Real Business Cycles

Stylized Facts about Business Cycles, Relevancy of RBC's, Solow Residuals

Outline

- Stylized facts about the business cycles: do they correspond to the RBC's?
- What is calibration of the RBC model
- Technology shocks – Solow residuals: How do they look like?

Stylized facts

- Stylized facts: a simplified presentation of an empirical finding - a broad generalization that summarizes complex behavior
- Examples: Kaldor's facts about economic growth
    - Shares of capital and labor on income constant
    - Rate of return on investment constant
    - Rate of growth of capital stock and output per worker constant
    - Growth of real wages
- Although essentially true these facts have a lot of inaccuracies in the detail.

Stylized facts

- Stylized facts: a simplified presentation of an empirical finding - a broad generalization that summarizes complex behavior
- Why useful?

Stylized facts

- Stylized facts: a simplified presentation of an empirical finding - a broad generalization that summarizes complex behavior
- Why useful?
- These generalizations help with building of theories and models – as thay have to be in accordance with these stylized facts.

Stylized facts about the business cycles

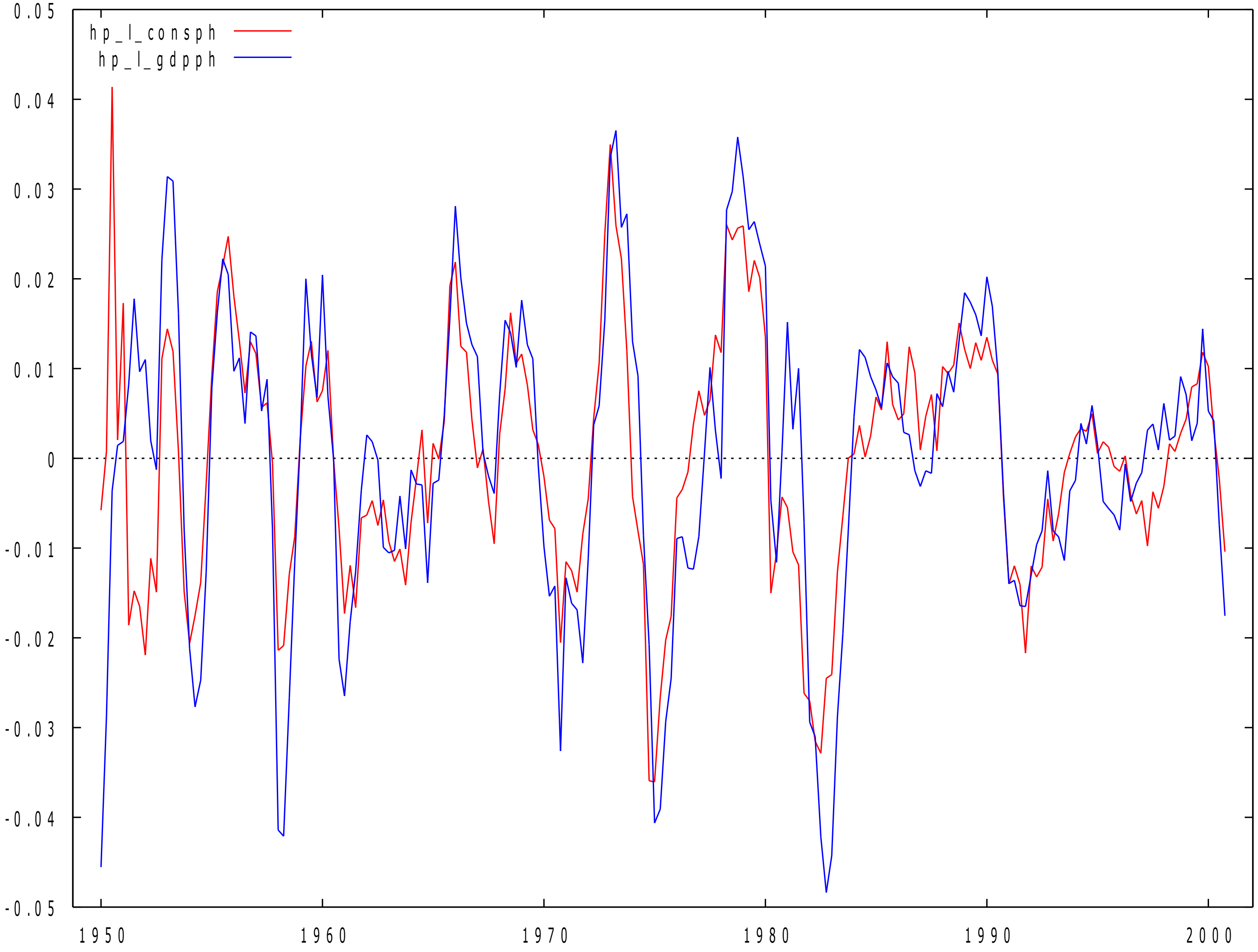
- What are we interested in:
  - Which variables are correlated with the cycles in GDP?
  - Which variables tend to evolve relatively smoothly and which ones are more volatile?
- Main ones with respect to business cycles:
- Persistence (prob of being above quarter if in q_{t-1} above very high)
- Cyclical variability (magnitudes of different variables differ, output and working higher same volatility; investment more volatile, consumption smoothing; several acyclical or even countercyclical).

