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Input – output modeling in the business analysis

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Abstract

Input - output (I – O) approach originated in the pioneering work of Wassily Leontief before the World War II. Traditionally, this approach has been used almost exclusively for macroeconomic analysis. Its use in the business decision making has been very modest in the past. This paper presents the two-stage optimization model in the business analysis, where the discrete dynamic deterministic programming is used in the first stage and enterprise I – O analysis together with the linear programming in the second stage of the business optimization. All three approaches, although each well known in literature and practice, bring possibilities of some new solutions if they are used in combination as suggested in this paper.

In the second part of the paper the application to aluminium plant in Slovenia is presented. The purpose of the two-stage optimization is to establish optimal purchasing quantities of input elements and production and selling quantities of outputs in the multiphase business process in aluminium industry for a certain number of time periods.

Keywords: decision analysis, dynamic programming, linear programming, input-output analysis

1 Introduction

The main idea of the two-stage optimization model presented in this paper is to combine the three methodological approaches into the decision making tool. The three methodological approaches are I–O modeling, linear programming and discrete dynamic deterministic programming. Although each of them is well known, there are some important new features of their combined use, presented in this paper.

Discussion of the literature on enterprise I–O modeling (we found it very modest) is presented in section 2. The two-stage optimization model is described in section 3.

On the other hand we also tried to present the real world application to aluminium plant in Slovenia. Taking into account the agreement with the aluminium plant management the application is presented in a reduced form in section 4. Inclusion of the optimization model into the decision making process in practice is presented in section 5. Main characteristics and advantages and disadvantages of the methodology utilized as well as the implication for future work is presented in the last section.

2 Literature survey on enterprise I–O modeling

In the decision making process in an enterprise the need to establish optimal purchasing quantities of input elements and production and selling quantities of outputs for a certain number of time periods, often arises. Business optimization is, especially in some industries, performed on the basis of mathematical models, where the idea of traditional Leontief's I–O modeling is used. These models are presented a bit later in this section.

Let us first describe some basic characteristics of the production process, since they dictate the mathematical approach that can be used. Production process may be trivial, simple, or complex, regarding the I – O relations between the sectors of the production process. Each sector represents the production center, producing single product, which can be used as intermediate output and/or can be sold on the market. The structure of the production process can be represented by the matrix P, with the elements p_{ij}, $i,j=1,2,\dots,n$, where n equals the number of sectors. If the i-th and the j-th sector are connected through the I – O relations, p_{ij} equals 1. Elements of the matrix P of the trivial production process fulfill the condition

$$\perp \neq P \subseteq \{(i,j)/i=j;i,j=1,2,...,n\}$$

of the simple production process the condition

$$\perp \neq P \subseteq \{(i,j)/i \leq j;i,j=1,2,...,n\} \text{ or}$$

$$\perp \neq P \subseteq \{(i,j)/i \geq j;i,j=1,2,...,n\}$$

and the elements of the matrix P of the complex production process the condition

$$\exists (i,j): i<j \wedge (i,j),(j,i) \in P$$

The trivial production process, where the matrix P is diagonal, is not likely to appear in practice. The matrix P of the simple production process is triangular, while the complex production process implies at least one pair of sectors that are connected as the supplier and as a consumer at the same time.

The production processes may consist of a single phase or of two or more phases – multiphase production process. A single-phase production process can not be divided into more steps of production. But the first phase of the multiphase production process represents the production (from the raw materials) of the intermediate outputs, which are the input elements to the next phases, for the production of another intermediate outputs or final products.

I–O model of W. Leontief (1936)

I – O approach originated in the pioneering work of Wassily Leontief (1936). The original static I – O problem originally consisted in the search for an "equilibrium" output vector for a certain number of sectors comprising the economic system of national economy, which meets the predetermined final demand vector and the induced industrial demand.

