Let

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \qquad |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} \qquad |\alpha\rangle = \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \qquad |\beta\rangle = \frac{|0\rangle - i|1\rangle}{\sqrt{2}} \qquad .$$

(1) Show that $\{|+\rangle, |-\rangle\}$ and $\{|\alpha\rangle, |\beta\rangle\}$ are orthonormal bases of $\mathbb{H}_2$.
(2) Express the vectors of bases $\{|0\rangle, |1\rangle\}$, $\{|+\rangle, |-\rangle\}$ and $\{|\alpha\rangle, |\beta\rangle\}$ in terms of each other.
(3) Decide whether which of the following matrices define a projection onto a vector. If so, which one?

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}, \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}, \begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix}.$$

(2) Consider the matrix

$$H_2 = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

Find its eigen-values, eigen-vectors, diagonal form, and spectral decomposition.
(3) Compute the inner product of the real vectors (0) and (0)
(4) Compute the inner product of the quantum states $|0\rangle$ and $|0\rangle$

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