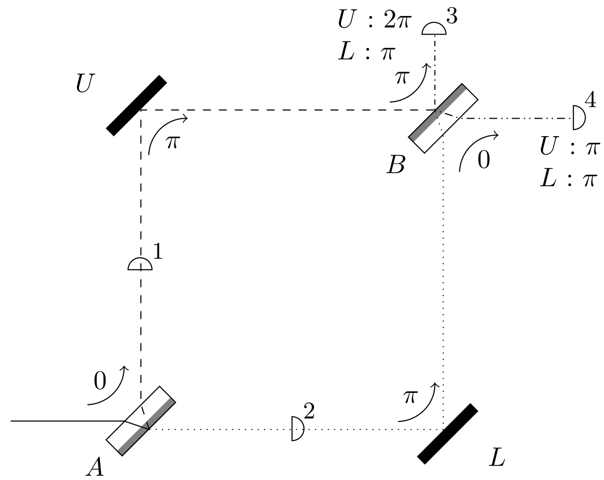


OBRÁZEK 1. Mach-Zehnder interferometer.

EXAMPLE OF QUANTUM BEHAVIOR

The following example illustrates the peculiar behavior of light, which not only contradicts the classical notion of its wave character, but raises much deeper doubts about the relationship between theoretical description, measurement, and physical reality. The device is shown in Figure 1.

A and B are semi-transparent mirrors through which half of the arriving beam passes and half is reflected. Such a mirror is simply a glass plate on which a layer of a substance, such as aluminum, of appropriate thickness is applied on one side. U and L are ordinary mirrors (in the experiment they only serve to direct the beam, the effect itself does not depend on them). Numbers 1 to 4 indicate the positions of detectors (obviously, if we want the light to reach the mirror B, we must remove detectors 1 and 2).

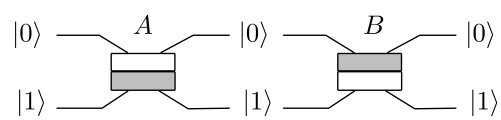
Light that is reflected by a surface changes its phase depending on whether it is reflected by an environment with a lower or higher optical density (corresponding to the speed of light in that environment). If light is reflected from an optically denser environment, it changes its phase by π (which corresponds to a shift of half a wavelength); when reflected from an optically thinner environment, the phase does not change. (In addition, the passage of light through the glass shifts the phase by a small value of φ, which is irrelevant to the experiment and can be neglected.)

There are four options for the passage of light: A - U - B - 3, A - U - B - 4, A - L - B - 3 and A - L - B - 4. If we measure the intensity of light on detector 1 or 2, in both cases we measure half the input intensity, in accordance with the assumed properties of the mirror. Measurements on detectors 3 and 4 (after removing detectors 1 and 2) show that there is no signal on detector 3, only on detector 4. This is due to light interference. Light traveling along the path A - U - B - 3 is canceled by destructive interference with light traveling along the path A - L - B - 3, because the first beam is shifted by half a phase relative to the second due to reflection on the mirror A. Conversely, there is constructive interference between the beams A - U - B - 4 and A - L - B - 4, because they were both reflected twice.

This is the classical wave description of the experiment. However, if the energy of light is reduced to a certain amount (i.e. a certain *quantum* denoted by the famous word *photon*), the light begins to behave as a particle in the sense that the signal is captured either on detector 1 or on the detector 2, the division into two half-intensity detection is excluded. The probability of both possibilities is equally one half. At first glance, this rehabilitates the old notion of light as a stream of particles. The classical measurement result can thus be interpreted as the fact that half of the photons pass and half are reflected. However, such an interpretation would require the photon, regardless of its trajectory, to behave in the same way on the B mirror. Each of the four paths would then have the same probability of one fourth, and detectors 3 and 4 should detect the signal both with a probability of one half (similarly to detectors 1 and 2). However, the result of the experiment turns out to be the same as in the classical case: no signal on the detector 3, signal always on the detector 4.

This mysterious phenomenon is an example of why we speak about the wave-corpuscular nature of light: it behaves partly as a wave and partly as a particle. But the problem is deeper. How can a photon, which is always captured one path only (if we decide to measure at 1 or 2), somehow interfere with itself if we delay the measurement?

We will show the answer of quantum mechanics. For clarity, we will forget about ordinary mirrors L and U and display the mirrors A and B schematically as follows:



The symbols $|0\rangle$ and $|1\rangle$ indicate the state of the photon at three moments in the experiment. If the diagram is related to the more detailed Figure 1, at the beginning of the experiment the photon is in the state $|0\rangle$. After interacting with the mirror A we say that the photon is in the state $|0\rangle$ if it travels through the mirror U, and after interacting with the mirror B it is in the state $|0\rangle$ if it is heading to detector 3. Similarly for states $|1\rangle$.

The basis of the quantum description of the experiment is the assumption that a photon can be in a *superposition* of states, which is mathematically expressed by their linear combination. Thus, although the photon did not divide, it is nevertheless in a state that indicates some kind of division. After passing through the mirror A, the photon is in the state

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}}\,.$$

(We divide the sum by $\sqrt{2}$ because we want to work with vectors of norm one.) If the photon coming into the mirror were in the state $|1\rangle$, i.e. from below, it would go into the state

$$\frac{|0\rangle - |1\rangle}{\sqrt{2}},$$

where $-1 = e^{\pi i}$ expresses the phase shift of π, caused by the reflection. Overall, therefore, the matrix of action of the mirror A can be expressed by a matrix

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix},$$

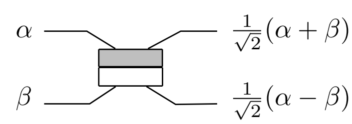
if we work in the basis

$$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}, \qquad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}.$$

This matrix describes the effect of the mirror on photons in a superposition of basis states. Therefore, if a photon enters the mirror in the state $\alpha\,|0\rangle + \beta\,|1\rangle$, it leaves in the state

$$\frac{1}{\sqrt{2}}(\alpha+\beta)\,|0\rangle + \frac{1}{\sqrt{2}}(\alpha-\beta)\,|1\rangle.$$

Schematically:



By the same considerations, we conclude that the action of the mirror B is expressed by the matrix

$$\frac{1}{\sqrt{2}}\begin{pmatrix}-1 & 1\\ 1 & 1\end{pmatrix}.$$

The overall effect of the system is then obtained by combining both mappings as

$$\frac{1}{\sqrt{2}}\begin{pmatrix}-1 & 1\\ 1 & 1\end{pmatrix}\cdot\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}.$$

This explains the result of the experiment: a photon entering in the state $|0\rangle$ exits in the state $|1\rangle$. The missing negative sign, indicating the phase shift by π, is caused by the fact that we neglected the ordinary mirrors U and L. The matrix of the action of these mirrors is obviously

$$\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix},$$

which completes the description. It remains to explain the results of measurements on detectors 1 and 2. This is done using the quantum postulate of measurement, which is one of the strangest and most controversial aspects of the mainstream interpretation of quantum mechanics.