. According to the comparison results, surgery schedules obtained by the proposed method have less idle time between surgical cases, much higher utilisation of operating rooms and produce less overtime.

1. Introduction

Over the past decade, in response to multiple challenges (such as the increase in the elderly population, the occurrence of new diseases, and stricter budgetary constraints), health care organisations have undergone increasing pressure of providing high quality surgery at as low as possible costs. Therefore, hospitals are always looking for the ways to not only improve patient care but also reduce operating costs. Since the operating theatre is a unit with highest cost and highest revenue as well and hence is of particular interest for hospitals (Health Care Financial Management Association (HCFMA), 2005; Macario, Vitez, Dunn, & McDonald, 1995), hospital managers are always interested in finding effective ways of running the operating theatre so as to improve the efficiency and quality of its services.

What can hospital managers do to improve the performance of their operating theatres? Good-quality patient care usually means excellent service and patient satisfaction, highly ensured patient safety, and first-class care and outcomes. Hospital managers should therefore take steps to improve quality in these three respects. With regard to costs, an efficient surgery schedule should not involve too much overtime, because the cost (especially the staffing cost) of each additional hour in the operating theatre is much greater than the cost of a regular working hour.

Obviously, it is difficult, if not impossible, to target at all these objectives in just one model. Oftentimes it is better to construct a soluble model with only the most important objectives considered. According to a recent review made by Cardoen, Demeulemeester, and Belien (2008), many researchers have tried to develop an efficient model for assigning surgical cases to the operating theatre in 2000 or later. This is considered as a solvable problem because patients are the only 'clients' of the operating theatre, and it seems reasonable to assign surgical cases to the surgical suite and then adjust hospital resources to minimise the wait in this sector. Among all those studies, two major classes of patients are involved: patients with elective cases and those with urgent cases. The surgery dates of the former are normally well planned in advance while those of the latter are usually unexpected and must be arranged urgently. Considering that the elective surgical cases compose an important part of the operating theatre's capacity, in this study, we restrict the focus to the construction of an efficient surgery schedule for the first kind of patients during a selected period.

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Before the surgery schedule is constructed, a decision has to be made about the operating theatre planning strategy. Three main planning strategies are used in hospitals:

- *Open scheduling strategy*. This was called the 'any workday' strategy by Dexter, Traub, and Macario (2003), where it meant that the surgeons could choose any workday for a case. Patterson (1996) simplified it as a 'the first-come first-served' strategy.
- *Block scheduling strategy*. Surgeons or groups of surgeons are assigned to a set of time blocks in which they can arrange their surgical cases. In theory, the surgeons or groups 'own' these time blocks, which are reserved in advance and cannot be released in the planning period even if some of them remain unused.
- *Modified block scheduling strategy*. The block scheduling strategy can be modified in two ways to provide more flexibility: either some of the operating rooms' opening hours are reserved while others are left open, or unused time blocks are released at some agreed time (e.g. 72 h) before the surgery.

In practice, the block scheduling or modified block scheduling strategies are widely used in hospitals. In general, the decision process for the surgery schedule consists of four steps: (1) forecast of the total demand of operating time for each department based on the past experiences during the past period (e.g. one trimester); (2) allocation of OR blocks and staff scheduling for the next trimester; (3) construction of the case schedule and optimization of the case schedule, given a specific scenario (normally for 1 or 2 weeks); (4) execution of the surgical schedule as well as scheduling of emergency and add-on elective cases (scheduled upon arrival).

In some Belgian hospitals, such as CHU Ambroise Paré, a variation of modified block scheduling strategy is implemented, where some time blocks are reserved for specific surgeons rather than specialties every trimester and surgeon are free to assign his surgical cases into blocks reserved to his specialty except that the priority of assigning surgical cases into the reserved blocks in the next week will be removed on upcoming Friday, i.e. either some empty blocks will be closed or some blocks will be assigned to another speciality with further demand. Furthermore, we find that within this modified block scheduling strategy, most of surgeons still prefer assigning as many as possible surgical cases into one block. Unfortunately, such kind of arrangement may cause inefficiency of the operating theatre. On the one hand, it is quite hard for one surgeon to focus on his operations without taking a break during all the working day! According to a further analysis about the rest time of surgeons between two successive surgical cases in CHU Ambroise Paré, surgeons take a rest for about 15 min after one surgery though some of them can start the next one within a few minutes. In general, the longer one surgical case is, the more time is needed by the surgeon for a rest. It can account for the phenomenon that if a surgeon is assigned with too many cases during 1 day, either some surgical case must be cancelled due to lack of time or much unexpected overtime will be needed. Although it is also true that some unexpected idle time could occur while patients are waiting for another surgeon in the operating room, we find if everything is well arranged, no time is needed for changing from one surgeon to another in one operating room; On the other hand, unexpected idle time may occur with block scheduling as long as one surgeon didn't fill his time blocks because the next surgeon could not begin his operations before the start of his time block.

Therefore we try to implement some ideas of the open scheduling strategy to surgery planning and scheduling in order to improve the performance in the operating theatre. In this study, we supposed that surgeons could assign their surgical cases into time blocks reserved with the block scheduling strategy as usual but the final surgery schedule of the coming week will be decided by an operating theatre management committee on Friday by applying open scheduling strategy targeted at optimizing some performance criteria of the involved operating theatre.