The idea could be used also if the business processes are analyzed. Of course, the theoretical assumptions of the classical I – O model, that each sector produces one product (or one service) only, that each sector uses only one production technology and that each product (or service) is produced in one sector only, must be met.

While the I – O models have been used widely for macro economic analysis, their use in the enterprise modeling has been very limited. The I – O analysis of the national economy is performed in monetary units (since the sectors in the national economy are of course aggregates), but materialized flows between sectors are the basis for the I – O modeling in an enterprise.

Enterprise I – O models

The basic I – O relations in the enterprise are expressed as follows

$$x = Ax + q \quad (1)$$

$$x^m = A^m x \quad (2)$$

The description of variables used is as follows:

x - vector of production quantities of sectors

q - vector of selling quantities of outputs

A - matrix of input coefficients of production. (The coefficient a_{ij} of the matrix A represents the quantity of the i-th good (produced in the sector i) that is needed to produce a unit of the j-th good (produced in the sector j)).

x^m - vector of input quantities of all input elements for production quantities defined by x.

A^m- matrix of input coefficients of all input elements

An output produced by the single sector can be used as input in other sectors of the production process or/and it can be sold on the market. These relations are expressed by equation (1). It is necessary to use the appropriate quantities of all input elements – equation (2).

The solution of (1) for x

$$x = (E - A)^{-1} q \quad (3)$$

is very suitable to analyze the impact of the different variants of sales quantities on the level of production.

The solution of (1) for q

$$q = X - Ax \quad (4)$$

is important for comparison of Leontief's model with others, described later in this section.

The matrix $(E–A)$ of the complex production process can not be proved to be invertible. But the matrix $(E–A^s)$, (where element a_{ij}^s of the matrix A^s represents the cost of the i-th good that is needed to produce a unit of the j-th good), is nonnegative and invertible (Vukman, 1976). It is essential, that the values of the coefficients of the matrix $(E-A^s)^{-1}$ depend upon the demand and therefore upon the production quantities of the sectors.

Gozinto model

The original Gozinto model (Mueller-Mehrabach, 1964) can be used, if the simple production process is analyzed, especially if the material planning in assembly industry is taken into account. Its use can be extended also to the complex production process (Kloock, 1969).

The Gozinto approach offers a very simple way to calculate the production quantities of sectors that are needed, if a unit production of a single sector is sold at the market. Let us describe this approach.

For sector i, i=1,2,...,n, of the production process, a following equation can be written

$$g_{ij} = a_{i1}g_{1j} + a_{i2}g_{2j} + \ldots + a_{in}g_{nj} + \delta_{ij}$$

where

$\delta_{ij} = 1$ if $i = j$

$\delta_{ij} = 0$ if $i \neq j$

g_{ij}- the total production quantity of output of sector i that is needed if a unit production of sector j is sold at the market

In matrix form we get

$$G = AG + E = (E - A)^{-1} \qquad (5)$$

Therefore the Gozinto model is equivalent to the Leontief's model. If q_j units of output of sector j should be sold at the market, therefore $x_i = g_{ij}q_j$ units of output of sector i are needed. This relation can be written as

$$x = Gq \qquad (6)$$

that is identical to equation (3) of the Leontief's model.

The appropriate quantities of all input elements can also be defined by the Gozinto model. Using equations (5), (6) and (4) it follows

$$x^m = G^m q$$

Element g_{ij}^m of the matrix G^m represents the quantity of the output of sector i that is needed, if a unit production of sector j should be sold at the market.

Vadnal's (1976) and Mesko's (1979) model

The model of Vadnal (1967) is appropriate for determination of optimal production program if the simple production is analyzed, with the assumption that $a_{ii} = 0$. His approach represents a special form of equations (3) of enterprise I–O model (based on Leontief) and (5) of the Gozinto model.

More general approach to the multiphase production analysis represents the model, developed by Mesko (1994). His model is appropriate for simple as well as for the complex production process. Both approaches use the exact methodology as the Leontief's I–O model.

