Tasks for credit

complete at least 5

deadline: one week before you want to have the credit

Task 1. Write a circuit which computes AND of 4 bits. Do not use ancilla qubits (i.e. use just 5 qubits).

Task 2. Write a circuit implementing a factorization of numbers 15 and 21 via Shor's algorithm. (You can use https://algassert.com/quirk and you can use the built-in modular multiplication and QFT, however it is recommended to at least try it separately).

Task 3. Factoring 15 via Shor's algorithm is rather simple comparing to most other composites, because the orders of all the elements of the multiplicative group have values dividing 2^n. How many odd square–free numbers like this are there? Write at least 5 examples.

Task 4. Suppose, that the number you want to factor is indeed odd, square–free, and all elements of $\mathbb{Z}_N^*$ have orders dividing 2^m (and you have exactly this information beforehand). Find an efficient (polynomial time or better) classical algorithm which factors this number.

Task 5. Write a circuit, which given an input $|0\rangle \otimes |0\rangle \otimes |0\rangle \otimes |0\rangle$ outputs an entangled state. Show, that the output is truly entangled.

Task 6. Compute the expected value of the observable X on the state $(2-i, i)$. (Do not forget to normalize).

Task 7. Compute the probability, that the measured value of the observable $H \otimes H$ on the state $(1+i, i) \otimes (1-i, 2i)$ will be one. (Hint: find the generating eigenvectors of the eigenspace, do not forget to normalize).