Stylized facts about the business cycles

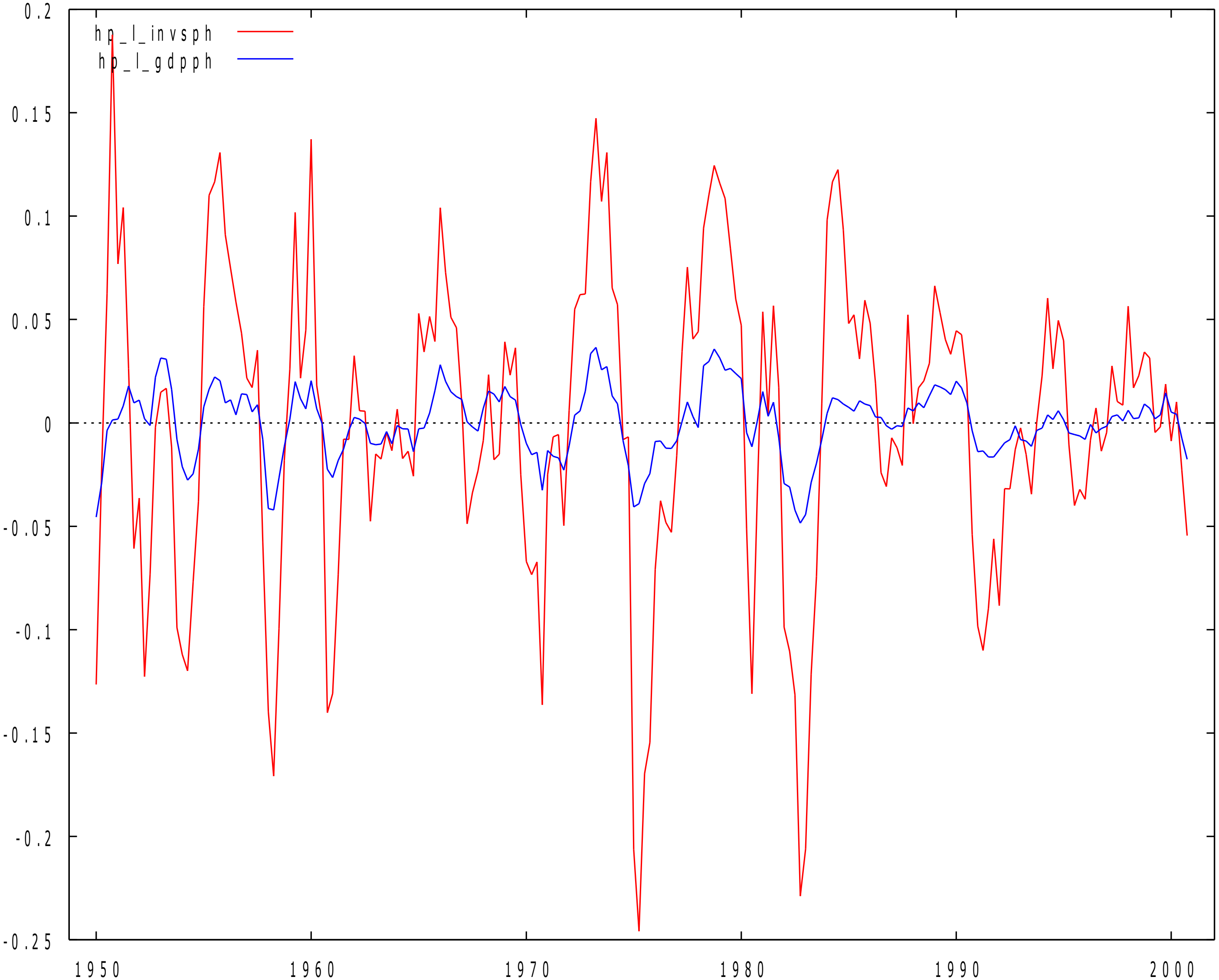
Derivation of stylized facts:

- Preparation of the data
- Variables should be “per head” (related to RBC's, growth models etc.)
- Variables should be in logs: in order to have the gaps in %.
- Gaps retrieved via Hodrick-Prescott filter (1600).