Let us mention also ***the Pichler's model.***

Pichler (1953, 1961) first discussed multiphase production process by the means of the I – O analysis. The model was formed with the purpose to perform the cost analysis in the chemical industry. It is suitable if simple or complex production processes are analyzed. The Pichler's model was discussed and extended by Teusch-Schluetter (1985).

The use of the enterprise I – O modeling, also found in the literature (S.W. Custer,1986), deals with the role of the I – O modeling at the enterprise level in the planning process, but it does not take into the account the changes of the matrix $(E-A^s)^{-1}$ due to the changes of demand and of the production quantities.

The use of the models mentioned above, depends upon the characteristics of the production process and the purposes of the analysis performed, but nevertheless they represent a specific forms of the Leontief's I – O modeling.

Where is the place of the two-stage model described in the next section of this paper, among models just presented? The main part of the two-stage model, presented in the next section, is the enterprise I-O model, developed on the basis of the Leontief's model of national economy (but it offers much more possibilities for decision making), like all models, described previously in this section. But it differs from them, since it combines the static and the dynamic approach, by bringing together I–O modeling with the linear programming and discrete dynamic deterministic programming.

The two-stage model is suitable as a decision making tool especially in the enterprises with characteristics of the multiphase processing or assembly production and limited storage capacities. I–O modeling is, on our opinion, successfully combined with linear programming and discrete dynamic deterministic programming and as such it enables us to solve special problems, which arises, especially with limited storage capacities. The two-stage model was first developed for business optimization in the aluminium plant in Slovenia.

3 The two-stage business optimization

The two-stage optimization model is formed with the purpose to obtain the optimal purchasing quantities of input elements and production and selling quantities of the intermediate outputs and outputs in the multiphase business process for a number of time periods, the demand is known and should be met. The discrete dynamic deterministic programming is used in the first stage and I – O analysis together with the linear programming in the second stage of the business optimization.

Working on problems presented here has started some years ago and some results have already been achieved, presented and published. The two-stage optimization methodology itself has already been published in the literature (Artenjak and Tominc, 1999), and therefore it is only briefly presented here.

The two-stage optimization model starts with the selection of products, which are of the greatest importance for the enterprise. Let us assume that there is only one product, product A, which is the most important for the business purposes of the enterprise. The methodology can be also easily used if two or more products are recognized as the most important for the enterprise.

In the first stage of the optimization, the optimal production quantities of the product A are obtained by using discrete dynamic deterministic programming, for each period t, t=1,2,…,T, having the cumulative production costs, the set-up and storage costs at the end of period T minimal. Production and storage capacities are fixed and the demand for each time period t is known, but varies and should be met.

After completing the dynamic stage of the optimization, the static second stage of the optimization, using enterprise I – O modeling and linear programming simultaneously for each time period t, t=1,2,...,T, begins with the structural analysis. It results in defining the elements and production, purchasing and selling activities.

Elements of the business process are purchasing elements (inputs), working hours, intermediate products, products and production capacities. Each production activity is representing the production of a single element, which can be used as intermediate output in production process and/or can be sold on the market. Production activities therefore represent the sectors of the production process. The I – O analysis and the linear optimization model on its basis are used simultaneously. They are described in the literature (Artenjak and Tominc, 1999), as well as the cost-volume-profit analysis of the multiphase business process on the basis of the suitable cost pricing I – O models (Artenjak, 1996). The linear optimization model of the multiphase business process for each time period t, t=1,2,…,T, must be properly modified, since the optimal producing quantity of the most important product in the time period t, defined in the first stage of the optimization by the dynamic programming, must be taken into account. Namely, the optimal producing quantities of all other outputs must be defined, considering already defined optimal producing quantity of the most important product, which reduces the available production capacities and the quantities of input elements.

