Problem 1

a) The individual's problem is to choose labor supply, L_i, to maximize expected utility, conditional on the realization of P_i. That is, the problem is:

$$\max_{L_i} E\left[\left(C_i - (1/\gamma)\cdot L_i^{\gamma}\right)\middle| P_i\right]$$

Substituting $C_i = P_i Q_i /P$ and $Q_i = L_i$ gives us:

$$\max_{L_i} E\left[\left(\frac{P_i L_i}{P} - \frac{1}{\gamma}\cdot L_i^{\gamma}\right)\middle| P_i\right]$$

Since only P is uncertain, this can be rewritten as:

$$\max_{L_i} E\left[\left(P_i/P\right)\middle| P_i\right]\cdot L_i - (1/\gamma)L_i^{\gamma}$$

The first-order condition is given by:

(1) $E[(P_i/P)|P_i] - L_i^{\gamma-1} = 0$

or:

$L_i^{\gamma-1} = E[(P_i/P)|P_i]$

Thus optimal labor supply is given by:

(2) $L_i = \left\{E\left[(P_i/P)\,|P_i\right]\right\}^{1/(\gamma-1)}$

Taking the log of both sides of equation (2) and defining $\ell_i \equiv \ln L_i$ yields:

(3) $\ell_i = [1/(\gamma-1)]\cdot \ln E[(P_i/P)|P_i]$

b) The amount of labor the individual supplies if she follows the certainty-equivalence rule is given by (in logs):

(4) $\ell_i = [1/(\gamma-1)]\cdot E[\ln(P_i/P)|P_i]$

Since $\ln(P_i/P)$ is a concave function of (P_i/P), then by Jensen's inequality $\ln E[(P_i/P)|P_i] > E[\ln(P_i/P)|P_i]$

Thus the amount of labor the individual supplies if she follows the certainty-equivalence rule is <u>less</u> than the optimal amount derived in part (a).

c) We are given that:

(5) $\ln(P_i/P) = E[\ln(P_i/P)|P_i] + u_i \qquad u_i \sim N(0, V_u)$

Taking the exponential function of both sides of equation (5) yields:

(6) $P_i/P = e^{E[\ln(P_i/P)|P_i]}\cdot e^{u_i}$

Now take the expected value, conditional on P_i, of both sides of equation (6):

(7) $E\left[\left(P_i/P\right)\middle| P_i\right] = e^{E[\ln(P_i/P)|P_i]}\cdot E\left(e^{u_i}\,|\,P_i\right)$

Taking the natural log of both sides of equation (7) yields:

(8) $\ln E\left[\left(P_i/P\right)\middle| P_i\right] = E\left[\ln(P_i/P)|P_i\right] + \ln E\left(e^{u_i}\,|\,P_i\right)$

Note that $\ln E\left(e^{u_i}\,|\,P_i\right)$ is just a constant whose value is independent of P_i. Substituting equation (8) into equation (3) – the optimal amount of (log) labor supply – gives us:

$$\ell_i = \left[1/(\gamma-1)\right]\cdot\left\{E\left[\ln(P_i/P)|P_i\right] + \ln E\left(e^{u_i}\,|\,P_i\right)\right\}$$

or simply:

(9) $\ell_i = \left[1/(\gamma-1)\right]\cdot E\left[\ln(P_i/P)|P_i\right] + \left[1/(\gamma-1)\right]\cdot\left\{\ln E\left(e^{u_i}\,|\,P_i\right)\right\}$

The first term – $[1/(\gamma-1)]\cdot E[\ln(P_i/P)|P_i]$ – is the certainty-equivalence choice of (log) labor supply, while the second term is a constant. Thus the ℓ_i that maximizes expected utility differs from the certainty-equivalence rule only by a constant.