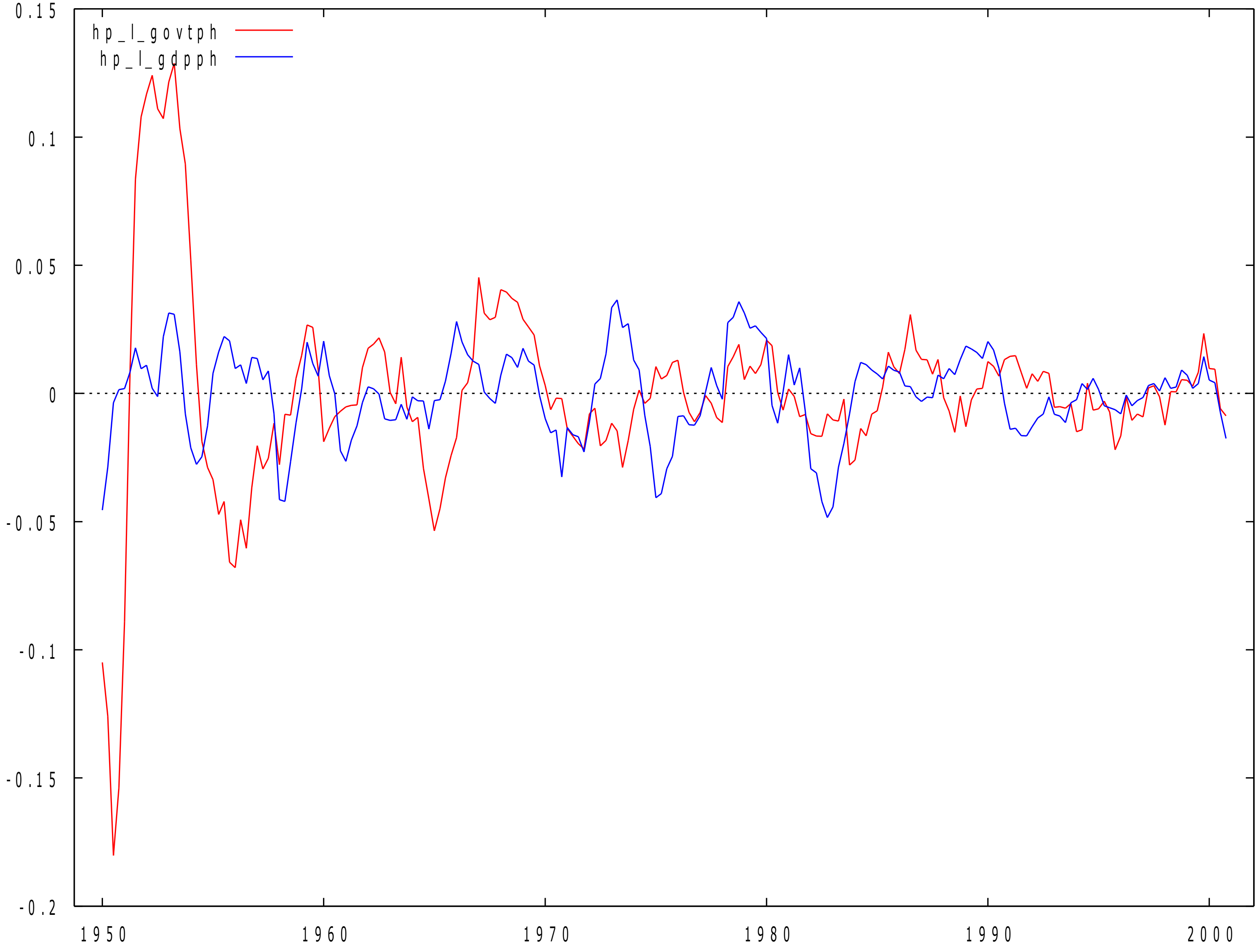
Consumption/Output



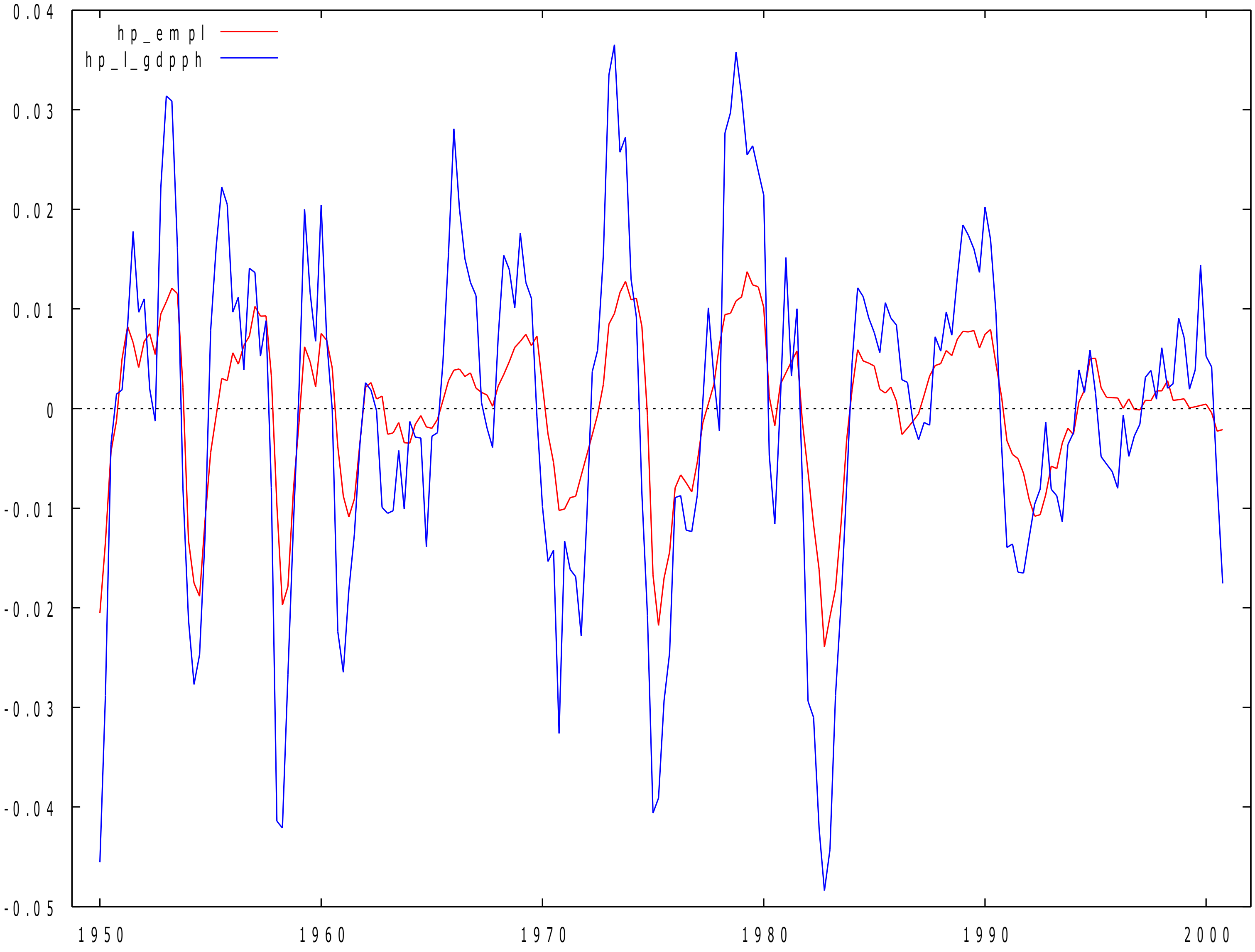
Investment/Output



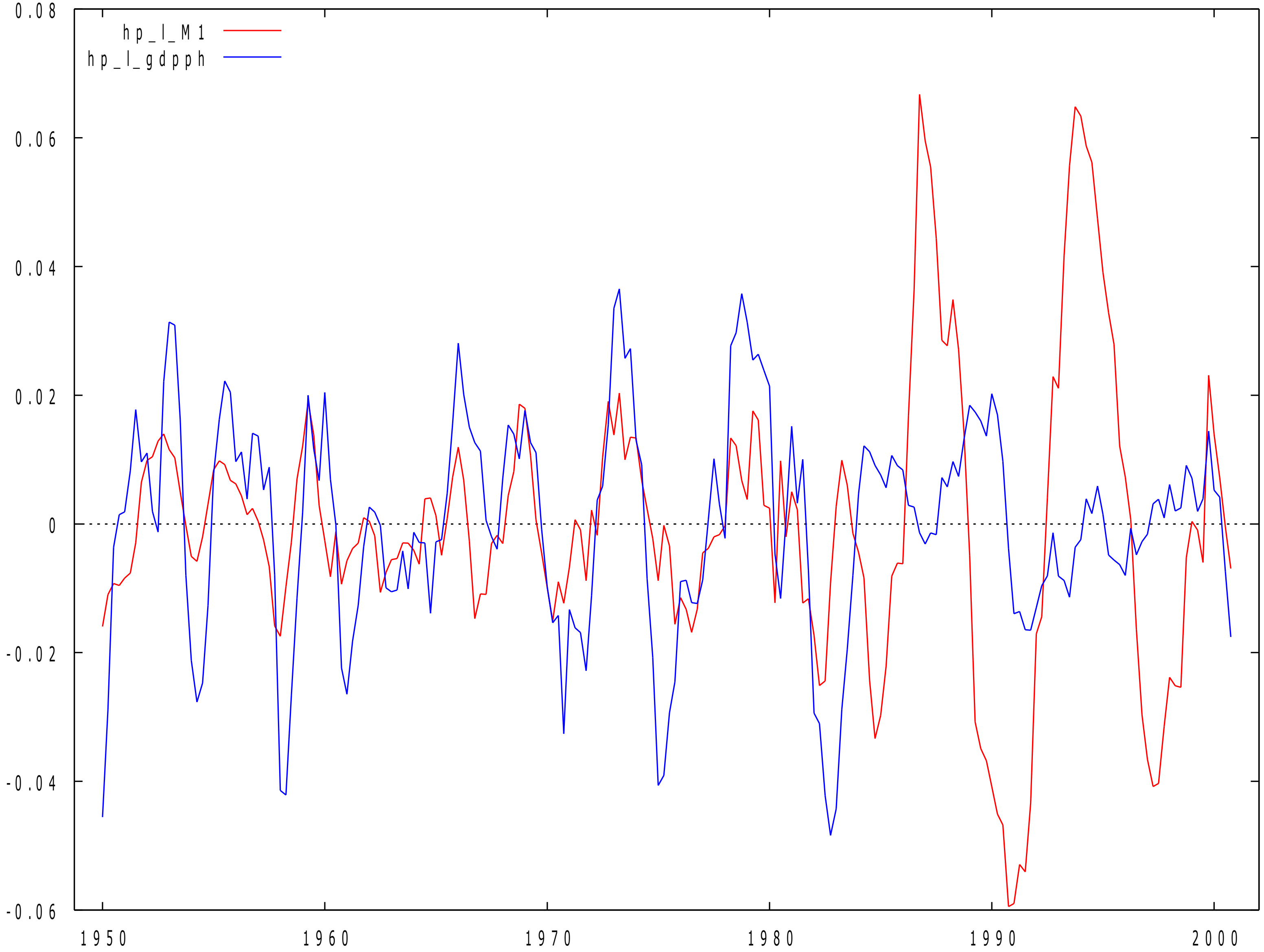
GovtExp/Output



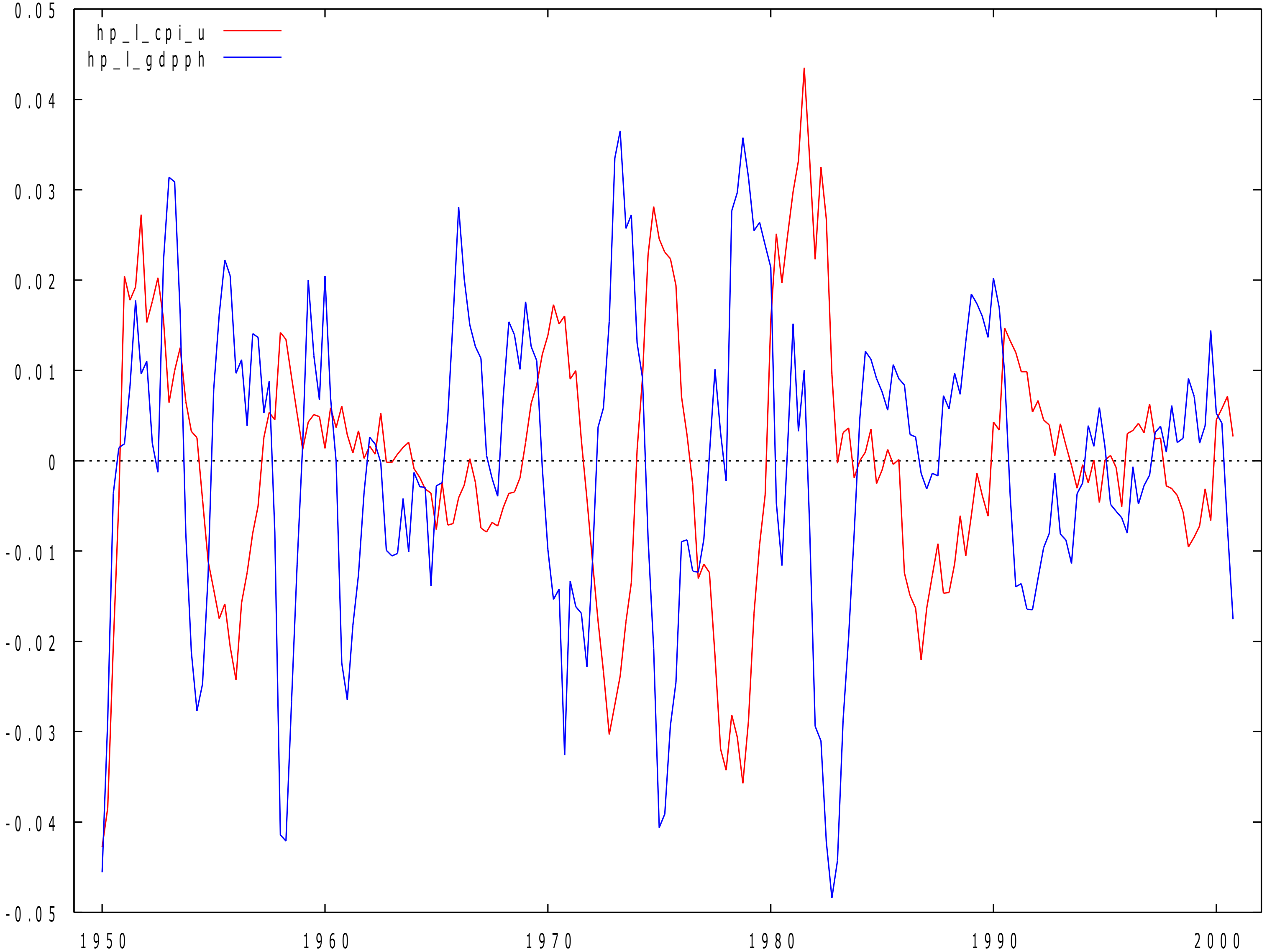
Employment/Output



MoneySupply/Output



PriceLevel/Output



Properties of Cyclical Deviations

	Std. Dev.	Ratio to GDP	Correlation to GDP
M1	0,02	1,3	0,19
C	0,01	0,8	0,78
I	0,07	4,42	0,84
G	0,04	2,25	0,21
P	0,01	0,84	-0,32
L	0,01	0,46	0,88
Y	0,02	1	1

Summary of Stylized Facts

- Components of GDP have strongly correlated cycles
- Investment are more volatile and consumption less than the output
- Labor: pro-cyclical
- Money aggregate M1: change in recent 20 years
- Price level: counter cyclical
- Missing? Industrial production, wages, interest rates, disposable income, M2...

Additional Readings

- Kydland, F. - Prescott, E. (1990): Business cycles: Real facts and a monetary myth.
- Hartley, J. (1999): Real myths and a monetary fact. (A critical response to the Kydland's and Prescott's article)

Part II

Calibration of a simple RBC model
for the Czech economy

Simple RBC model

- Not surprisingly, the RBC's fit relatively well to the U.S. and sometimes to other developed economies.
- How successful they can be when applied on other than the U.S. data?
- To some extent easily, because of flexible framework and dynamics determined by calibrated coefficients, but some regularities in the data for example in Czechia different from the benchmark neoclassical model.

Simple RBC model

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

$$u(c_t, l_t) = \log c_t + \psi \log l_t$$

$$y_t = e^{z_t} k_t^{\alpha} n_t^{1-\alpha}$$

$$z_t = \rho z_{t-1} + \varepsilon_t$$

$$k_{t+1} = i_t + (1-\delta) k_t$$

$$n_t + l_t = 1$$

$$y_t = c_t + i_t$$

- Intertemporal utility function
- Intratemporal substitution of C and L
- Production function
- Stochastic shocks on technology
- Motion of capital
- Labor + leisure = 1
- Output + investment = 1
- All variables in HP gaps.
- Why simple: many current macro models contain more than 30 or even 50 such equations or identities.