The second stage of the optimization model must be repeated for each time period t, t=1,2,...,T. The advantage of this model is that all possible changes in the market (demand, prices,..) may be taken into account. The first stage of the optimization can always be repeated, considering the latest changes; the optimal producing quantities of the most important product (products) can be corrected if any change arises.

The methodology presented in the previous chapter has already been used in practice. A simple case occurs if the most important product (or products) is produced only from the raw materials without any intermediate outputs. In this case, the optimal purchasing and selling quantities in the time period t, t=1,2,…,T, in the second stage of the optimization can be easily obtained by solving the linear optimization models. This procedure has been used in practice in an enterprise in the building industry, which produces Ferro-concrete prefabricated elements (Artenjak, Tominc, 1999)

If the selected important product (or products) is produced also from the intermediate outputs, the modified optimization model must be used.

More complex problem may occur if there are two or more outputs that are of the greatest importance for the business and are technologically dependent. One (or more) of them can be sold on the market, but at the same time, it is also used as input for the production of another product, which is also treated as very important for the business. The case described arises in the aluminium industry, where alumina and aluminium are produced and sold in the market. At the same time, alumina is the input in the production of aluminium. This application is presented in the paper.

4 An application to aluminium plant in Slovenia

Consider the aluminium plant in Kidricevo (Slovenia), with the capacity of 40.000 metric tons of raw aluminium per year and 120.000 metric tons of alumina per year. The aluminium plant was built in early sixties in Kidricevo in Slovenia, but due to high production costs as well as because of the pollution, the aluminium plant was closed some years ago.

The business process, which is presented in this paper and described in the following application, is simplified. But let us first describe it a little bit accurately.

The basic source for the production of aluminium is alumina – aluminium oxide. Alumina is extracted from bauxite ore, which contains up to 50-60% of aluminium oxide in the form of hydroxide. Approximately two tons of alumina are needed for extraction of a ton of aluminium and approximately 4,5 tons of bauxite are needed for two tons of alumina. Alumina is extracted on the basis of a modified Bayer procedure by dissolving in a hot, concentrated solution of caustic soda. Setting and filtration separate remaining impurities from the alumina solution. Cooling and agitating transform hydrated alumina into crystals, which are filtered, washed and heated to drive off the moisture. Calcination process converts the moist matter into the dry aluminium oxide – alumina.

The Hall-Heroult method in the electrolyzing process produces primary aluminium. For this purpose electrolytic cells with carbon electrodes are used. In the electrochemical reaction, which runs at temperatures between 950^0C and 970^0C, alumina is decomposed into oxygen and aluminium. Extruded molten aluminium metal sediments on the carbon lined bottom of the cell. There it accumulates and is pumped out. It is transferred into the foundry. The consumption of electric energy in this process is about 14.500 kWh per ton of primary aluminium. Carbon electrodes – anodes are also produced in the aluminium plant. Used anodes, which are removed out of the electrolytic cells, are also returned into the process of production of the new anodes. The capacity of the carbon anode plant is about 55.000 tons per year.

Molten primary aluminium is transferred into the foundry which produces two main sorts of cast products: *classical cast products* (ingots in various forms, billets and slabs) and cast products which are produced by the *continuos casting process* (cast strip, cast wire).

The manufacturing of ingots involves alloying, preparing and treating the melt, and casting. Casting ingots on cast-machines is done continuously. Products of the foundry are based on the usage of secondary aluminium from the own manufacturing process and of the alloyed aluminium ingots, which are used as high quality input. All inputs are melted together, with the purpose to produce wide assortment of aluminium alloys needed. For the production of low alloyed aluminium alloys, gas-fire furnaces are used. For alloys of top quality, smaller electric-induction furnaces are used. Next phases in the foundry are alloying, preparing and treating the melt, and at the end, casting billets and slabs. Turning and milling removes their rough surface layers.

