Tutorial 2

26.3.2021

Task 1. Let us have observables:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

Compute their expected values on states:

$$|0\rangle, |1\rangle, |+\rangle, |-\rangle, |0\rangle + (1+i)|1\rangle.$$

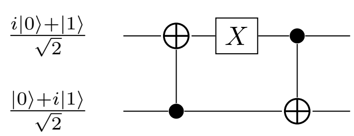
Task 2. Rewrite linear operators

$$H_2 = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \quad B = \frac{1}{\sqrt{2}}(\beta_{00}, \beta_{01}, \beta_{11}, \beta_{10}) = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

as linear compbinations of projections onto their eigenspaces.

Task 3. The state $|\varphi\rangle$ is measured with respect to basis $\{|0\rangle, |1\rangle\}$, next with respect to basis $\{|+\rangle, |-\rangle\}$ and once again in basis $\{|0\rangle, |1\rangle\}$. What are the possible results of the last measurement?

Task 4. Compute the output of the following circuit and rewrite it into a matrix.



Task 5. Show, that the state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ is entangled.

Task 6. Show that vectors $|\beta_{00}\rangle$, $|\beta_{01}\rangle$, $|\beta_{10}\rangle$, $|\beta_{11}\rangle$ form an orthonormal basis of $\mathbb{H}_4$, where

$$|\beta_{x,y}\rangle = \frac{|0y\rangle + (-1)^x |1\overline{y}\rangle}{\sqrt{2}}.$$

Task 7. Describe the results of measuring the expected value of the observable H_2 and compute its expected value on states:

a) $|00\rangle$,$|01\rangle$,$|10\rangle$,$|11\rangle$

b) $|\beta_{00}\rangle$, $|\beta_{01}\rangle$, $|\beta_{10}\rangle$, $|\beta_{11}\rangle$

c) $|u\rangle = (1, -1, 1, 1)$