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- Steps in evaluating the model:

1) Computational issues (maximization and finding optimum, solving the model and log-linearizing it around steady state, see sections 6.5 in the lecture; computers can do)
2) Deriving stylized facts about the Czech business cycle
3) Calibration of parameters in the model
4) Simulating the model
5) Comparing „stylized facts“ of the simulated model with empirical ones.
6) Studying predicted dynamics of the model

Simple RBC model

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$$k_{t+1} = i_t + (1-\delta) k_t$$

$$n_t + l_t = 1$$

$$y_t = c_t + i_t$$

$$\psi \frac{c_t}{l_t} = (1-\alpha) e^{z_t} \left(\frac{k}{n}\right)^{\alpha}$$

$$\frac{1}{c_t} = \beta \frac{1}{c_{t-1}} E_t \left[1 + \alpha e^{z_{t+1}} \left(\frac{n_{t+1}}{k_{t+1}}\right)^{1-\alpha} - \delta \right]$$

$$y_t = e^{z_t} k_t^{\alpha} n_t^{1-\alpha}$$

$$z_t = \rho z_{t-1} + \varepsilon_t$$

$$y_t = c_t + i_t$$

$$n_t + l_t = 1$$

$$k_{t+1} = i_t + (1-\delta) k_t$$

Caveats

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

$$u(c_t, l_t) = \log c_t + \psi \log l_t$$

$$y_t = e^{z_t} k_t^{\alpha} n_t^{1-\alpha}$$

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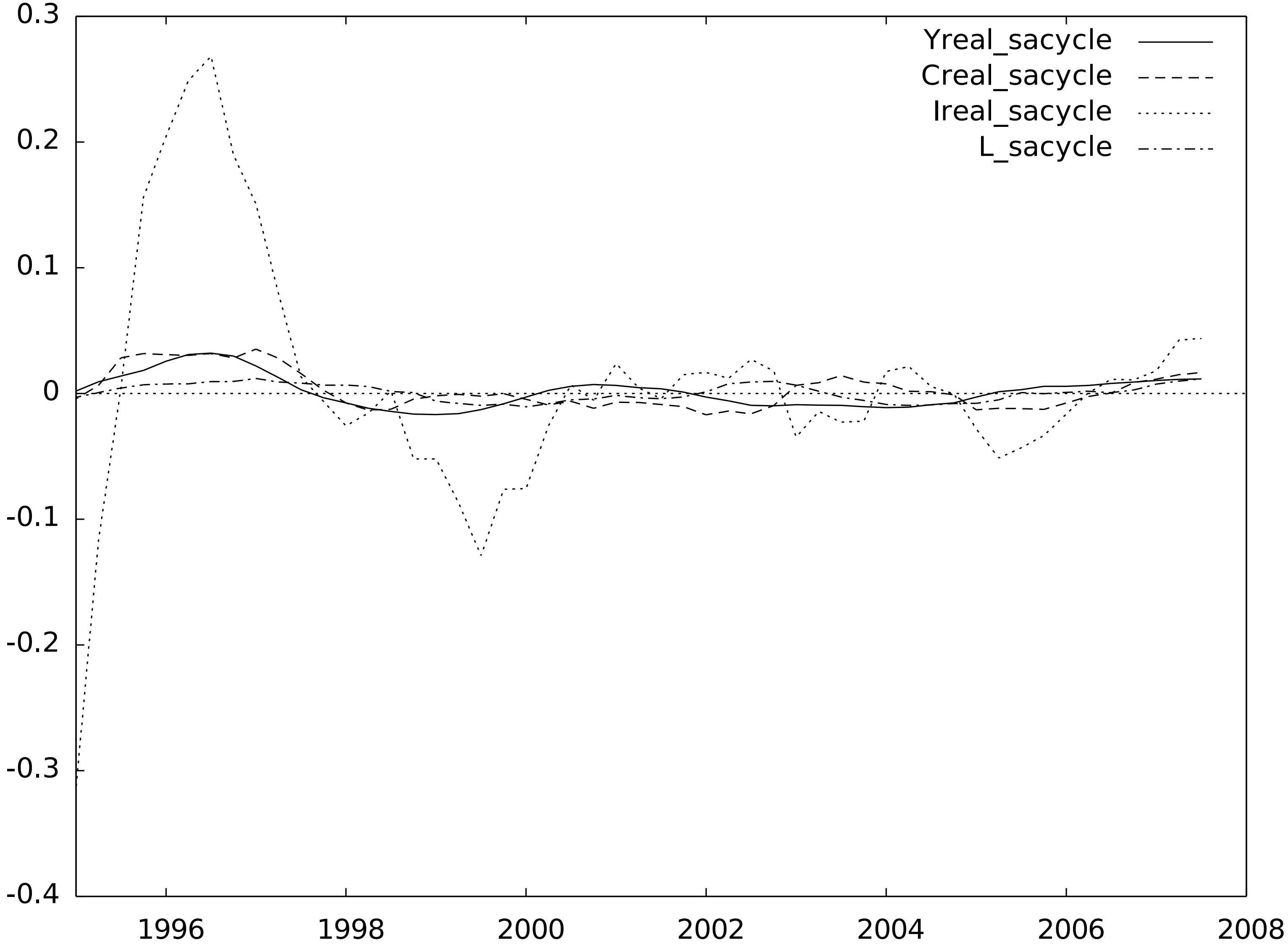
- Caveat 1: Closed vs. small open economy
  - Simplification 1: Let's adjust the income for the current account deficits (although we know that the share of exports or imports on income is around 70%)
- Caveat 2: Detrended data
  - Simplification 2: Let's assume that the shocks are temporary and do not have significant permanent effect on growth
- This is academic example, a „true model“ of the Czech economy should definitively capture foreign shocks and more sophisticated macro models do account for them, some of them introduced in the International Macro Class.

Stylized facts



Stylized facts

- Are the Czech business cycles consistent with the real business cycles hypothesis?
- (Procyclical aggregates, employment, volatility ratios...)
- Evidence: mixed results with a lot of uncertainties, but consumption and investment procyclical, labor procyclicalm but not much and investment and consumption uncorrelated.