Cast strip and cast wire production on continuos lines involves alloying, preparing and treating the melt, programmed casting and rolling, and careful winding of the strip. The wide cast strip is further processed by hot and cold rolling in rolling mills. The whole procedure involves the following operations: milling, warming and homogenizing the rolling slabs, hot rolling, breaking and final cold rolling, final thermal treatment, leveling and cutting the material. Two hot-rolling mills with a 170-m long rolling line perform hot rolling, while the cold rolling runs on four high rolling machines. This procedure is used either for the production of thin strips and foil, or for the production of the pre-rolled strip for the purpose of manufacturing the roll-bond sheet. Aluminium foil is also processed further. Paper, cardboard, plastic foil etc. are added in the lacquering, painting, laminating and printing procedures with the purpose to increase the applicability of the raw aluminium foil.

Aluminium plant in Kidricevo produces also the extruded and drawn products (about 15.000 tons per year): rods, tubes, profiles etc. Inputs for this production are cast billets of various dimensions. The manufacturing process consists of the following procedures: preparing and warming the billets, extruding rods and tubes, cold and thermal treating (to equalize structural differences and assure optimal working characteristics), leveling and cutting.

According to particular needs and demands, the surface of finished aluminium products can be mechanically treated, lacquered and painted or anodic oxidized.

Besides the standard products described, also some special products can be manufactured, according to demand: aluminium windows, doors, front-wall elements, equipment for laboratories, wire-fences etc..

For the purpose of this paper, the simplified business process is decomposed into production, purchasing and selling activities. Elements of the business process are purchasing elements (inputs), working hours, intermediate outputs, production capacities and outputs. They are presented in the Table 1.

The purchasing and selling prices as well as the maximal possible purchasing quantities of input element and maximal possible quantities of output elements refer to the time period $t = 1$.

Each production activity represents the production of a single element, which can be used as an intermediate output in production process and/or it can be sold on the market. Production activities are described in Table 2.

Alumina and aluminium are the most important outputs of the aluminium plant and are technologically dependent. Alumina is the basic source for producing aluminium and at the same time alumina can be sold on the market. The structure of the production process and technological dependencies are represented by Figure 1.

Table 1: Elements of the business process, purchasing and selling activities.

Input elements		Unit	Purchas. activity	Purch. Price	Max.possible purch.quantity
E1	Prebaked anodes	Ton	Z1	240	
E2	Electric energy	Kwh	Z2	0,03	
E3	Labour	Hour	Z3	0,50	
E4	Bauxite ore	Ton	Z4	30	
E5	Metal supplements	Ton	Z5	3200	
E6	Dissolving device	Hours	Z6		252
E7	Electrolytic cells	Hours	Z7		252
E8	Cast-machine 1	Hours	Z8		252
E9	Cast-machine 2	Hours	Z9		252
E10	Rolling machine	Hours	Z10		252
Intermediate outputs					
D2	Molten Al	Ton			
D3	Remnant 1	Ton			
D4	Remnant 2	Ton			
Output elements			Selling Activity	Selling price USD	Max. possible sell. quantity
D1	Red mud	Ton	/	/	/
P1	Alumina	Ton	Y1	300	4000
P2	Vanadium salt	Ton	Y2	1000	∞
P3	Alloyed ingots	Ton	Y3	2200	200
P4	Pure ingots	Ton	Y4	1900	800
P5	Al blocks	Ton	Y5	2000	0
P6	Al foil	Ton	Y6	2950	0

Table 2: Production activities

Production activities	
X1	Production of alumina
X2	Electrolysis
X3	Foundry
X4	Foundry1
X5	Production of Al blocks
X6	Rolling mill
X7	Recycling 1
X8	Recycling 2

The purpose of the optimization in time period *t*, *t*=1,2,3,4, is to establish optimal purchasing, production and/or selling quantities of the elements with the maximal contribution to cover fixed costs. The demand for alumina and aluminium in each time period should be met.