According to the literature, studies about the surgery planning and scheduling vary between the techniques and the objectives. As for the techniques, among a wide range of methodologies introduced from the domains of industrial operations research, mathematic programming models and discrete-event simulation tool are the two most commonly used techniques; the former are used not only to construct the master surgery plan, i.e. allocation of OR time blocks for each surgeon or specialty (e.g. Belien & Demeulemeester, 2007; Blake, Dexter, & Donald, 2002) but also to specify a surgery date for each patient, i.e. assigning surgical cases into operating rooms (e.g. Cardoen, Demeulemeester, & Belien, 2006; Fei, Chu, & Meskens, 2009; Fei, Chu, Meskens, & Artiba, 2008; Guinet & Chaabane, 2003; Hans, Wullink, Van Houdenhoven, & Kazemier, 2008; Jebali, Hadj Alouane, & Ladet, 2006; Kuo, Schroeder, Mahaffey, & Bollinger, 2003; Lamiri, Xie, Dolgui, & Grimaud, 2008a, 2008b; Mulholland, Abrahamse, & Bahl, 2005; Ogulata & Erol, 2003; Perez, Arenas, Bilbao, & Rodriguez, 2005; Pham & Klinkert, 2008) while the latter is normally used for improving the surgery scheduling (e.g. Bowers & Mould, 2005; Dexter, Macario, & Lubarsky, 2001; Dexter & Traub, 2002; Sciomachen, Tanfani, & Testi, 2005; Testi, Tanfani, & Torre, 2007). In addition, some researchers treat the surgery scheduling problem as the workshop scheduling problems and therefore some meta-heuristics used to solve the workshop problems are adapted to the healthcare system (Fei, Meskens, & Chu, 2006; Fei, Meskens, Combes, & Chu, 2006; Hans et al., 2008). As for the objectives, we noticed in the some studies have attempted to optimize a single performance criterion such as maximisation of operating room utilisation and minimisation of the related cost (e.g. Belien & Demeulemeester, 2008; Chaabane, Meskens, Guinet, & Laurent, 2008; Kuo et al., 2003; Testi et al., 2007; Van Houdenhoven, Van Oostrum, Hans, Wullink, & Kazemier, 2007), while many others have included several performance criteria in their study (e.g. Cardoen et al., 2006; Fei, Chu, Meskens, & Artiba, 2008; Fei, Chu, & Meskens, 2009; Fei, Meskens, Combes, & Chu, 2006; Guinet & Chaabane 2003; Hans et al., 2008; Jebali et al., 2006; Lamiri et al., 2008a, 2008b; Mulholland et al., 2005; Ogulata & Erol, 2003; Pham & Klinkert, 2008). This study is aimed at scheduling surgical cases to the involved operating theatre with the intent of both maximising the operating room utilisation and minimising the overtime cost of the operating theatre.

As many other researchers did (such as Guinet & Chaabane, 2003; Jebali et al., 2006; Van Houdenhoven et al., 2007), the considered problem is divided into two-stages. At the first stage, a weekly surgery planning problem is solved by assigning a surgery date to each surgical case. At the second stage, the surgery schedule on each day is finally obtained by solving a daily surgery scheduling problem.

The rest of this paper is organised as follows. Firstly, a mathematic programming model is constructed for the weekly operating room planning problem, and it is solved by a column-generation-based heuristic (CGBH) procedure. Secondly, a daily surgery scheduling problem is transformed to a hybrid flow-shop scheduling problem and solved by a hybrid genetic algorithm (HGA) to determine the final sequence of the surgical cases that have been assigned to that day in the planning phase. Thirdly, the experimental results with data collected from CHU Ambroise Paré, one university hospital in Belgium, are used to evaluate the performance of the proposed method. The paper is ended up with some conclusions and perspectives.

2. Operating theatre weekly planning problem

Given that the final surgery schedule is normally decided on Friday before the coming week in many Belgian hospitals, we focus on the assignment of surgical cases within 1 week in this study as well. As mentioned in the previous section, this considered surgery assignment problem is solved in two steps: first, each surgical case is assigned with a date for surgery; second, the start time of each surgical case is determined, and the objective is both to maximise the operating room utilisation and to minimise the overtime cost of the operating theatre.

When trying to solve an assignment problem in two phases, the operating theatre planner normally assigns a surgery date to each patient (the planning phase) and decides in which operating room (OR) the patient will be operated on, and when he or she will be transferred to that OR (the scheduling phase). In this section, we deal with the problem at the first phase, i.e. the operating theatre weekly planning problem.

Before going further, we would like to first introduce some background information: in practice, the patient chooses a surgeon at the consultation stage, which will often make the surgery for the patient later. Therefore, we assume that the surgeon for each surgical case is determined in advance and cannot be changed. Normally, when a surgical case is assigned with a date, the other surgical team members will be specified by the operating theatre planners. Thus, in this paper, we assume that the human and instrumental resources, except for the surgeons, are always available whenever they are needed.

The other hypotheses, adopted to define the weekly planning problem, are as follows:

- all operating rooms are multifunctional, i.e. a patient can be operated on in any available operating room by the specific surgeon before the given deadline;
- an open scheduling strategy is used, i.e. no surgeon can decide the final order of surgical cases in the coming week;
- emergency cases are not taken into consideration because patients admitted from the emergency department are usually operated on immediately, and hence only planned surgical cases are involved in this study;
- once a surgical case gets started in an operating room, it cannot be interrupted, i.e. there is no pre-emption.

With such hypotheses, the planning phase problem can be regarded as a resource-constrained bin-packing problem (Van Houdenhoven et al., 2007) and can be formulated as a binary-integer problem. Binary-integer-programming, whose decision version was one of Karp's 21 NP-complete problems (Karp, 1972), is classified as NP-hard and so that the planning problem under consideration is NP-hard as well. Considering that no polynomial algorithm has been found yet able to systematically obtain its optimal solution and hospital managers often prefer a good-quality solution that requires a reasonable running time to an optimum one that needs several hours or even days for execution, we are interested in devising a heuristic procedure to find an approximate but good-quality solution with reasonable running time.

Also employed in this study is Column Generation (CG) procedure, known as an efficient technique to deal with this kind of problem (Barnhart, Johnson, Nemhauser, Savelsbergh, & Vance, 1998; Belien & Demeulemeester, 2008; Fei, Chu, Meskens, & Artiba, 2008; Fei, Chu, & Meskens, 2009; Fei, Meskens, Combes, & Chu, 2006; Lamiri et al., 2008b) and a column-generation-based heuristic (CGBH) procedure is developed to obtain an efficient assignment of surgical cases in the planning phase. Since the CG procedure is often used to solve problems involving set-partitioning constraints. In the remainder of this section, we will first introduce the constructed set-partitioning integer-programming formulation for the planning problem under consideration and then present the CGBH procedure.

