(*c*) Suppose that monetary policy is initially determined as above, with $a > 0$, and that the monetary authority then announces that it is switching to a new regime where a is 0. Suppose that private agents believe that the probability that the announcement is true is ρ. What is y_t in terms of m_{t-1}, m_t, ρ, $\overline{y}$, b, and the initial value of a?

(*d*) Using these results, describe how an examination of the money-output relationship might be used to measure the credibility of announcements of regime changes.

10.6. Regime changes and the term structure of interest rates. (See Blanchard, 1984, Mankiw and Miron, 1986, and Mankiw, Miron, and Weil, 1987.) Consider an economy where money is neutral. Specifically, assume that $\pi_t = \Delta m_t$ and that r is constant at zero. Suppose that the money supply is given by $\Delta m_t = k\Delta m_{t-1} + \varepsilon_t$, where ε is a white-noise disturbance.

(*a*) Assume that the rational-expectations theory of the term structure of interest rates holds (see [10.6]). Specifically, assume that the two-period interest rate is given by $i_t^2 = (i_t^1 + E_t i_{t+1}^1)/2$. i_t^1 denotes the nominal interest rate from t to $t+1$; thus, by the Fisher identity, it equals $r_t + E_t[p_{t+1}] - p_t$.

(*i*) What is i_t^1 as a function of Δm_t and k? (Assume that Δm_t is known at time t.)

(*ii*) What is $E_t i_{t+1}^1$ as a function of Δm_t and k?

(*iii*) What is the relation between i_t^2 and i_t^1; that is, what is i_t^2 as a function of i_t^1 and k?

(*iv*) How would a change in k affect the relation between i_t^2 and i_t^1? Explain intuitively.

(*b*) Suppose that the two-period rate includes a time-varying term premium: $i_t^2 = (i_t^1 + E_t i_{t+1}^1)/2 + \theta_t$, where θ is a white-noise disturbance that is independent of ε. Consider the OLS regression $i_{t+1}^1 - i_t^1 = a + b(i_t^2 - i_t^1) + e_{t+1}$.

(*i*) Under the rational-expectations theory of the term structure (with $\theta_t = 0$ for all t), what value would one expect for b? (Hint: For a univariate OLS regression, the coefficient on the right-hand-side variable equals the covariance between the right-hand-side and left-hand-side variables divided by the variance of the right-hand-side variable.)

(*ii*) Now suppose that θ has variance σ_θ^2. What value would one expect for b?

(*iii*) How do changes in k affect your answer to part (*ii*)? What happens to b as k approaches 1?

10.7. (Fischer and Summers, 1989.) Suppose inflation is determined as in Section 10.3. Suppose the government is able to reduce the costs of inflation; that is, suppose it reduces the parameter a in equation (10.11). Is society made better or worse off by this change? Explain intuitively.

10.8. Solving the dynamic-inconsistency problem through punishment. (Barro and Gordon, 1983.) Consider a policymaker whose objective function is

$\sum_{t=0}^{\infty} \beta^t(y_t - a\pi_t^2/2)$, where $a > 0$ and $0 < \beta < 1$. y_t is determined by the Lucas supply curve, (10.10), each period. Expected inflation is determined as follows. If π has equaled $\hat{\pi}$ (where $\hat{\pi}$ is a parameter) in all previous periods, then $\pi^e = \hat{\pi}$. If π ever differs from $\hat{\pi}$, then $\pi^e = b/a$ in all later periods.

(*a*) What is the equilibrium of the model in all subsequent periods if π ever differs from $\hat{\pi}$?

(*b*) Suppose π has always been equal to $\hat{\pi}$, so $\pi^e = \hat{\pi}$. If the monetary authority chooses to depart from $\pi = \hat{\pi}$, what value of π does it choose? What level of its lifetime objective function does it attain under this strategy? If the monetary authority continues to choose $\pi = \hat{\pi}$ every period, what level of its lifetime objective function does it attain?

(*c*) For what values of $\hat{\pi}$ does the monetary authority choose $\pi = \hat{\pi}$? Are there values of a, b, and β such that if $\hat{\pi} = 0$, the monetary authority chooses $\pi = 0$?

10.9. Other equilibria in the Barro–Gordon model. Consider the situation described in Problem 10.8. Find the parameter values (if any) for which each of the following is an equilibrium:

(*a*) **One-period punishment.** π_t^e equals $\hat{\pi}$ if $\pi_{t-1} = \pi_{t-1}^e$ and equals b/a otherwise; $\pi = \hat{\pi}$ each period.

(*b*) **Severe punishment.** (Abreu, 1988, and Rogoff, 1987.) π_t^e equals $\hat{\pi}$ if $\pi_{t-1} = \pi_{t-1}^e$, equals $\pi_0 > b/a$ if $\pi_{t-1}^e = \hat{\pi}$ and $\pi_{t-1} \neq \hat{\pi}$, and equals b/a otherwise; $\pi = \hat{\pi}$ each period.

(*c*) **Repeated discretionary equilibrium.** $\pi = \pi^e = b/a$ each period.

10.10. Consider the situation analyzed in Problem 10.8, but assume that there is only some finite number of periods rather than an infinite number. What is the unique equilibrium? (Hint: Reason backward from the last period.)

10.11. More on solving the dynamic-inconsistency problem through reputation. (This is based on Cukierman and Meltzer, 1986.) Consider a policymaker who is in office for two periods and whose objective function is $E[\sum_{t=1}^{2} b(\pi_t - \pi_t^e) + c\pi_t - a\pi_t^2/2]$. The policymaker is chosen randomly from a pool of possible policymakers with differing tastes. Specifically, c is distributed normally over possible policymakers with mean $\bar{c}$ and variance $\sigma_c^2 > 0$. a and b are the same for all possible policymakers.

The policymaker cannot control inflation perfectly. Instead, $\pi_t = \hat{\pi}_t + \varepsilon_t$, where $\hat{\pi}_t$ is chosen by the policymaker (taking π_t^e as given) and where ε_t is normal with mean 0 and variance $\sigma_\varepsilon^2 > 0$. ε_1, ε_2, and c are independent. The public does not observe $\hat{\pi}_t$ and ε_t separately, but only π_t. Similarly, the public does not observe c.

Finally, assume that π_2^e is a linear function of π_1: $\pi_2^e = \alpha + \beta\pi_1$.

(*a*) What is the policymaker's choice of $\hat{\pi}_2$? What is the resulting expected value of the policymaker's second-period objective function, $b(\pi_2 - \pi_2^e) + c\pi_2 - a\pi_2^2/2$, as a function of π_2^e?

(*b*) What is the policymaker's choice of $\hat{\pi}_1$ taking α and β as given and accounting for the impact of π_1 on π_2^e?