In the first stage of the optimization the optimal producing quantities of both the most important outputs are obtained using discrete dynamic deterministic optimization, for each time period *t*, *t*=1,2,3,4, having the cumulative production, set-up and storage costs at the end of the last time period, minimal. The production and storage capacities are fixed. The data are in tables 3, 4 and 5.

It must be considered that two units of alumina are needed for every unit of aluminium.

The stock at the beginning of the first time period, *t*=1, equals a unit of aluminium and a unit of alumina. The stock at the end of the time period t=4 should be equal to 0 for alumina and 0.5 unit of aluminium. Using the discrete dynamic deterministic programming, the optimal producing quantities of alumina and of aluminium are to be obtained for each time period *t*, *t*=1,2,3,4.

Table 3: The demand for alumina and aluminium (1000 tons)

Time period t	1	2	3	4
The demand for ***alumina***	4	1	2	3
The demand for ***aluminium***	1	2	3.5	3

Table 4: Production and set-up costs

Possible produced quantities of ***alumina*** in every time period (1000 tons)	0	1	2	3	4	5	6	7
Production cost (USD)	70	360	710	1035	1340	1625	1932	2240
Possible produced quantities of ***aluminium*** in every time period (1000 tons)	0	0.5	1	1.5	2	2.5		
Production cost (USD)	240	1100	1750	2475	2900	3125		

Table 5: Storage costs

Possible storage quantities of ***Alumina*** in every time period (1000 tons)	1	2	3	4	5
Storage cost (USD)	24	30	38	45	51
Possible storage quantity of ***Aluminium*** in every time period (1000 tons)	0	0.5	1	1.5	2
Storage cost (USD)	24	33	55	150	25 0

The recursion equations are as follows

$$C(y(1)_t, y(2)_t) = \min_{x(1)_t, x(2)_t} \begin{bmatrix} (c(y(1)_{t-1}) + +(c(y(2)_{t-1}) + c(x(1)_t)) \\ + c(x(2)_t) + C(y(1)_{t-1}, y(2)_{t-1}) \end{bmatrix}$$

for t = 1,2,3,4

subject to

$C_0\,(y(1)_0, y(2)_0) = 0$

$y(2)_t = y(2)_{t-1} + x(2)_t - d(2)_t$ $\qquad$ $y(1)_t = y(1)_{t-1} + x(1)_t - d(1)_t - 2x(2)_t$

$y(2)_0 = 1$ $\qquad$ $y(2)_4 = 0{,}5$

$y(1)_0 = 1$ $\qquad$ $y(1)_4 = 0$

$y(1)_t \in \{1,2,3,4,5\}$ $\qquad$ $y(2)_t \in \{0, 0.5, 1, 1.5, 2\}$

$x(1)_t \in \{0, 1, 2, 3, 4, 5, 6, 7\}$ $\qquad$ $x(2)_t \in \{0, 0.5, 1, 1.5, 2, 2.5\}$

Index (1) at all variables denotes alumina, index (2) denotes aluminium and the description of the variables used is as follows

- y_{t-1} - stock at the beginning of time period t
- y_t - stock at the end of time period t
- x_t - production quantity in time period t
- d_t - demand in time period t
- $c(y_{t-1})$ -storage cost at period t related to the stock at the beginning of time period t
- $c(x_t)$ - production and set-up costs in time period t, related to the producing quantity x_t
- $C(y_t)$ - minimum cumulative production, set-up and storage cost at the end of time period t,
- C_0 - sum of production, set-up and storage cost at the beginning of time period t=1

The result obtained is in table 6.