2.1. Model of the weekly operating theatre planning problem with open scheduling

The binary set-partitioning model consists of two parts: one is a master problem describing the main constraints with the desired objective and the other is an auxiliary problem used to determine the values of parameters of the master problem. In this section, we will first introduce the master problem and then the auxiliary problem. Since the existence of a basic feasible solution is pre-condition of the auxiliary problem, the construction of the initial set of feasible plans will be introduced at the end of this section.

2.1.1. Set-partitioning model for the master problem

In the binary set-partitioning model of the master problem, each column corresponds to a feasible case plan for one operating room in 1 day, i.e. a feasible sub-surgery schedule, named as a feasible plan in the rest of this paper, generated by one of the auxiliary problems (details are given in Section 2.1.2).

Parameters used in the set-partitioning model for the master problem are

N_D number of days over the planning period: in our model $N_D = 5$, i.e. the planning period is 1 week
N_S^d number of surgeons available on day d
Ω set of all surgical cases awaiting assignment
t_i predicted duration of surgical case i
D_i days left before the deadline for surgical case i, i.e. the number of days within which surgical case i must be performed
M number of operating rooms in the involved operating theatre
R_k^d number of regular opening hours of operating room k on day d. If operating room k is unavailable on day d, R_k^d is set as 0
S_k^d maximum number of overtime hours for operating room k on day d
A_l^d maximum working hours for surgeon l on day d. If surgeon l is unavailable on day d, A_l^d is set as 0
Ω_l set of surgical cases to be treated by surgeon l
β cost ratio of a regular working hour to an overtime hour, i.e. the penalty cost of the overtime
C_j operating cost of either unused opening hours or overtime hours for the operating room if the feasible plan j is adopted
Ξ set of all feasible plans for the planning period
a_{ij} 1 if surgical case i is assigned to feasible plan j; otherwise = 0
b_j^d 1 if feasible plan j is scheduled on day d; otherwise = 0
e_{kj} 1 if operating room k is used by feasible plan j; otherwise = 0
Decision variables
x_j 1 if feasible plan j is accepted; otherwise = 0

The set-partitioning model for the considered master problem is as follows:

$$\text{Min} \sum_{j\in\Xi} C_j x_j \tag{1}$$

Subject to

$$\sum_{j\in\Xi} a_{ij}x_j = 1, \quad i\in\Omega, \quad D_i \le N_D; \tag{2}$$

$$\sum_{j\in\Xi} a_{ij}x_j \le 1, \quad i\in\Omega, \quad D_i > N_D; \tag{3}$$

$$\sum_{j\in\Xi} b_j^d e_{kj} x_j \le 1, \quad k\in\{1,\cdots,M\}, \quad d\in\{1,\cdots,N_D\}; \tag{4}$$

$$\sum_{j\in\Xi} b_j^d \left(\sum_{i\in\Omega_l} a_{ij}t_i\right) x_j \le A_l^d, \quad l\in\{1,\cdots,N_S^d\}, \quad d\in\{1,\cdots,N_D\}; \tag{5}$$

$$x_j\in\{0,1\} \quad j\in\Xi. \tag{6}$$

The operating cost of feasible plan j is calculated as

$$C_j = \max\left\{\left(\sum_{d=1}^{N_D}\sum_{k=1}^{M} b_j^d e_{kj} R_k^d - \sum_{i\in\Omega} a_{ij} t_i\right), \beta\left(\sum_{i\in\Omega} a_{ij} t_i - \sum_{d=1}^{N_D}\sum_{k=1}^{M} b_j^d e_{kj} R_k^d\right)\right\}, \; j\in\Xi \tag{7}$$

The objective function seeks to minimise the cost of the total unexploited opening hours and overtime (calculated by Formula (7)). Since the cost of a regular opening hour of the operating room can be treated as constant, it is omitted from this formula of the objective function.

Constraints (2) and (3) ensure that, during the planning period, each surgical case, the deadline of which is no later than the end of the planning period (1 week), is treated exactly once before the end of the planning period, while the other cases are treated at most once. Constraint (4) shows that each operating room can be occupied by at most one accepted feasible plan in 1 day. Constraint (5) ensures that the total operating time assigned to each surgeon per day cannot exceed his or her maximum working hours in that day. This is necessary because one surgeon cannot work on two surgical cases simultaneously. Although it seems that a surgeon can be assigned to two or more surgical cases in parallel in this planning model, a feasible operating programme can be always guaranteed if the patient sequence is well determined at the daily scheduling stage as long as this constraint is respected at the planning phase.

As mentioned above, each column in this set-partitioning model corresponds to a feasible plan constructed while an auxiliary problem is being solved. Once a set of feasible plans for assigning all involved surgical cases have been generated, i.e. values of parameters a_{ij}, b_j^d and e_{kj} having been determined for each column, this model is apparently a binary-integer linear programming, the linear relaxation of which can be easily solved by the linear programming solver.

2.1.2. Generation of feasible plans with auxiliary problem

In column j of the set-partitioning model of the master problem described in Section 2.1.1, parameters a_{ij}, b_j^d and e_{kj} must respect the following constraints in order to ensure that this column corresponds to a feasible plan:

$$\sum_{i\in\Omega} a_{ij} t_j \le \sum_{d=1}^{N_D}\sum_{k=1}^{M} b_j^d e_{kj}(R_k^d + S_k^d), \quad j\in\Xi; \tag{8}$$

$$\sum_{d=1}^{N_D}\sum_{k=1}^{M} b_j^d e_{kj} = 1, \quad j\in\Xi; \tag{9}$$

$$\sum_{d=\bar{D}}^{N_D}\sum_{k=1}^{M} b_j^d e_{kj} = 0, \quad \text{if } N_D > \bar{D} = \min\{a_{ij}D_i \,|\, i\in\Omega\}, \quad j\in\Xi. \tag{10}$$

Constraint (8) implies that the total operating time of each plan would not exceed the maximum opening hours of the operating room where the plan is carried out; Constraint (9) ensures that each plan corresponds to just one available operating room during the planning period; Constraint (10) implies that each plan containing surgical cases with a deadline no later than the end of the planning period is implemented before its deadline.

