JEM004 Macroeconomics
IES, Fall 2010
Lecture Notes
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6 Real-Business-Cycle (RBC) theory

- shift from long term growth to short run fluctuations
- **goal** = explain the causes

6.1 Stylized facts about economic fluctuations

- Fluctuations **do not exhibit simple regular or cyclical pattern**
  - output changes vary considerably in size and duration => not outcome of combination of deterministic cycles of different lengths ; rather disturbances of various types and sizes at more or less random intervals
  - question: where they come from and how are they transmitted?
- Fluctuations are **distributed very unevenly over the components of output**
  - stable: consumption of non-durables and services, government purchases, net exports
  - unstable: consumption of durables, housing, inventories investment
- **No large asymmetries between rises and falls** in output
  (but usually long time slightly above and then short time far below mean)
- Behavior of **important macroeconomic variables**:
  - employment falls, unemployment rises
  - average workweek falls
  - output per worker (productivity) falls - as declines in employment + hours are smaller than fall in the output
  - **Okun's law:** 3 percent decline in GDP growth (relative to normal growth) produces 1 percentage point increase in the unemployment rate (current 2:1)
  - inflation - no clear pattern
  - real wage - slight fall
  - nominal and real interest rates - slight decline
  - real money stock - no clear pattern

6.2 Competing theories of fluctuations

Can fluctuations be modeled using **Walrasian model** (competitive market, no market imperfections - i.e. no externalities, asymmetric information, missing markets, etc.)?

If yes => no departure from conventional micro analysis = no problem :-)

- natural candidate = **Ramsey model**: we need to make **2** modifications (otherwise smooth convergence):
- **Modification 1** = introduction of the **source of disturbances**
  - shocks to the economy's technology (production function)
  - shocks to government purchases
- both **real** (not monetary - nominal) disturbances => Real Business Cycle models
- **Modification 2** = **endogenous labor supply**
  - hours worked enter household's utility function
- **Objections:**
  - based on technology shocks - need very high changes in short period to fit the data
  - propagation mechanism = intertemporal substitution in labor supply implies that people vary their labor supply between the periods with different productivity shock - BUT in reality low intertemporal elasticity of substitution
  - omission of monetary effects - either as a direct source or propagation mechanism of "real" effects
- extreme approach - perfectly competitive economy with random disturbances

Other extreme approach = pure **Keynesian models**:

- no optimization - aggregate relationships are "given"
- non-Walrasian features (price rigidity, imperfect competition, etc.) crucial for explaining fluctuations
- **Objections:**
  - unclear causal relations among variables

As a natural solution we have

- RBC models with non Walrasian features (nominal stickiness + real imperfections)
- Keynesian models with micro foundations (explaining price stickiness)

6.3 Basic RBC model - Assumptions:

- discreet time (t = 1,2,...)
- large number of identical households (infinitely lived) and firms
- Cobb-Douglas production function

$$Y_t = K_t^{\alpha}(A_t L_t)^{\alpha-1}, \quad 0 < \alpha < 1$$

- output can be used for three purposes

$$Y_t = C_t + I_t + G_t$$

- therefore new capital stock is determined by equation

$$K_{t+1} = K_t + I_t - \delta K_t \quad = K_t + Y_t - C_t - I_t - \delta K_t$$

- all taxes are lump sum, government budget is balanced every period
- labor and capital earn their marginal products

$$\begin{aligned} w_t &= (1-\alpha)K_t^{\alpha}(A_t L_t)^{-\alpha} A_t \quad = (1-\alpha)\Big(\frac{K_t}{A_t L_t}\Big)^{\alpha} A_t = (1-\alpha)\frac{Y_t}{L_t} \\ r_t &= \alpha\Big(\frac{A_t L_t}{K_t}\Big)^{1-\alpha} = \alpha\frac{Y_t}{K_t} \end{aligned}$$

- **lifetime utility of representative household**

$$U = \sum_{t=0}^{\infty} e^{-\rho t} u(c_t, 1 - l_t)\frac{N_t}{H}$$

  - $u(\cdot,\cdot)$ - instantaneous utility function in consumption ($c_t = C_t/N_t$) and leisure (time endowment = 1, numbers of hours worked $l_t = L_t/N_t$), log-linear

$$u_t = \ln c_t + b\ln(1 - l_t)$$

  - ρ - discount rate (exponential discounting because of log-linear utility function)
  - N - population growing at exogenous rate n

$$N_t = e^{\bar{N}+nt} \quad \Rightarrow \quad \ln N_t = \bar{N} + nt$$

  - H - number of households in the economy

- evolution of **technology** A_t - subject to random disturbances[1]

$$\begin{aligned} \ln A_t &= \bar{A}_t + gt + \tilde{A}_t \\ \tilde{A}_t &= \rho_A \tilde{A}_{t-1} + \epsilon_{A,t} \quad \rho_A \in (-1,1) \end{aligned}$$

  i.e. $\tilde{A}_t$ follows first order autoregressive process, while $\epsilon_{A,t}$'s are white noise disturbances (uncorrelated, $E[\epsilon_{A,t}] = 0$)