Y	C	Ih	N	
1,0000	0,5560	0,7875	0,3665	Y
	1,0000	0,0269	0,1640	C
		1,0000	0,3102	Ih
			1,0000	N

Calibration

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

$$u(c_t, l_t) = \log c_t + \psi \log l_t$$

$$y_t = e^{z_t} k_t^{\alpha} n_t^{1-\alpha}$$

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- “Choose parameter values on the basis of microfoundations and then compare the model's predictions with the data”
- Calibration of parameters of life-time utility function:
  - the subjective discount factor β
  - the leisure share of instantaneous utility ψ.
- The production side contains the parameters of
  - the capital share on output α,
  - the depreciation rate δ and
  - the structure of the technology shocks ρ and σ.
- How find these numbers?
- Comparison with other micro studies; it is not just estimation, we can go to wide range, even cross-country studies and upon a broad set even non quantified information we can set values of parameters.

Calibration

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

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- Practically: $\beta = \frac{1}{1+r}$
- Discount factor : Usually taken as average interest rate on deposits (if economy in the equilibrium), implying 0.99
- The capital share on output α – estimated K/Y, 0.43 for the Czech economy (much higher than in developed countries)
- Depreciation rate δ set to 0.015 – corresponds to 6% per year, this is in line with micro studies estimating the depreciation rate using micro data from firms.
- …
- Similarly, leisure share of instantaneous utility ψ leads to value 1.21 (reasoning beyond the scope of the presentation) and properties of shocks into technology derived from Solow residuals.

Technology shocks – Solow residuals

- Stochastic shocks based on the Solow Residuals.

$$y_t = A_t e^{z_t} k_t^{\alpha} n_t^{1-\alpha}$$

$$\log SR_t = \log y_t - \alpha \log k_t - (1-\alpha) \log n_t$$

- Resulting SR's detrended using linear trend and then decomposed in order to find the structure of shocks following the AR(1) process:

$$\log SR_t = \log A_t + z_t$$

$$z_t = \rho z_{t-1} + \varepsilon_t$$

- Consequence: Shocks represent all shocks that affect the economy, not only *pure* technology shocks.

Solow residuals

Figure 4.1: Estimated Solow Residuals

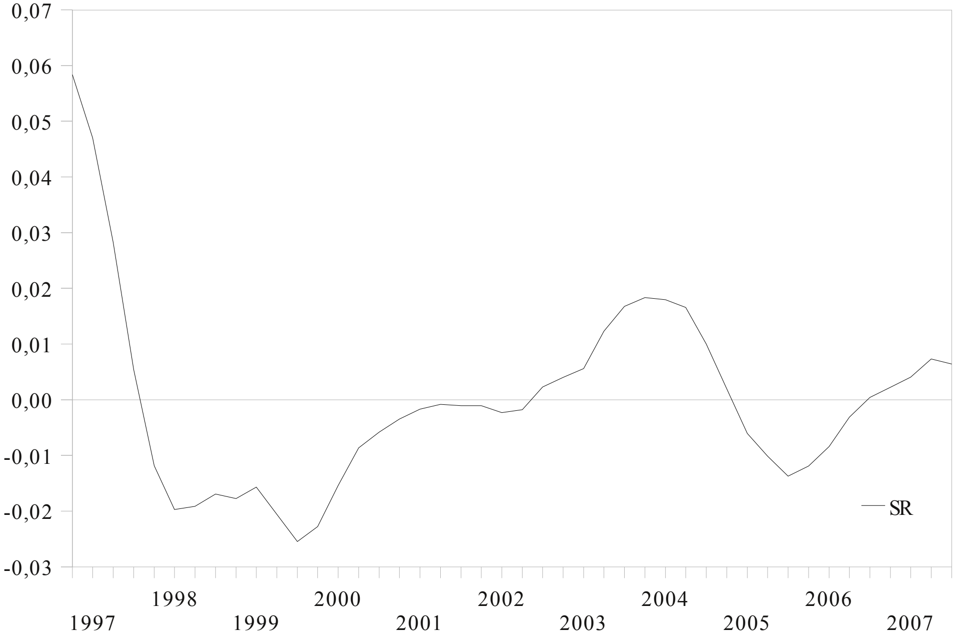
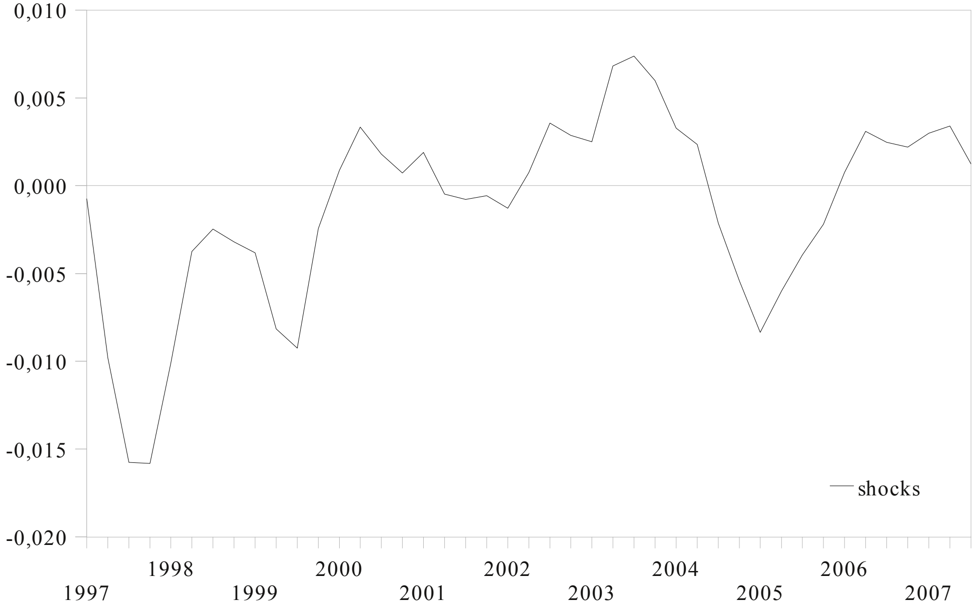


Figure 4.2: Productivity Shocks



Calibration

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

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$$k_{t+1} = i_t + (1-\delta) k_t$$