Set X_B as a basic feasible solution (BFS) to the linear relaxation of the master problem (LMP) with corresponding basic matrix B and objective value z. According to the theory of simplex method (Dantzig & Wolfe, 1960), if there exists a column A_j corresponding to a feasible plan but not in B whose reduced cost $\sigma_j < 0$ and at least one element in $B^{-1}A_j > 0$, then it is possible to obtain a new basic feasible solution by replacing one column in B with the new column, and the new value of the objective function is no smaller than the previous one. Therefore, given an existing BFS to the LMP, the auxiliary problem can determine values of parameters a_{ij}, b_j^d and e_{kj} for one column, with an objective of minimising the corresponding reduced cost, so that it can be either inserted into the current BFS to improve the current solution or used as the indicator of obtaining the optimal solution of the LMP when the minimum reduced cost is non-negative. This is the basic the Column Generation (CG) procedure. In our study, the reduced cost corresponding to column $A_j = (a_{1j}, \cdots, a_{|\Omega|j}, b_j^1 e_{1j}, \cdots, b_j^{N_D} e_{Mj}, b_j^1 \sum_{i\in\Omega_{N_S^1}} a_{ij}, \cdots, b_j^{N_D} \sum_{i\in\Omega_{N_S^{N_D}}} a_{ij})^T$ is

$$\sigma_j = C_j - \sum_{i\in\Omega} a_{ij}\pi I_i - \sum_{d=1}^{N_D}\sum_{k=1}^{M_d} b_j^d e_{kj}\pi II_k^d - \sum_{d=1}^{N_D}\sum_{l=1}^{N_S} b_j^d e_{kj}\left(\sum_{i\in\Omega_l} a_{ij} t_i\right)\pi III_l^d \tag{11}$$

where $\pi I_i (i\in\Omega, D_i \le N_D)$ represent the dual variables corresponding to the part $(a_{1j}, \cdots, a_{|\Omega|}j)$, $\pi II_k^d (d = 1,\ldots,N_D, k = 1,\ldots,M)$ corresponds to the part $(b_j^1 e_{1j}, \cdots, b_j^{N_D} e_{Mj})$ and $\pi III_l^d (l = 1,\ldots, N_S^d, d = 1,\ldots,N_D)$ corresponds to the rest part of column A_j. This auxiliary problem can be regarded as one kind of knapsack problems and solved by dynamic programming procedures. Since details of those dynamic procedures can be found in a previous study that deals with the similar planning problem (Fei, Chu, & Meskens, 2009), they are not introduced in this paper.

2.1.3. Construction of the initial set of feasible plans

As mentioned above, an existing basic feasible solution is necessary for determining the value of parameters in the objective function of the auxiliary problem. In our algorithm, a heuristic based on the Best Fit Descending with Fuzzy constraint (BFDFC) (Dexter, Macario, & Traub, 1999) is used to generate an initial set of feasible plans, with which a restricted master problem (RMP) can be constructed. This BFDFC procedure works as follows.

Step 1: Surgical cases waiting for assignment are sorted by deadlines in ascending order. Surgical cases with the same deadline are sorted by operation duration from the longest to the shortest. In addition, surgical cases are considered in such an order that the longest case with the nearest deadline is assigned to an operating room at first.
Step 2: Each surgical case is assigned to the operating room that (1) is available on 1 day before the deadline of this case; (2) has sufficient regular open time available for inserting this case; (3) has the lowest amount of available regular open time.
Step 3: If no operating room has sufficient regular open time available for the current case, but sufficient open time is available in the operating room with the most remaining time provided that the surgical case duration is shortened by $\leqslant$ min {15 min, its maximal overtime}, the case is assigned to the operating room with the most remaining time.
Step 4: If no operating room has sufficient regular open time available even with the fuzzy constraint, a dummy plan will be constructed for each case not yet arranged, i.e. a single case plan will be constructed where the case is assigned to one operating room that is available before its deadline and has sufficient regular open time. When all cases have been assigned to an operating room, an initial set of feasible plans have been obtained. Since each surgical case has been included in one feasible plan, i.e. one column of the initial RMP, a feasible operating programme can always be guaranteed with the solution obtained from the current RMP.

2.2. Column-generation-based heuristic (CGBH) procedure

So far we have described the set-partitioning model for the planning problem and the generation of the initial set of feasible plans. Started with an initial set of feasible plans generated by the BFDFC procedure, a column-generated-based heuristic procedure (CGBH) proposed in previous study (Fei, Chu, & Meskens, 2009) is implemented for obtaining a feasible weekly surgery plan with good quality. The general steps of the CGBH procedure are as follows:

- *Step 0:* Make the surgical-cases-assignment problem described in Section 2.1.1 the current problem.
- *Step 1:* Solve the linear relaxation of the current problem (LMP) by an explicit CG procedure.
- *Step 2:* If no feasible solution of the LMP is obtained, the CGBH procedure is ended because no feasible weekly operating programme can be obtained by the CGBH procedure; otherwise, an optimal solution of this LMP is obtained; If the solution obtained in Step 1 respects all the integer constraints, meaning that the decisions variables x is either zero or one, the CGBH procedure will be stopped. If this is the first iteration, the optimal weekly surgery plan is obtained; otherwise, a feasible solution with acceptable quality is found.
- *Step 3:* If the solution obtained in Step 1 does not respect all the integer constraints (because the integer constraints were relaxed in the LMP), only a lower boundary of the current problem has been identified. In that case, one plan is selected by the MaxXMinC criterion which is shown to have the largest robustness in Fei, Chu, and Meskens (2009). This criterion works as follows: if several decision variables are equal to one, the plan with the smallest operating cost is selected; if all the decision variables are fractional, the plan with the largest decision variable, i.e. closest to one, is selected. Ties are broken by selecting the plan with the lowest operating cost.
- *Step 4:* Add the plan selected in Step 3 to the list of final plans, which is empty at the beginning of this procedure, and remove the surgical cases assigned and the operating room used by the selected plan from the current problem. A reduced planning problem is thus obtained.
- *Step 5:* When all the surgical cases have been assigned, or all the operating rooms have been planned, the CGBH procedure is complete. If all those surgical cases whose deadlines are earlier than the end of the planning duration are assigned, a feasible solution is then obtained; if not, no feasible solution is obtained by the CGBH procedure. If all the surgical cases have been assigned but some rooms still remain free, a set of empty plans are constructed for the unused rooms, i.e. those operating rooms are supposed to be closed in the next week. If there are still some unassigned cases and unallocated rooms, the reduced planning problem becomes the current problem, and the procedure is repeated, starting from Step 1.

