Tutorial 3

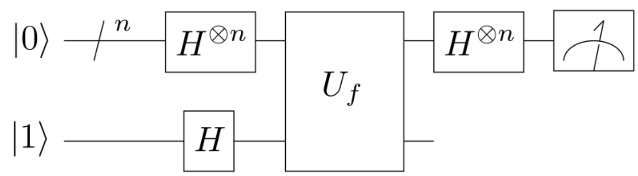
9.4.2021

Task 1. Find the eigenvalues and spectral decomposition of the operator $X \otimes Z$, where

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Use the properties of the tensor product.

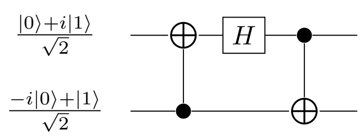
Task 2. Recall the Deutch–Jozsa algorithm (the following circuit)



We are measuring n qubits, however, we only extract one bit of usable information (i.e. the function is balanced/constant). Improve the algorithm by modifying its promise – find a bigger family of functions (or their classes) that can be distinguished.

Find a way to use this algorithm to distinguish between boolean affine function.

Task 3. Compute the output of the following circuit and rewrite it into a matrix.



Task 4. FInd a circuit implementing SWAP gate. Use only one qubit gates and CNOTs. Try to find such a circuit, that the CNOT gates are only used in one way.

Task 5. Find 2^X, where X is the known Pauli matrix.