Let

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \qquad |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}.$$

(1) Show that $|+\rangle$, $|-\rangle$ is an orthonormal basis of $\mathbb{H}_2$.
(2) Express $|0\rangle$ and $|1\rangle$ in terms of $|+\rangle$ and $|-\rangle$.

Pauli matrices are defined by

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

and the Hadamard matrix by

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

(3) Find inverses of X, Y, Z and H.
(4) Verify whether Pauli matrices commute.
(5) Find eigen-values, eigen-vectors, diagonal form, and spectral decomposition of X, Y, Z and H.
(6) Compute the inner product of the real vectors (0,1,0,1) and (0,1,1,1)
(7) Compute the inner product of the quantum states $|0101\rangle$ and $|0111\rangle$

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