3. Daily operating theatre scheduling problem

When the surgery schedule in the operating theatre for the coming week has been decided, the patient should be informed of both the surgery date and the starting time, i.e. his or her position in the sequence of operations, of the given day. However the order of surgical cases is not yet optimized by taking into account the constraint that a surgeon cannot operate on two surgical cases at the same time. Whereas the operating theatre consists of two parts: a set of operating rooms and a recovery room containing several recovery places, the efficiency in the operating theatre depends not only on the efficiency of operating rooms but also on the efficiency of the recovery room. Therefore, an efficient surgery schedule should also consider the availability of places in the recovery room. In this section, a daily operating theatre scheduling model is constructed to build a daily surgery schedule by taking into account these two constraints mentioned above with the aim of minimising the daily operating cost.

3.1. Description of the scheduling model

In order to construct a soluble model of the scheduling phase, some hypotheses are made:

- Surgical cases treated by one specific surgeon can be inserted into the sequence of the cases that will be made by other surgeons in an operating room.
- Human resources and all material resources but recovery beds and surgeons are always available whenever needed. We allow for the facts that no surgeon can operate on more than one patient at the same time; similarly, no recovery bed can be occupied by more than one patient at the same time.
- As in practice, all the operating rooms open simultaneously, and all recovery beds are empty at the beginning.
- All the scheduled patients are ready for their surgery on the given day, i.e. their arrival time is not taken into account in the model.
- Once started, an operation cannot be interrupted until it is finished. Moreover, once transferred to a recovery bed, a patient will stay in that recovery bed until the pre-defined recovery time elapses.
- The induction time for each operation and the clean-up time before leaving the operating room are included in the operating time, operation duration.

According to the literature, some researchers have treated the operating theatre scheduling problem as "hybrid flow-shop" problems (e.g. Guinet & Chaabane, 2003; Jebali et al., 2006) since an analogy can be drawn between these two kinds of problems. Many studies of hybrid flow-shop situations have been carried out in industrial fields. However, to the best of our knowledge, no previous studies have allowed for the fact that the recovery time after an operation can be shared between the operating room and the recovery room although it has been in practice in most hospitals.

In this section, we regard the daily scheduling problem as a two-staged hybrid flow-shop problem (with the operating rooms as the first stage and the recovery room as the second stage) and yet take account of the fact that the recovery time after an operation can be shared between the operating room and the recovery room. The objective of this scheduling phase is to determine an operation sequence that minimises the daily operating cost including the cost of both the operating rooms and the recovery room. Since this scheduling problem is also an NP-hard one, we are interested in developing an efficient heuristic procedure to solve the problem due to the same consideration in the planning phase. The hybrid genetic algorithm, proposed by Fei, Meskens, & Chu (2006) for solving the daily scheduling problem with block scheduling, performs quite well, and a similar hybrid genetic algorithm (HGA), therefore, is proposed for solving the daily open scheduling problem under consideration.

The notation used in the scheduling phase is

N	number of surgical cases (patients) awaiting scheduling on the given day
$C_i^{(s)}$	completion time for operation $i(i \in \{1, \cdots, N\}$ at stage s. In the scheduling model, the starting time of the operating theatre is set as 0, so the completion time is the moment when the patient is leaving the operating room (for $s = 1$) or the recovery room (for $s = 2$)
E_k	the time at which the last patient leaves the operating room k ($k \in \{1, \cdots, M_1\}$) where M_1 represents the number of operating rooms available on the given day
π	a feasible daily surgery schedule, namely a sequence of patients passing through the operating theatre
$C_{\max}^{(1)}$	the time at which the last patient leaves the first stage (operating rooms), $C_{\max}^{(1)} = \max\{C_i^{(1)} \vert i \in \{1, \cdots, N\}$. In addition, this indicator can also be calculated by the formula $C_{\max}^{(1)} = \max\{E_k \vert k \in \{1, \cdots, M_1\}\}$
$C_{max}^{(2)}$	the time at which the last patient leaves the second stage (the recovery room). This also represents the time at which the last patient leaves the operating theatre. $C_{\max}^{(2)} = \max\{C_i^{(2)} \vert i \in \{1, \cdots, N\}\}$

f the objective value used in the HGA algorithm. In order to obtain the most balanced surgery schedule with small operating cost for operating rooms available on that day, we employ the makespan at the first stage and the objective function is formulated as $\omega C_{\max}^{(1)} + C_{\max}^{(2)}$. The HGA algorithm aims to obtain a surgery schedule with the smallest f so that the operating theatre can be closed as early as possible

f' an auxiliary criterion used to break the tie when several surgery schedules reach the smallest value of f. Since the cost of one open hour in the operating room is much higher than that of the recovery room (the ratio of these two costs is set as ω in this study), this auxiliary criterion is formulated as $f = \omega \sum_{k=1}^{M} E_k + C_{\max}^{(2)}$ so that a surgery schedule with smaller amount of open hours in the operating rooms tends to be chosen

With notation above, our model can be described as follows. On one given day, N patients are ready to enter the operating theatre, consisting of M_1 multifunctional operating rooms and a recovery room with M_2 recovery places. In general, the operation duration of patient i in the operating room is pre-estimated as $t_i^{(1)}$, and the recovery duration after the operation is foreseen as $t_i^{(2)}$. If no place is available in the recovery room when the operation is completed, the patient will be 'blocked' in the operating room until a place becomes available in the recovery room or he comes round, i.e. a patient's recovery process can be shared between the operating room and the recovery room in our model. If one surgeon is needed simultaneously by patients in several operating rooms, he will be able to just operate on one of them and the others must be "blocked" in their operating rooms until their surgeon becomes available for that patient's surgery again, leading to the waste of open hours of those operating rooms. The daily scheduling problem considered in this phase aims at determining the surgery sequence of patients in both operating rooms and the recovery room with minimum daily operating cost f. If a set of daily surgery schedules with the same operating cost emerge, the most balanced surgery schedule, defined above, will be chosen.