Table 6: Result after the first stage of the optimization

	y_{t-1}		x_t		d_t		y_t		$C(y_t)$
t	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)+(2)
1	1	1	7	1,5	4	1	1	1,5	4794
2	1	1,5	6	2,5	1	2	1	2	10025
3	1	2	7	2,5	2	3,5	1	1	15664
4	1	1	7	2,5	3	3	0	0,5	21108

It is optimal to produce 7, 6, 7 and 7 thousand tons of alumina and 1.5, 2.5, 2.5 and 2.5 thousand tons of aluminium in time periods t = 1,2,3,4, to met the demand in each time period. The cumulative production, set-up and storage costs are in this case minimal and equal 21108 USD at the end of the time period $t = 4$.

The second stage of the optimization must be performed for each time period t, t = 1, 2, 3, 4. *Using the data in tables1 and 2 along with the results of the dynamic* stage, the linear optimization model for time period $t = 1$ is formed.

Maximal possible contribution to cover fixed costs is 666272,90 USD. It is obtained if the purchasing quantities of input elements, producing quantities of intermediate outputs and outputs and selling quantities of outputs, written in the table 7 are considered, having met the demand for alumina and aluminium in the time period $t = 1$.

Table 7: Optimal solution for time period $t = 1$.

Element	Optimal purchasing quantity	Element	Optimal producing quantity	Element	Optimal selling quantity
E1	750,00	D2	1500,00	P1	4000,00
E2	27.927.270,00	D3	0,00	P2	350,00
E3	2.485.455,00	D4	0,00	P3	200,00
E4	17.500,00	D1	9800,00	P4	800,00
E5	18,18	P1	7000,00	P5	0,00
E6	252,00	P2	350,00	P6	0,00
E7	165,00	P3	200,00		
E8	88,00	P4	800,00		
E9	52,73	P5	0,00		
E10	0,00	P6	0,00		

The second stage of optimization must be repeated for time period t = 2,3,4, having in mind the optimal producing quantities of alumina and aluminium, which were obtained in the first stage of the optimization by the dynamic programming. If any change occurs, for example if market conditions (selling quantities, demand etc.) in next time periods change, the first stage of the optimization can easily be modified before performing the second stage of the optimization.

5 Inclusion of the optimization model into the decision making process in practice

The basic time period for planning was one month. The optimal plan, formed on the basis of the two-stage optimization model described, took into the account the number of the basic time periods that were interesting for the management. This decision making tool was used in the aluminium plant (at first in the production of the primary aluminium only) for the last four years.

The planning department in the plant had two main functions: elaboration of the optimal plan and analysis of the plan implementation. Regarding to the needs, the team of external experts in the field of operational research was occasionally gathered.

All relevant information from all business functions in the plant were collected in the planning department (about maximal possible and minimal (necessary) purchasing and selling quantities, purchasing and selling prices, about manpower, production capacities and procedures). In the next step the planning department drew up the outline plane that was checked by the plant management. According to changes, demanded by the management, more detailed plan was drawn up. Before its implementation, the functional division of the plan was accomplished (purchasing, production, selling and finance).

As already mentioned in the previous section, there was always the possibility to up-to-date the plan, if any changes, that were relevant for the implementation of the plan, occurred, in the business process itself or in its business environment (at the market). Both stages of the two-stage optimization model were repeated for all time periods in which the changes of business conditions could effect the previously determined optimal plan. The two-stage model and the plan were modified in the planning department, taking into the account all relevant information.

It should be mentioned that, especially under the conditions of the underdeveloped market, the production process was not always performed in accordance with the objectively determined (on the basis of mathematical model) plan. It was often replaced by the administrative directives as well as with subjective decision making.

But it is not possible to evaluate the financial effect of the implementation of the plan, drawn up on the basis of the methodology, described in this paper, with comparison to the situation before its use, since the data is not available.

6 Conclusions

Enterprise I-O modeling together with the linear and discrete dynamic programming in the optimization model, is presented in this paper. This approach can be used as a decision making tool especially in the enterprises with the characteristics of the multiphase processing/assembly production and limited storage capacities.