- evolution of **government purchases** G_t - also subject to random disturbances

$$\begin{aligned} \ln G_t &= \bar{G}_t + (n+g)t + \tilde{G}_t \\ \tilde{G}_t &= \rho_G \tilde{G}_{t-1} + \epsilon_{G,t} \quad \rho_G \in (-1,1) \end{aligned}$$

  i.e. $\tilde{G}_t$ follows first order autoregressive process, while $\epsilon_{G,t}$'s are white noise disturbances (uncorrelated, $E[\epsilon_{A,t}] = 0$)

6.4 Household optimization

6.4.1 Labor supply decision

One-period model: one household member, no initial wealth

Household's problem:

$$\begin{aligned} \max_{c,l} \quad & \ln c + b\ln(1-l) \\ s.t \quad & c = wl \quad \text{(B.C)} \end{aligned}$$

Using Lagrangian we can solve this problem

$$\begin{aligned} L &= \ln c + b\ln(1-l) + \lambda[wl - c] \\ (c:) \quad & \frac{1}{c} = \frac{1}{wl} = \lambda \\ (l:) \quad & \frac{b}{1-l} = \lambda w \end{aligned}$$

Therefore, in equilibrium l^* can be found as a solution of

$$\frac{b}{1-l} = \frac{1}{l}$$

- wage does not affect labor supply decision
- due to log-utility and zero initial wealth, income and substitution effect of change in wage cancel out

[1] In deterministic case, the evolution of A_t would be defined as $A_t = e^{\bar{A}+gt}$ or $\ln A_t = \bar{A} + gt$.

Two-period model: one household member, no initial wealth

Household's problem:

$$\max_{c_1,l_1,c_2,l_2} \quad \ln c_1 + b\ln(1-l_1) + e^{-\rho}[\ln c_2 + b\ln(1-l_2)]$$

$$s.t \quad c_1 + \frac{1}{1+r}c_2 = w_1 l_1 + \frac{1}{1+r}w_2 l_2 \quad \text{(intertemporal B.C)}$$

Using Lagrangian we can solve this problem

$$L = \ln c_1 + b\ln(1-l_1) + e^{-\rho}[\ln c_2 + b\ln(1-l_2)] + \lambda[w_1 l_1 + \frac{1}{1+r}w_2 l_2 - c_1 - \frac{1}{1+r}c_2]$$

$$(l_1 :) \qquad \frac{b}{1-l_1} = \lambda w_1$$

$$(l_2 :) \qquad \frac{e^{-\rho}b}{1-l_2} = \frac{1}{1+r}\lambda w_2$$

When we express λ from both equations, equalize and rearrange, we obtain expression

$$\frac{1-l_1}{1-l_2} = \frac{1}{e^{-\rho}(1+r)}\frac{w_2}{w_1}$$

- relative wage affects relative labor supply decision - if wage in the first period w_1 rises relative to wage in second period w_2, agent increases first-period labor supply l_1 relative to second period labor supply l_2
- due to log-utility elasticity of substitution between leisure (as well as labor) in two periods is 1
- rise in r increases the first period labor supply relative to second period labor supply - because it is more profitable to work now, save and get higher interest on savings in the next period
  - CRUCIAL for transmission of shocks in RBC model

6.4.2 Optimization under uncertainty:

Consumption:

- uncertainty in evolution of A_t and G_t implies uncertainty in the future value of r_{t+1}
- we need "new" Euler equation that considers uncertainty
- in period t, suppose household reduces its current consumption per member by small Δc and saves it for the future. Marginal utility of current consumption per member is $e^{-\rho t}\frac{1}{c_t}\frac{N_t}{H}$. This implies a current utility cost per member is

$$e^{-\rho t}\frac{\Delta c_t}{c_t}\frac{N_t}{H}$$

- in period $t+1$, this translates into additional income of $e^{-n}(1+r_{t+1})\Delta c$ (as there will be e^n times more members of the household at period $t+1$ than in period t). Marginal utility of consumption in the period $t+1$ is $e^{-\rho(t+1)}\frac{1}{c_{t+1}}\frac{N_{t+1}}{H}$. Therefore, the expected utility gain per member of household, as perceived in period t is

$$E_t\Big[e^{-\rho(t+1)}\frac{1}{c_{t+1}}\frac{N_{t+1}}{H}e^{-n}(1+r_{t+1})\Delta c\Big]$$