3.2. A hybrid genetic algorithm for the daily operating theatre scheduling problem

Encouraged by the good performance of genetic algorithms for NP-hard problems (Goldberg, 1989) and motivated by the advantages of Tabu search procedures in local improvement (Glover, 1986), we have combined both methods in our study, and a hybrid genetic algorithm (HGA) is proposed for the daily open scheduling problem.

3.2.1. Framework for the hybrid genetic algorithm

The framework of the HGA employed in this study is similar to that used in a previous study (Fei, Meskens, & Chu, 2006) for solving the block scheduling problem, except for the construction of the initial population, the calculation of fitness, crossover and mutation operators, and the implementation of Tabu search procedure for local improvement of one selected solution. This algorithm proceeds as follows:

- *Step 1:* Construct an initial population, each constituent of which represents a daily surgery schedule generated with a decomposition procedure, and set it as the current population. Such decomposition procedure works as follows: (a) given that patients are sorted randomly or in ascending order of their indices, they will be first successively scheduled into a random-chosen operating rooms available on the given day; (b) when all patients have been scheduled into the operating rooms, they are re-sorted by their completion time in ascending order, i.e. by the time of leaving their operating room; (c) afterwards, each of them is scheduled into a randomly-chosen place in the recovery room. It should be noted that it is possible that one patient is assigned to a place in the recovery room while he comes around in the operating room because the place assigned in the recovery room is always unavailable; in this case, he has a dummy passage in the recovery room, since the length of this dummy passage is set zero and hence does not influent the passages of the following patients assigned to this place, this surgery plan will be still regarded as a feasible one.
- *Step 2:* Calculate the fitness value of constituent s of the current population as: $F(S) = \sqrt{f_w f_s}$ where f_s represents the objective value of constituent s and f_w represents the maximum objective value in the current population, i.e. the daily operating cost of the "worst" surgery schedule found so far. With such definition of fitness, we ensure that the greater the fitness value of one constituent is, the more likely this constituent is to be kept for the next generation (population).
- *Step 3:* Select two constituents by applying a roulette-wheel selection process. The classical idea of the roulette-wheel selection process is to first assign each constituent with a probability of being selected based on its fitness value and then stochastically select from the current population to create the basis of the next generation. Since the natural requirement is that the constituent with better fitness has a greater chance of survival than the weaker ones, in our study, an individual with a greater fitness value has a higher probability of being selected. This can be viewed as each constituent having a space on a roulette-wheel proportional to its fitness value and then spinning the wheel to select one member of the population (Goldberg, 1989). In order to avoid the identification of a 'super constituent', characterised by much better fitness, which is selected much more often than others, the classical roulette-wheel selection process has been improved in our algorithm by eliminating those constituents that have been once selected from the population. The two constituents selected in this step are used as 'parents' by genetic operators described in following steps.
- *Step 4:* Recombine the two parents selected in Step 3 with a probability P_c and generate two new constituents ("children"). In this step, one of two crossover operators is randomly applied: either an OX crossover operator (Davis, 1985), used for patient sequencing in the operating rooms, or a two-point crossover operator (Goldberg, 1989), used for patient assignment in the recovery room. Since the crossover operators work on just one stage, the feasibility of each "child", i.e. a newly generated surgery schedule obtained by the crossover operators, can be always held by carefully modifying the part not touched by the crossover operators to respect the constraints concerning the sequence of patients' passing through both operating rooms and the recovery room.
- *Step 5:* Randomly select one of the children generated in Step 4, and then mutate it with a probability P_m to generate a new constituent. In this step, three mutation operators are randomly used for different parts of the constituent (details are given in Section 3.2.2).
- *Step 6:* Use the Tabu search procedure as a local improvement operator for another child obtained in Step 4 to generate another new constituent.
- *Step 7:* Select constituents in the current generation, according to their fitness values, in order to regenerate the initial population for the next generation. In this step, an elitism mechanism is applied to ensure that the constituent with the best current fitness value will be always selected.

$$\text{A constituent}\quad \underbrace{1\,2\,3\,4\,5\,6\,7}_{V_1}\,\Big|\,\underbrace{1\,2\,3\,2\,3\,2\,1}_{V_2}\,\Big|\,\underbrace{3}_{V_3}\,\Big|\,\underbrace{1\,7\,4\,2\,6\,3\,5}_{V_4}$$

Fig. 1. An example of the coding scheme for the scheduling phase problem.

- *Step 8:* Return to Step 2 and repeat the procedure until one of the termination conditions is satisfied.

For a better understanding of this HGA, some key points are explained in more detail below.

3.2.2. Coding scheme

In our study, each constituent, i.e. a feasible daily surgery schedule, is coded as an integer array, consisting of four parts:

- V_1: a vector of size N, recording the order of patients' passing through the operating rooms. Each member of V_1 represents the index of a patient.
- V_2: a vector of size N, containing the indices of the recovery beds in the same order as the patients (surgical cases) given in V_1.
- V_3: a vector of size $(M_1 - 1)$, indicating the delimitation positions at which the patients in V_1 are assigned to the different operating rooms.
- V_4: vector of size N, containing the order of patients in the recovery beds. The indices of the patients are given in V_1.

Fig. 1 shows an example of the coding scheme for a scheduling phase problem. In this surgery schedule, two operating rooms and three places in the recovery room are available on the given day. Patients (surgical cases) 1–3 are scheduled in the first operating room in the order {1–2–3}, and the other four patients pass through the second operating room in the order {4–5–6–7} (i.e. the delimitation position is right after patient 3). After their operations in the operating rooms, patients 1 and 7 are scheduled to be transferred to the first place in the recovery room in the order {1–7}, patients 2, 4 and 6 to the second recovery place in the sequence {4–2–6} and patients 3 and 5 to the third place in the recovery room in the order {3–5}.