The two-stage optimization may be extended also by the inclusion of the ecological component into the model. The ecological component is strongly expressed in the aluminium industry. Consider the elements P2 - vanadium salt and D1 - red mud, which both appear as by-products in the production of alumina. Vanadium salt is a desirable by-product, but the production of red mud is not. Red mud is very destructive for the ecosystem of the aluminium plant environment. There are possibilities to include appropriate cost and price charges into the model. At the other hand, the question of the inclusion of the national economy to bear the part of these costs must be taken into the account if such an industry is important for the national economy.

Some of the problems, which may arise in the enterprise production process according to the assumptions of the classical I-O model are:

(i) some products can be produced by different production technologies

(ii) it is possible to produce more than one product in a single sector

(iii) production capacities of sectors are not independent
(iv) more than one product is produced in a single sector (for example main product and by-product).

The problems (i) and (iii) may be solved by the use of the linear optimization model of the production process, but for pricing and cost analysis empirical data are usually used (Artenjak, 1992). If the problem (ii) arises, it is necessary to introduce an additional sector for each such product. By-products (problem (iv)) can be considered as final products, which strength the competitive position of the main product, therefore the expected sales revenue from the by-product appear as deductive costs in the price model.

The enterprise I–O approach can be used in a process of decision making for many purposes. As mentioned earlier the method can be used also to perform cost accounting analysis. It can be used either as a full-absorption costing method (in this case all manufacturing costs are assigned to units produced) or as a variable costing method (which assigns only variable costs to products). The enterprise I–O cost accounting models can be very useful in the process of establishing gross margin per unit or contribution margin per unit. This is also the topic of our future work.

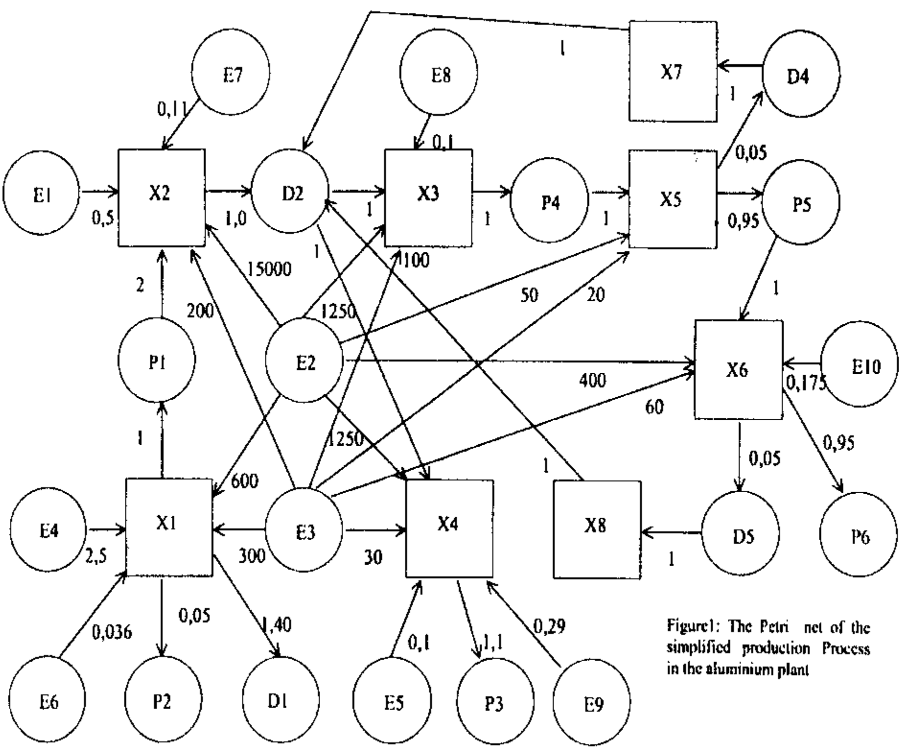

Figure1: The Petri net of the simplified production Process in the aluminium plant

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