- Optimal behavior of the household implies, that for small Δc expected utility gains must be equal to current utility loss

$$e^{-\rho t}\frac{\Delta c_t}{c_t}\frac{N_t}{H} = E_t\Big[e^{-\rho(t+1)}\frac{1}{c_{t+1}}\frac{N_t}{H}e^{-n}(1+r_{t+1})\Delta c\Big]$$

- Since $e^{-\rho(t+1)}\frac{N_t}{H}e^{-n}$ is not uncertain, and since we can write $N_{t+1} = N_t e^n$ we can simplify it into following Euler equation

$$\frac{1}{c_t} = e^{-\rho}E_t\Big[\frac{1}{c_{t+1}}(1+r_{t+1})\Big]$$

Labor supply

- by the same token we can derive equation that describes the intratemporal choice of labor input - assume that households increases labor supply by Δl in the current period and use the resulting income to increase its consumption in the same period => NO INTERTEMPORAL EFFECTS = NO UNCERTAINTY

- equating costs and benefits of such decision gives us equation that relates choice of leisure and consumption

$$\begin{aligned} e^{-\rho t}\frac{N_t}{H}\frac{b}{1-l_t}\Delta l &= e^{-\rho t}\frac{N_t}{H}\frac{1}{c_t}w_t\Delta l \\ \frac{c_t}{1-l_t} &= \frac{w_t}{b} \end{aligned}$$

6.5 A special case of the model

- model cannot be solved analytically (due to the mixture of linear and log-linear components)

- 2 changes to the model:
  - eliminate government purchases
  - 100 % depreciation (no capital accumulation)

$$\begin{aligned} K_{t+1} &= Y_t - C_t = s_t Y_t \\ r_t + \delta &= r_t + 1 = \alpha\Big(\frac{A_t L_t}{K_t}\Big)^{1-\alpha} = \alpha\frac{Y_t}{K_t} \end{aligned}$$

6.5.1 Solving the model

- competitive markets, no externalities, finite number of individuals => Pareto optimality of the competitive equilibrium
- rewrite everything in log-linear form, focus on two variables:
  - labor supply per person - l
  - fraction of output being saved - s (therefore we rewrite $C_t = (1-s_t)Y_t; c_t = (1-s_t)\frac{Y_t}{N_t}$)

$$\begin{aligned}
\frac{1}{c_t} &= e^{-\rho}E_t\Big[\frac{1}{c_{t+1}}(1+r_{t+1})\Big] \\
\ln\left(\frac{1}{c_t}\right) &= -\rho + \ln E_t\Big[\frac{1}{c_{t+1}}(1+r_{t+1})\Big] \\
-\ln\Big[(1-s_t)\frac{Y_t}{N_t}\Big] &= -\rho + \ln E_t\Big[\frac{1+r_{t+1}}{(1-s_{t+1})\frac{Y_{t+1}}{N_{t+1}}}\Big] \\
&= -\rho + \ln E_t\Big[\frac{\alpha\frac{Y_{t+1}}{s_tY_t}}{(1-s_{t+1})\frac{Y_{t+1}}{N_{t+1}}}\Big] = -\rho + \ln E_t\Big[\frac{\alpha N_{t+1}}{s_t(1-s_{t+1})Y_t}\Big] \\
-\ln(1-s_t) - \ln\left(\frac{Y_t}{N_t}\right) &= -\rho + \ln\alpha - \ln\left(\frac{Y_t}{e^n N_t}\right) - \ln s_t + \ln E_t\Big[\frac{1}{1-s_{t+1}}\Big] \\
\ln s_t - \ln(1-s_t) &= -\rho + n + \ln\alpha + \ln E_t\Big[\frac{1}{1-s_{t+1}}\Big]
\end{aligned}$$

- there is a **constant saving rate** $s = \hat{s}$ that satisfies this equation. Then there is no uncertainty about future $\hat{s}$ and we can rewrite the equation as

$$\begin{aligned}
\ln\hat{s} &= \ln\alpha + n - \rho \\
\hat{s} &= \alpha\, e^{n-\rho}
\end{aligned}$$

- let us now solve for labor supply l_t

$$\begin{aligned}
\frac{c_t}{1-l_t} &= \frac{w_t}{b} \\
\frac{(1-\hat{s})\frac{Y_t}{N_t}}{1-l_t} &= \frac{(1-\alpha)\frac{Y_t}{l_tN_t}}{b} \\
\ln(1-\hat{s}) - \ln(1-l_t) &= \ln(1-\alpha) - \ln l_t - \ln b \\
l_t &= \frac{1-\alpha}{(1-\alpha)+b(1-\hat{s})} \equiv \hat{l}
\end{aligned}$$

- thus, **labor supply is also constant** over time. Why?