3.2.3. Mutation operators

According to the coding scheme described above, the sequence in V_4 takes account of both the time of patients' leaving the operating rooms and the assignment of patients to the recovery beds; in other words, it respects the constraints of both V_1 and V_2. Therefore, many adjustments will be needed if there is any change in this vector, and thus no mutations are planned for V_4. However, mutation operators are constructed for the other three parts of the 'chromosome' as follows.

3.2.3.1. A. Mutation operator for V_1. Randomly select two elements in V_1, and exchange them. Make sure that all the elements in V_4 still respect the constraints implied by the modification of V_1 and modify V_4, if necessary, to make it a feasible solution.

3.2.3.2. B. Mutation operator for V_2. Select two elements with different values in V_2, and exchange them. As the mutation operator for V_1 is proceeded with, examine V_4 to ensure that it is still feasible after the mutation in V_2 and modify it if necessary.

3.2.3.3. C. Mutation operator for V_3. Randomly generate an integer $k \in \{1, \cdots, M_1\}$. Delete the kth element in V_3, and make V_3 a vector of size $M_1 - 2$. Randomly generate an integer $j \in \{1, \cdots, N+1\}$ and insert this integer into V_3 so that the elements of V_3 are in ascending order of size. Make sure that V_4 is still feasible, and modify it if necessary.

4. Experimental results

In order to evaluate the proposed method in improving the practical arrangement of surgical cases in the operating theatre, real data of a Belgian university hospital, CHU Ambroise Paré, are used in this study.

4.1. Data

In the University Hospital of Amrboise Paré, there are nine surgical specialties: Stomatology, Gynecology, Urology, Orthopedic surgery, ENT/Oto-rhino-larynogology, Ophthalmology, Pediatric surgery, Plastic surgery and Abdominal surgery. In practice, a variation of block scheduling strategy is implemented, i.e. most of the time blocks are assigned to specific surgeons while some of them are assigned to specialties (e.g. Plastic surgery, Ophthalmology and Stomatology). In the latter case, any surgeon can book a case under the blocks reserved for his specialty. The operating theatre in this hospital is composed of six operating rooms and one recovery room with 10 places. Normally, all the operating rooms are open from 8:00 a.m. to 4:00 p.m. The recovery room open simultaneously with operating rooms and remain open until the last patient is possible to be transferred out of the operating theatre.

In this study, the experiments are based on 6321 records from the operating theatre, which were collected by a student performing field work at CHU Ambroise Paré over a 1-year period (from 1st November 2006 to 31st October 2007, i.e. 52 weeks). The data mainly consists of date of surgery, induction time, the start time and end time of surgery, time of the patient's leaving operating room, corresponding surgeon and specialty for each surgical case and admittance reason together with some personnel information (such as the patient's birthday, gender, etc.). After eliminating the urgent cases according to the admittance reason and the surgical cases with incomplete data, 5427 records collected from 49 weeks (number of surgical cases awaiting assignment in 1 week varies from 53 to 131) are finally available to be used in our experimentation. In order to respect the initial assignment, i.e. the real surgery schedule generated by surgeons, as much as possible, the deadline for each surgical case is set as its surgery date, standardized from 1(Monday) to 5 (Friday), which is given in the real surgery schedule. Even if some planed surgical cases were cancelled on the surgery date, they are still taken into account in our study and have a deadline of 6. Since no data about the recovery time was collected from CHU Ambroise Paré (because it has been impossible for the current information system to record this data) though the delay in recovery area could not be neglected, we applied the distribution, proposed in (Jebali et al., 2006), that the recovery time for any surgical case, generated from a lognormal distribution rule, ranges from 30 to 60 min, with a mean equal to the operation duration in the operating room minus 10 min as well as with a standard deviation of 15 min. The available working time of one surgeon is set as follows: if one surgeon has assigned cases to 1 day, he is supposed to be available for the whole day (8 h); otherwise, his available time on 1 day is set as either 0 or 8 h with a probability of 50%.

4.1.1. Cost ratios used in our model are defined as follows

Ratio of standard cost to overtime cost, β, is supposed to be decided by labour costs and is set as 1.5, such as the figure reported in Tessler, Kleiman, and Huberman (1997).

Ratio of the cost in an operating room to that in the recovery room, ω, is set as a 10.9, an average ratio obtained from costs reported in Schuster et al. (2004) for specialties involved in our study.

The proposed method has been executed with a programme coded with Microsoft VC++ 2005 Express Edition on a DELL Latitude D830 (Dual CPU: PM 2.6 GHz, memory: 2.0 GB, Operating System: Windows Vista). The linear relaxation of RMP, involved in the CG procedure, is solved by a linear programming solver COIN-OR, which can be downloaded from the website http://projects.coin-or.org/Clp as open source code.

4.2. Performance Indicators

In order to evaluate the performance of the proposed method, three indicators are employed:

- Occupancy Rate of Operating Rooms (OROR): ratio of the number of those operating rooms with at least one surgical case assigned in to the total number of operating rooms available in 1 week.
- Utilisation Rate of Operating Rooms (UROR): ratio of the number of open hours that are occupied by patients in the operating rooms with at least one surgical case assigned in to their regular open hours in 1 week.
- Percentage of Scheduled Patients (PPS): ratio of the number of patients scheduled into the involved week to the number of patients awaiting the assignments in 1 week.
- Overtime (OT): total number of overtime hours in 1 week.
- Idle time (IT): total number of idle hours during the regular open period of operating rooms in 1 week.
- Execution time (CPU): Time needed to find a weekly surgery schedule with the proposed method.

4.3. Experimental results

As shown in Tables 1–3, comparisons are made on three surgery schedules with 53 (the smallest group), 101 (medium-size group) and 131 (the largest group) patients awaiting assignment for the coming week. Thirteen, 27 and 31 surgeons are in charge of the three groups of surgical cases, respectively. Comparison between the theoretical surgery schedules obtained by the proposed method and the actual surgery schedules for those scenarios are performed with indicators introduced in Section 4.2 and experimental results are shown in Tables 1–3, respectively.