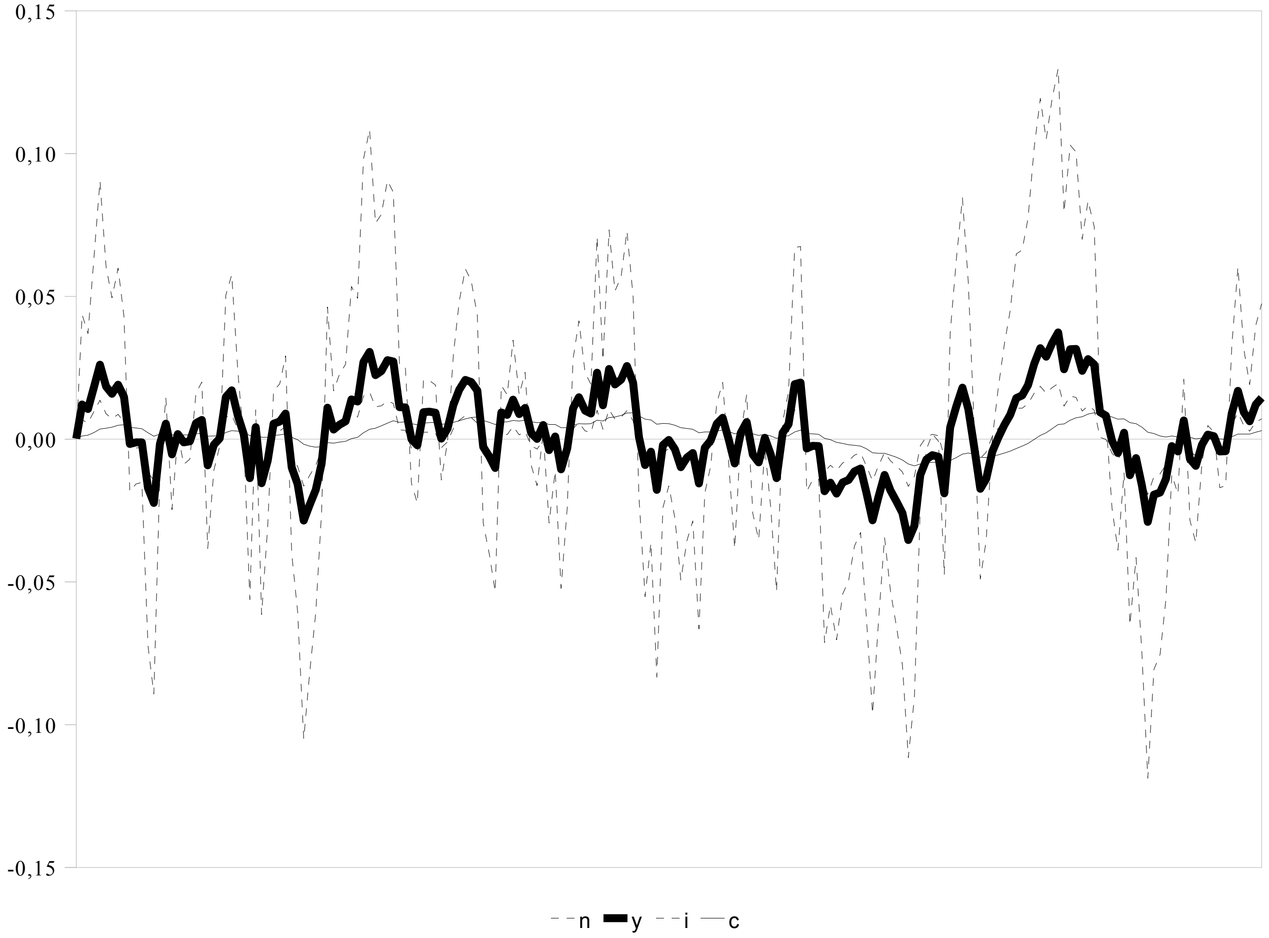
$$n_t + l_t = 1$$

$$y_t = c_t + i_t$$

- After setting all parameters let's simulate the model!
- Done in Matlab
- Evaluation of the model and its dynamics.

Plot of simulated variables

Figure 5.1: Plot of Simulated Variables



Comparing calibrated model with the data

Descriptive Statistics of Observed Variables

	mean	st. dev.	var x/var y	1st autocorr	2nd autocorr
y	0,0000	0,0143	1,0000	0,8906	0,6416
i	0,0000	0,0506	3,5348	0,6681	0,3665
c	0,0000	0,0117	0,8173	0,6436	0,4274
n	0,0000	0,0060	0,4191	0,8566	0,6324

Descriptive Statistics of Simulated Variables

	mean	st. dev.	var x/var y	1st autocorr	2nd autocorr
y	0,0000	0,0127	1,0000	0,7592	0,5921
i	0,0000	0,0436	3,4391	0,7222	0,5333
c	0,0000	0,0044	0,3500	0,9899	0,9723
n	0,0000	0,0067	0,5307	0,7159	0,5238

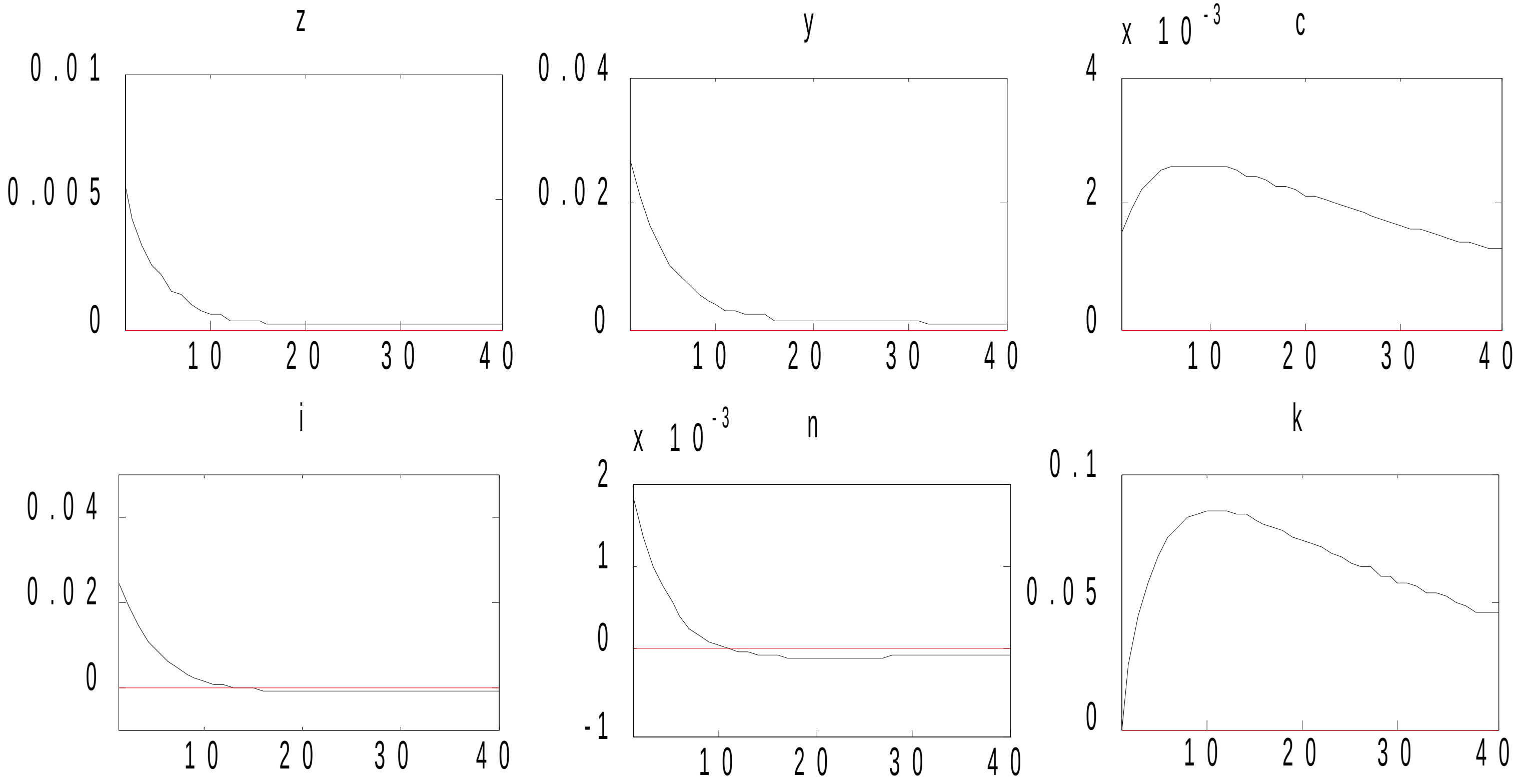
- I more volatile than Y in both, N and C less, but cons. smoothing not that strong in observed data => people react more with their consumption on changes in output than the theory would predict and reactions of employment are lower than predicted.
- Observed variables have lower persistence in all variables but Y.