- households are willing to optimize intertemporally
- with simplified set-up, movements in technology or capital have offsetting impacts on the real wage and interest rate effects on labor supply
- Ex.: better technology raises current wage and leads to higher labor supply. But by raising amount saved it decreases expected interest rate which reduces labor supply
- in this particular set-up, two effects are in balance

6.5.2 Discussion

- real shocks drive economic fluctuations
- fluctuations are optimal responses to these shocks (and do not reflect market failures) - i.e. government intervention would lead to lower welfare
- observed aggregate output movements represent the time-varying Pareto optimum

$$\begin{aligned}
Y_t &= K_t^{\alpha}(A_t L_t)^{1-\alpha} \\
\ln Y_t &= \alpha \ln K_t + (1-\alpha)(\ln A_t + \ln L_t) \quad /K_t = \hat{s} Y_{t-1};\ L_t = \hat{l} N_t \\
\ln Y_t &= \alpha \ln \hat{s} + \alpha \ln Y_{t-1} + (1-\alpha)(\ln A_t + \ln \hat{l} + \ln N_t) \\
&= \alpha \ln \hat{s} + \alpha \ln Y_{t-1} + (1-\alpha)(\bar{A} + gt) + (1-\alpha)\tilde{A}_t + (1-\alpha)(\ln \hat{l} + \bar{N} + nt)
\end{aligned}$$

Let us now assume situation without shocks, i.e. when $\ln A_t = \bar{A} + gt$ and denote the value of output in this situation as $\{Y_t^*\}_{t=1}^{\infty}$. In this situation we can rewrite previous equation as

$$\ln Y_t^* = \alpha \ln \hat{s} + \alpha \ln Y_{t-1}^* + (1-\alpha)(\bar{A} + gt) + (1-\alpha)(\ln \hat{l} + \bar{N} + nt)$$

If we subtract this from the previous one, we obtain equation

$$\ln \frac{Y_t}{Y_t^*} = \alpha \ln \frac{Y_{t-1}}{Y_{t-1}^*} + (1-\alpha)\tilde{A}_t$$

Note that we can rewrite this equation in terms of deviations of output from the deterministic steady state, as

$$\ln \frac{Y_t}{Y_t^*} = \ln \frac{Y_t^* + \Delta Y_t}{Y_t^*} \simeq \Delta Y_t \equiv \tilde{Y}_t$$

new equation being

$$\tilde{Y}_t = \alpha \tilde{Y}_{t-1} + (1-\alpha)\tilde{A}_t$$

This holds in every time period, thus we can express one period lagged technological progress as combination of past values of output

$$\tilde{Y}_{t-1} = \alpha\tilde{Y}_{t-2} + (1-\alpha)\tilde{A}_{t-1} \quad \Rightarrow \quad \tilde{A}_{t-1} = \frac{1}{1-\alpha}(\tilde{Y}_{t-1} - \alpha\tilde{Y}_{t-2})$$

If we substitute this into the main equation, together with the assumption on the evolution of $\tilde{A}_t$ we obtain

$$\begin{aligned}\tilde{Y}_t &= \alpha\tilde{Y}_{t-1} + (1-\alpha)(\rho_A\tilde{A}_{t-1} + \epsilon_{A,t}) = \alpha\tilde{Y}_{t-1} + \rho_A(\tilde{Y}_{t-1} - \alpha\tilde{Y}_{t-2}) + (1-\alpha)\epsilon_{A,t} \\ &= \underbrace{(\alpha+\rho_A)}_{>0}\tilde{Y}_{t-1} \underbrace{-\alpha\rho_A}_{<0}\tilde{Y}_{t-2} + (1-\alpha)\epsilon_{A,t}\end{aligned}$$

- log output deviations from balanced path follow second order AR process - linear combination of previous two values + white noise disturbance
- possible hump shaped response of output - consistent with reality
- dynamics od output is largely determined by ρ_A
- no mechanism to translate temporary shocks into long-lasting output movements

Empirical relevance:

- does no really match facts about fluctuations
  - constant saving rate - consumption and investment are equally volatile (FACT - investment varies more than consumption)
  - labor supply is constant (FACT - employment and hours worked are cyclical)
  - highly procyclical wage - moves with Y_t (FACT: moderately procyclical wage)
- very simplified model - in real macro analysis, need to look at general model, or models with non-Walrasian features
- RBC models are usually studied in terms of analyzing the deviations from steady state - method = log-linearisation around steady state