According to the results shown in Table 1, we find that all patients can be scheduled into the coming week with both methods (PPS = 100%) while fewer operating rooms are needed by our surgery schedule. When examining the indicators IT and OT, we find operating rooms have undergone much overtime while a lot of idle hours existing in the actual schedule and this problem has been greatly improved in our surgery schedule. In addition, the operating cost of our surgery schedule is much lower than that of the actual one. If the cost of recovery time per minute is $1.39, as mentioned in Windisch and Worsham (2002), our surgery schedule can save about $19,000 in 1 week ((46422.5 − 32794.5) $*$ $1.39 = $18942.92) if the staff can collaborate to fulfil our schedule, a tremendous improvement in cost-effectiveness.

When the number of patients awaiting assignment increases to about 100, the actual surgery schedule employed in CHU Ambroise Paré becomes denser and less idle time is found (see Table 2) and we find that some planned surgical cases are cancelled (PPS% = 91.08%) in the studied case. Although several reasons can cause those cancellations, it looks possible that some of them are caused by the lack of open hours in operating rooms though overtime is always allowed in hospitals. With the proposed method, we have re-scheduled those cases to the coming week without signif-

Table 1
Numerical results for the scenario with 53 cases awaiting assignment (13 surgeons).

Method		IT (minutes)	UROR (%)	OT (minutes)
Actual surgery schedule	Mean	114.44	89.69	50
	Std-dev	137.05	41.73	129.78
	Max	420	182.29	395
Schedule obtained by the proposed method	Mean	5.71	84.97	3.57
	Std-dev	13.05	30.33	7.48
	Max	35	104.17	20
	f	f'	OROR (%)	PPS (%)
Actual surgery schedule	27402	46422.5	36.67	100
Schedule obtained by the proposed method	17589	32794.5	23.33	100
CPU (seconds)	354.92			

Table 2
Numerical results for the scenario with 101 cases awaiting assignment (27 surgeons).

Method		IT (minutes)	UROR (%)	OT (minutes)
Actual surgery schedule	Mean	71.17	89.90	40.22
	Std-dev	75.54	29.98	76.60
	Max	320	162.5	300
Schedule obtained by the proposed method	Mean	6.58	111.90	90.26
	Std-dev	18.64	26.95	80.34
	Max	80	143.75	210
	f	f'	OROR (%)	PPS (%)
Actual surgery schedule	32976.5	114399.5	73.33	91.09
Schedule obtained by the proposed method	38876.5	112342.5	83.33	100
CPU (seconds)	891.08			

Table 3
Numerical results for the scenario with 131 cases awaiting assignment (31 surgeons).

Method		IT (minutes)	UROR (%)	OT (minutes)
Actual surgery schedule	Mean	73.63	95.20	39.37
	Std-dev	88.53	27.39	77.47
	Max	390	175	360
Schedule obtained by the proposed method	Mean	2.5	107.86	54.32
	Std-dev	4.56	14.78	48.16
	Max	15	123.96	115
	f	f'	OROR (%)	PPS (%)
Actual surgery schedule	38,154	138194.2	90.00	98.47
Schedule obtained by the proposed method	33502.5	127406	73.33	100
CPU (seconds)	1056.11			

icant increase in overtime. As in the first scenario, both idle time and overtime found in our surgery schedule are less than the actual ones.

No big different have been found between the results shown in Tables 2 and 3. With further analysis, we find more operating rooms are opened for the assignment in the scenario and slightly more idle time is reported in Table 3, possibly resulting from the unavailability of the places in the recovery room. Our surgery schedule tends to open fewer operating rooms and maximise the utilisation of regular open hours of the operating rooms, and therefore almost all the operating rooms are filled. Similar to the other scenarios, surgery schedules obtained by our method have lower operating cost and less idle time as well.

The computer time (CPU) needed to find a weekly surgery schedule with the proposed method increases from about 6 min (354.92 s in scenario 1) to about 20 min (1056.11 s in scenario 3) and hence can be still considered reasonable.

5. Conclusions and perspectives

The objective in this paper is to explore the possibility of improving the efficiency of operating theatre with open scheduling strategy. Assuming that times blocks are assigned to surgeons or specialties in advance and surgeons are free to assign surgical cases into their time blocks until the Thursday evening before the coming week. Every Friday, with the set of patients that have been assigned to the coming week by their surgeons, a management committee will finally decide the weekly surgery schedule with an open scheduling strategy by taking account of necessary constraints.

In this paper, a two-staged heuristic method has been developed to construct weekly surgery schedules with an open scheduling strategy by taking account of the availabilities of both surgeons and places in the recovery room. In the first phase, a weekly planning problem is defined by a set-partitioning model and solved by a CGBH procedure so that each surgical case can be assigned with a surgery date. In the second stage, a daily scheduling problem is regarded as a two-staged hybrid flow-shop model, and solved by a hybrid genetic algorithm, using a Tabu search procedure as the local improvement operator.

The proposed method is executed with a set of real data collected from a Belgian university hospital, where a variation of block scheduling strategy is employed. Comparing surgery schedules generated by our method with several actual surgery schedules, we find our schedules can theoretically outperform the actual ones because they have less idle time and overtime as well as much higher utilisation of operating rooms. Since the CPU execution time needed for the scenario with the largest number of cases (131 cases per week), in the observation period, is reasonable (less than 20 min), we conclude that if the staff can collaborate well to fulfil the proposed surgery schedule (e.g. surgeons can be so motivated to perform their operations in several days that they can stay available over those days rather than assign as many as possible cases to one time block), considerable operating cost of the operating theatre can be saved.

Since the hospital involve in this study only consists of six operating rooms and a recovery room with 10 places, no large-size scenario has been found in the observation period. Therefore, this study will continue to test the proposed method with data from other hospitals. Considering that the method employed in this paper is a decomposition one and it is possible that a bad assignment of surgical cases in the first phase will influence the efficiency of the final operating programme, we also plan to continue our work to eliminate the possibility of such a weakness. Moreover, we wish to collaborate with practitioners in hospitals and to develop a more realistic model by taking account of their demands. In one word, our future work is to make the results of our research more practical.

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