Problem 2

a) Substitute the aggregate price level -- $p = qp^r + (1 - q)p^f$ -- into the expression for the price set by flexible-price firms -- $p^f = (1 - \phi)p + \phi m$ -- to yield:

(1) $p^f = (1 - \phi)[qp^r + (1 - q)p^f] + \phi m$

Solving for p^f yields:

(2) $p^f [1 - (1 - \phi)(1 - q)] = (1 - \phi)qp^r + \phi m$

Since $1 - (1 - \phi)(1 - q) = q + \phi - \phi q = \phi + (1 - \phi)q$, equation (2) can be rewritten as:

(3) $p^f [\phi + (1 - \phi)q] = (1 - \phi)qp^r + \phi m$

and thus finally:

(4) $$p^f = \frac{(1-\phi)q}{\phi+(1-\phi)q}p^r + \frac{\phi}{\phi+(1-\phi)q}m = p^r + \frac{\phi}{\phi+(1-\phi)q}\left(m - p^r\right)$$

b) Since rigid-price firms set $p^r = (1 - \phi)Ep + \phi Em$, we need to solve for Ep -- the expectation of the aggregate price level. Taking the expected value of both sides of $p = qp^r + (1 - q)p^f$ gives us:

(5) $Ep = qp^r + (1 - q)Ep^f$

Thus we have:

(6) $p^r = (1 - \phi)[qp^r + (1 - q)Ep^f] + \phi Em$

The rigid-price firms know how the flexible-price firms will set their price. That is, they know that flexible-price firms will use equation (4) to set their prices. Thus the rational expectation of the price set by the flexible-price firms is:

(7) $$Ep^f = p^r + \frac{\phi}{\phi+(1-\phi)q}(Em - p^r)$$

Substituting equation (7) into equation (6) yields:

(8) $$p^r = (1-\phi)\left\{qp^r + (1-q)\left[p^r + \frac{\phi}{\phi+(1-\phi)q}(Em - p^r)\right]\right\} + \phi\, Em$$

which implies:

(9) $$p^r = (1-\phi)p^r + \phi Em + \frac{(1-\phi)(1-q)\phi}{\phi+(1-\phi)q}(Em - p^r)$$

Defining $C = [(1 - \phi)(1 - q)\phi]/[\phi + (1 - \phi)q]$, we can rewrite equation (9) as:

(10) $p^r [1 - (1 - \phi) + C] = (\phi + C)Em$

or:

(11) $p^r (\phi + C) = (\phi + C)Em$

and thus finally:

(12) $p^r = Em$

Rigid price firms simply set their price equal to the expected value of the nominal money stock.

c) The aggregate price level is given by:

(13) $p = qp^r + (1 - q)p^f$

Substituting equation (4) for p^f into equation (13) yields:

(14) $$p = qp^r + (1-q)\left[p^r + \frac{\phi}{\phi+(1-\phi)q}\left(m - p^r\right)\right] = p^r + \frac{(1-q)\phi}{\phi+(1-\phi)q}\left(m - p^r\right)$$

Finally, from equation (12), we know that $p^r = Em$. Thus the aggregate price level is:

(15) $$p = Em + \frac{(1-q)\phi}{\phi+(1-\phi)q}(m - Em)$$

We know that $y = m - p$. Adding and subtracting Em to the right-hand side of this expression yields:

(16) $y = Em + (m - Em) - p$

Substituting equation (15) into equation (16) yields:

(17) $$y = (m - Em) - \frac{(1-q)\phi}{\phi+(1-\phi)q}(m - Em) = \frac{\phi + (1-\phi)q - (1-q)\phi}{\phi+(1-\phi)q}(m - Em)$$

which simplifies to:

(18) $$y = \frac{q}{\phi+(1-\phi)q}(m - Em)$$

i) From equations (15) and (18), we can see that <u>anticipated</u> changes in m affect only prices. Specifically, consider the effects of an upward shift in the entire distribution of m, with the realization of $m - Em$ held fixed. From equation (18) we can see that this will have no effects on real output. In this case, rigid-price firms get to set their price knowing that m has changed and thus incorporate it into their price setting decision.

ii) <u>Unanticipated</u> changes in m affect real output. That is, a higher value of m given its distribution -- that is, given Em -- does raise y as we can see from equation (18). In this case, the rigid-price firms do not get to observe the higher realization of m and cannot incorporate it into their price setting decision and hence the economy does not achieve the flexible-price equilibrium.

In addition, flexible-price firms are reluctant to allow their real prices to change. One can show that:

$$\frac{\partial y}{\partial \phi} = \frac{-(1-q)q}{[\phi+(1-\phi)q]^2}[m - Em] < 0 \quad \text{for } m > Em$$

Thus a lower value of ϕ -- that is, a higher degree of "real rigidity" -- leads to a higher level of output for any given positive realization of $m - Em$. This means that the impact on real output of an unanticipated increase in aggregate demand is larger the larger is the degree of real rigidity -- the more reluctant are flexible-price firms to allow their real prices to vary.