Dynamics

- Another analytical tool: the impulse-response functions.
- Assume one-unit shock into one variable and simulate, what happends with the others.
- What it tells: how big the impact is, how long it lasts, positive/negative effect of shock
- Here: technology shock and effects on C, Y, I, K, L.

Impulse response functions

- Technology shock *z* and effects on C, Y, I, K, L
- Response of C and N to technology shock small, K very persistent, GDP rises and then decreases.
- Effect of propagation mechanism (N below 0 after some time) very small



Extensions and Limitations

- See the section 4.10 in Romer's textbook
- Extensions: open economy, more sectors, unemployment
- Limitations: Technology shocks so big! Are innovations really so fast? The Solow residuals are correlated with many things (changes in G, M...)
- Intertemporal substitution of labor: the elasticity of substitution in labor supply usually not significant
- No monetary sector – monetary policy, inflation...
- Policy implications: low costs of business cycles (the economy is always in optimum) and no involuntary unemployment (business cycles are times of chronical laziness... - Mankiw, 1989)

RBC's and crisis?

- Is RBC model able to replicate behavior of the main macro variables during crisis like this?
- Studies aimed at the Great Depression or Argentinian crisis show they can: because of calibration.
- So flexible framework that can mimic almost everything, however parameters uncertain and their changes contradict the philosophy of theoretically constant „deep structural parameters“ of microfoundations of the model.
- If interested, see E. Prescott's Nobel Lecture on web.

Key Points

- Understanding the stylized facts
- Cyclical behavior of variables
- Calibration vs. Estimation
- Solow residuals and their interpretation
- Impulse response functions

1. Derivation of First Order Conditions

Households maximize life time utility function

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \,, \qquad \text{A.1}$$

where

$$u(c_t, l_t) = \log c_t + \psi \log l_t \,. \qquad \text{A.2}$$

Their choice is constrained by budget constraint:

$$y_t = c_t + i_t = c_t + k_{t+1} - (1-\delta) k_t \,. \qquad \text{A.3}$$

To find an optimal path we form a Lagrangeian using A.1-A.3:

$$L = E_0 \sum_{t=0}^{T} \beta^t u(c_t, l_t) + E_0 \sum_{t=0}^{T} \lambda_t \left[e^{z_t} k_t^{\alpha} n_t^{1-\alpha} - c_t - k_{t+1} + (1-\delta) k_t \right] . \qquad \text{A.4}$$

Optimal paths are then derived from first order conditions given by:

$$\frac{\partial L}{\partial c_t} = \beta^t u'(c_t) - \lambda_t = 0 \,, \qquad \text{A.5}$$

$$\frac{\partial L}{\partial l_t} = \beta^t u'(l_t) - \lambda_t (1-\alpha) e^{z_t} \left(\frac{k_t}{n_t} \right)^{\alpha} = 0 \qquad \text{A.6}$$

$$\frac{\partial L}{\partial c_{t+1}} = \beta^{t+1} u'(c_{t+1}) - \lambda_{t+1} = 0 \quad \text{and} \qquad \text{A.7}$$

$$\frac{\partial L}{\partial k_{t+1}} = -\lambda_t + \lambda_{t+1} E_0 \left[\alpha e^{z_{t+1}} \left(\frac{n_{t+1}}{k_{t+1}} \right)^{1-\alpha} + 1 - \delta \right] = 0 \,. \qquad \text{A.8}$$

Rearranging:

$$\beta^t u'(c_t) = \lambda_t \,, \qquad \text{A.9}$$

$$\beta^{t+1} u'(c_{t+1}) = \lambda_{t+1} \,, \qquad \text{A.10}$$

$$\lambda_t = \lambda_{t+1} E_0 \left[\alpha e^{z_{t+1}} \left(\frac{n_{t+1}}{k_{t+1}} \right)^{1-\alpha} + 1 - \delta \right] . \qquad \text{A.11}$$

Now A.9 and A.11 form an identity

$$\beta^t u'(c_t) = \lambda_{t+1} E_0 \left[\alpha e^{z_{t+1}} \left(\frac{n_{t+1}}{k_{t+1}} \right)^{1-\alpha} + 1 - \delta \right] . \qquad \text{A.12}$$

Now λ_{t+1} can be substituted using A.9 and using the fact that $\beta^{t+1} = \beta^t$ holds in steady state an intertemporal optimality condition, referred as Euler equation, arises:

$$u'(c_t) = \beta u'(c_{t+1}) E_0 \left[\alpha e^{z_{t+1}} \left(\frac{n_{t+1}}{k_{t+1}} \right)^{1-\alpha} + 1 - \delta \right] . \qquad \text{A.13}$$

Resulting form A.13 is equivalent to the Euler equation 2.9 if proper functional form of utility function is substituted within A.13.

Similarly condition A.6 together with A.5 form the intratemporal optimality condition equivalent to 2.8

$$\frac{u'(l_t)}{u'(c_t)} = (1-\alpha) e^{z_t} \left(\frac{k_t}{n_t} \right)^{\alpha} . \qquad \text{A.14}$$